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REINFORCED CONCRETE DESIGNERS' HANDBOOK

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REINFORCED CONCRETE DESIGNERS' HANDBOOK

BY

CHAS. E. REYNOLDS
B.Sc. (Eng.), Assoc.M.Inst.C.E.
A.Am.C.I.

A Reference Book giving the Essential Data and Method of Designing any type of Reinforced Concrete Structure

THIRD EDITION

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A list of other practical and useful books on concrete and reinforced concrete design and construction, pre-cast concrete, cement, and allied subjects is given on page 321.

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PREFACE

THE principal object of this volume is to provide a Handbook that will give the average reinforced concrete designing engineer the data that he may normally require in the ordinary course of his duties. The book is divided into three principal Parts, the second of which comprises a series of tables that endeavour to present in a concise form, and in accordance with modern practice, the necessary information and formulæ. The first Part gives explanatory notes upon, and subject matter additional to, that given in the tables, and thus by not interspersing the text among the tables the latter are supplemented without sacrificing completeness or compactness.

The text deals with the factors of design in the following sequence : (*a*) loads and pressures ; (*b*) calculations of moments and forces ; (*c*) determination of stresses and design of sections. There is no pretence of dealing with the wide subject of the construction of reinforced concrete structures except in so far as the work on the site affects that of the engineer at the drawing board, and in the endeavour to avoid duplicating much material that is fully and admirably dealt with elsewhere the mathematical processes by which the various formulæ are derived have been omitted. The third Part takes the form of a descriptive bibliography to which the reader is referred for fuller information on those parts of the subject that are briefly dealt with in this volume.

With this as his aim, the author feels that no apology is needed for adding this volume to the literature of reinforced concrete design. Although it is hoped that the presentation of some aspects of the subject may strike a new note, information has had to be obtained from many sources, and acknowledgment must be made to those engineers who are mentioned in the text.

The author's thanks are also due to Mr. R. A. Manthei, A.C.G.I., D.I.C., and Mr. R. Hicks for their assistance in preparing the work for publication.

Although this volume is primarily addressed to concrete designers, it is hoped that it may be of some value to those engineers whose practice only occasionally brings them in touch with reinforced concrete, and that even the expert may find the tables interesting and of assistance.

C. E. R.

STAFFORD, 1932.

PREFACE TO SECOND EDITION

A NEW edition of this book is now necessary to include the requirements of the London County Council's Building By-laws issued in 1938, the Recommendations for a Code of Practice for the Use of Reinforced Concrete in Buildings, and other Codes and technical advances since the publication of the first edition. The inclusion of this new data has necessitated the complete rewriting of many of the chapters and tables, while many additions have been made to other parts of the original text. For example, the new material includes pressures due to materials submerged in liquids, bending moments and shear forces due to triangular loading, torsion, temperature stresses, etc. Additional examples, more details of specification clauses, and particulars of alternative methods of measuring quantities have been inserted, together with the requirements of the " Code of Practice for Reinforced Concrete Structures for the Storage of Liquids " issued by the Institution of Civil Engineers, whose permission is gratefully acknowledged. The bibliography included in the first edition is now omitted.

The opportunity is taken to thank all those who have suggested corrections or amendments, many of which have been incorporated in this edition.

C. E. R.

LONDON, 1939.

PREFACE TO THIRD EDITION

THE need for a further reprinting has presented an opportunity to make additions to the subject matter, especially in respect to the analysis of framed structures. Consideration of moment distribution has been introduced, together with additional data relating to the design of bridges and impermeable construction.

During the war certain relaxations of stresses have been allowed; for example, the permissible working stress in tension on mild steel bars has been increased from 18,000 to 20,000 lb. per square inch. Such temporary expedients are not included in this edition. Neither has any attempt been made to anticipate post-war revisions to stresses, superimposed floor loads, or wind pressures, which in this edition remain in accordance with the regulations and recommendations current prior to the war. The text dealing with cement and reinforcement has, however, been brought into line with certain recent British Standards.

C. E. R.

LONDON, 1945.

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PART I

DESCRIPTIVE TEXT

CHAPTER I

INTRODUCTION

1.—Design.

UNLIKE other structural designers, who usually have only one material to consider, the reinforced concrete designer has to think almost simultaneously of steel, cement, sand, stone, water, and timber, which have to be combined to fulfil his requirements efficiently. When he makes his calculations, instead of adopting elementary mechanics, he is assailed by the apparently abstruse mathematics of the elastic theory. The concrete engineer realises that his detail design is as important as his main planning. Efficiency and economy—related but not identical—must always be uppermost in his mind. Efficient detailing, by which is meant much more than providing in an economical manner the computed area of reinforcement, is of paramount importance, implying as it does that the calculated amount of steel can be placed practically so that it can do the work it will be called upon to perform, and that reinforcement is provided to take those many incalculable secondary tensile forces that are inherent in a monolithic concrete structure. Towards the production of safe and economical structures good judgment will do almost as much as calculations and especially in the matter of the assessment of loading can the designer display this judgment. It is of little use to make calculations with the effective depth of a section carried to two decimal places, if the loads are 25 per cent. under- or over-estimated. It should be remembered in connection with estimates of loadings, costs, and other quantities that the more items that can be included at their exact value, the smaller will be the overall percentage of error due to the inclusion of those items whose exact magnitude is indeterminate.

Competent designing must be associated with efficient supervision on the site if, in the interests of his profession and his client, the reinforced concrete engineer is to take full advantage of the latest research results and refinements in calculation and of the high uniform quality of modern British cements and steel. The practical side of concrete engineering is not dealt with in the present volume, but an attempt has been made to bridge the gap between the drawing office and the site in the author's companion volume " Concrete Construction."

The tread of modern design is such that, where proper precautions are taken to ensure that the assumed loading and specified quality of concrete are realised, high working stresses can be employed, and the more factors allowed for in the calculations the higher the working stresses may be, and vice versa. At the same time complicated and pseudo-exact mathematics should not be allowed to confuse the sense of what is good engineering. If low stresses of the order of 16,000 lb. and 600 lb. per square inch on the reinforcement and concrete

respectively are adopted, then approximate loadings, bending-moment coefficients and resistance-moment factors are undoubtedly sufficient. The object of technical advances should be to introduce refinements and simplifications into the methods of calculation that the engineer makes to aid his judgment. The refinements should render the stress survey more comprehensive and therefore more satisfactory. Much practical design is to-day controlled by regulations, but even within the bounds of such regulations, the concrete engineer must still exercise considerable judgment in his interpretations of the requirements, endeavouring to grasp the spirit of the regulations rather than to design to the minimum allowed by the letter of the by-laws. Design practice in this country is to-day largely influenced by two recent codes, the "Recommendation for a Code of Practice for Reinforced Concrete" and the "London Building By-laws" (1938) together with the Memorandum issued in connection with the latter. Design in compliance with the Code and the By-laws is dealt with fully in the author's "Practical Examples of Reinforced Concrete Design." Although the appropriate recommendations and requirements are given in the present volume, the greater part of the book deals with the many problems and structures which are outside the scope of these documents.

The tables given in the second part of the present volume have been planned to enable the designer to reduce the amount of mathematical processes to a minimum. The use of such tables not only makes for speed but also eliminates inaccuracies when the tables being used are thoroughly understood and their basis and limitations realised. In the succeeding chapters the basis of the tabulated material is described. A certain amount of general information is also tabulated; for example on *Table* No. 38 are given fractions of an inch expressed in decimals of a foot. *Table* No. 37 is a conversion table for metric units, whilst *Table* No. 40 gives the trigonometrical functions of angles to a degree sufficient for design purposes. In conjunction with this table, fundamental trigonometrical formulæ are given.

The duties of the reinforced concrete designer are considerably varied, extending from the initial stages of site inspection and general planning of a structure through the intermediate stages involving the preparation of the detail designs, cost estimates, bills of quantities, and specifications to the final stage of inspection of works in progress, measuring-up, and amendments to designs due to unforeseen circumstances arising during the course of the work. In the performance of these duties an engineer is continually using, and at the same time augmenting, his store of knowledge and experience of structural materials and design, as well as of construction methods and costs. A competent reinforced concrete engineer is one who blends sufficient theory with practice. At one extreme is the mathematician who can calculate precisely the stresses and strains produced by an assumed system of loads, and who is of considerable value when his theoretical knowledge is coupled with experience of the limitations of the materials he is hypothetically handling. At the other end of the scale is the essentially practical engineer who possesses little advanced mathematical knowledge but knows by experience what can be done with a given material, and who can skilfully peg-out an intricate setting-out, make good concrete, use modern labour-saving contractor's plant efficiently, organise the work, and economically cause the mathematician's dreams to materialise.

The engineer between these extremes must, at his best, have sufficient technical knowledge to control and interpret the mathematician's conceptions and to translate them into projects that the contractor can erect ; he must know where he can safely sacrifice theoretical accuracy for economy of construction ; he must be able to anticipate probable loadings and conditions other than those specified by the client ; he must be able to comprehend the requirements of his non-technical clients and to draw up a practical specification that will defend his client against unscrupulous contractors without being unfair to conscientious builders. He must know sufficient of the practical man's ways and of the materials' market to estimate with precision the probable cost of a project, and he must have an acquaintance with the business side of engineering.

It is reasonable that, for the reinforced concrete engineer who intends making designing his principal business, in his blending of practical mathematics and field experience his study of theory should predominate, and an endeavour should be made to proceed along both roads at once in order that in an intensive study of one side the importance of the other may not be obscured. His studies should obviously include the theory of structures, strength of materials, and such other subjects as are usually included in professional examinations. These, however, can be valuably supplemented by the less abstract studies of building construction, geology, and the elements of architecture. A knowledge of languages is useful, since much that is of value in the literature of concrete engineering is often presented in German or French, and on the purely practical side, clear draughtsmanship is as important as in any other branch of engineering. An insight into such occupations as tracing and the keeping of office and site records is useful to an engineer, whatever technical diplomas he may possess. Outdoor studies should embrace surveying and familiarity with such details as timber sizes, the amount of concrete that can be made and placed in a given time, and the difficulties experienced in the bending and placing reinforcement.

2.—Design Economics.

There are few criteria of economy in reinforced concrete design, since besides the obvious effect of the relative cost prices of concrete, steel, and timber on the economic proportions of the quantity of concrete, reinforcement, and shuttering, there are possibly other factors in the designer's mind. For example, a designer engaged by a contracting-designing organisation may have to bear in mind that a particular piece of special plant, such as steel shuttering of certain sizes, will be available for use on a particular contract ; it is therefore economical to arrange the structural members so that this plant can be employed ; or, again, if a rise, say, in joiners' wage rates is expected, then economy may accrue by omitting complex shuttering at the expense of concrete.

Such considerations as whether less concrete of a rich mix is cheaper than a greater volume of a leaner mix ; whether the use of higher-priced bars of over-long lengths will offset the cost of the extra weight used in lapping cheaper bars of normal length ; whether, consistent with efficient detailing, a few bars of large diameter can replace a larger number of bars of smaller diameter ; whether the extra cost of rapid-hardening cement justifies the saving made by a greater number of uses of the shuttering ; or whether uniformity in the sizes of members

saves in shuttering what it may cost in extra concrete—these are a few among the many factors that a designer may have to consider in detail.

Then there is the wider aspect of the economical problem, such as whether the anticipated life and use of a proposed structure warrants the use of a higher factor of safety, or whether the extra cost of adopting an expensive type of construction will be commensurate with the improvement in internal facilities.

The wording of a contract and the experience of the contractor, the position of the site and the nature of the available materials, and even the method of measuring up the quantities, together with numerous other points, all have their effect, consciously or not, on the designer's attitude towards a contract. So many and varied are the collateral factors to consider that only experience and the study of the trend of design can give the designer any reliable guide. Attempts at determining the most economical proportions for a given member, basing that determination only on inclusive prices for concrete, steel, and shuttering, generally lead to dimensions that are usually unconventional. It is nevertheless possible to lay down certain broad principles that, coupled with the observations made in subsequent chapters, should be of some guidance.

For equal weights combined material and labour costs on small diameter bars are greater than those on large diameter bars, and within wide limits long bars are cheaper than short bars if there is a sufficient tonnage to justify special transport charges and facilities.

The lower the cement content the cheaper the concrete, but, other factors being equal, the lower the strength. Taking compressive strength and cost into account a rich mix is more efficient than a leaner mix, and what is usually known as a 1 : 1 : 2 mix may be 20 per cent. more efficient than a 1 : 2 : 4 mix. In beams and slabs, however, where much of the concrete is in tension and therefore neglected in design calculations, a lean mix is cheaper than a rich mix. In columns, where all the concrete is working in compression, the adoption of a rich mix leads to economy, since besides the more efficient concrete there is the saving in shuttering consequent upon the reduction in size of the member. The relative economy of various column designs is discussed in Chapter XIII.

The use of compression steel is always uneconomical when the cost of a single member is being considered, but advantages accruing therefrom by increasing headroom under beams or by reducing the size of columns may offset the extra cost of the individual member.

Bent-up bars are more economical than binders for resisting shear in beams.

Tee beams are cheaper if the rib is made as deep as possible; here again the increase in headroom consequent on reducing the depth may offset the small extra cost of the beam. An increase in overall depth of 33⅓ per cent. may result in a 20 per cent. decrease in cost.

Shuttering is obviously cheaper if all angles are right angles, if surfaces are plane, and if there is a fair amount of repetition. Thus splays, fillets, chamfers, etc., should be dispensed with unless essential to durability, and wherever possible architectural features in cast-in-situ work should be formed in straight lines with right-angle breaks. When shuttering costs are considered in conjunction with concrete and steel costs, the introduction of certain complications in the shuttering may lead to more economical construction. For instance, heavy continuous beams are more economical if haunched at the

supports, and circular tanks of medium capacity are cheaper than rectangular tanks of the same capacity. In some cases domed roofs and tank bottoms are more economical than flat beam and slab construction, although the shuttering may cost 100 per cent. more for spherical work. When shuttering can be used several times without alteration the advisability of employing steel shuttering should be considered, and, owing to the less adaptability of steel as compared with timber, the design should be modified if necessary to suit. Generally steel shuttering for beam-and-slab or column construction is only cheaper than timber shuttering if twenty or more uses can be assured, but for circular work half this number of uses may warrant the adoption of steel. Timber shuttering for slabs, walls, beams, column sides, etc., is usually good for four uses before repair, and six to eight times before the cost of repair equals the cost of remaking. Beam-bottom boards can be used at least twice this number of times.

A very essential aspect of the reinforced concrete engineer's work is an appreciation of the economical qualities of materials other than concrete. The judicious incorporation of such materials into his designs may lead to economies on a large scale, and because his structure is primarily built in his specialised material he should not be insensible to the applicability of other building materials in appropriate parts of his structure. Just as there is little excuse structurally for facing reinforced concrete bridges with stone, so there is no economic gain by constructing 4-in. reinforced concrete panel walls when a 4½-in. or 9-in. brick wall will serve the same purpose.

Other common cases of the consideration of the claims of other materials are that of asphalt road surfaces on concrete foundations compared with all-concrete roads, the installation of timber or steel bunkers in place of concrete structures when short life only is required, the erection of light steel framing for the superstructures of industrial buildings, the provision of pitched steel trusses with suitable roof coverings, compared with concrete roof slab and beam construction, suspension cables as against long-span concrete arches, masonry blocks as against concrete monoliths in marine work, and timber piles compared with reinforced concrete piles. Included in such economic comparisons should be such factors as fire-proofing, deterioration, depreciation, insurance, appearance, and constructional time, besides such structural considerations as foundations, convenience of construction, and availability of materials.

3.—Drawings.

The methods adopted in preparing drawings for reinforced concrete work vary considerably, and in most drawing offices a special practice has usually been developed to suit the particular class of work done. The following observations are based on experience and can be taken as a general guide when no precedent exists. One of the principal factors is to ensure that on all drawings for any one contract the same conventions should be adopted and that uniformity of appearance and size should be aimed at, thereby making easier the work of reading the drawings on the site.

In the preliminary stages a general drawing of the whole structure is usually prepared to show the principal sizes of beams, columns, slabs, walls, foundations, capacity of tanks, etc. Later this, or a similar drawing, is elaborated as a key

to the working drawings, and should then show precisely such particulars as the setting-out of the structure in relation to adjacent permanent buildings, etc., and the level of, say, the ground floor in relation to a predetermined datum. All principal dimensions such as the distance between columns, overall and intermediate heights of stories, etc., should be indicated in addition to any special requirements such as clearances, exceptional loads, etc., that are to be attended to in the design.

The most convenient scale for general drawings is usually $\frac{1}{8}$ in. to 1 ft. It is often of great assistance if the general drawing can be used as a key to the working detailed drawings by incorporating reference marks for each column, beam, slab panel, or other unit.

The working drawings should be large-scale details of the members shown on the general drawing. A suitable scale is $\frac{1}{2}$ in. to 1 ft., although plans of slabs and elevations of walls are often conveniently prepared to a scale of $\frac{1}{4}$ in. to 1 ft., while sections through beams and columns with complicated reinforcement are preferably drawn to a scale of 1 in. to 1 ft.

Separate sections and other views should be shown for reinforcement details of slabs, beams, and columns, since it is not advisable to show the reinforcement for more than one such member in a single view, except in the case of framed structures. An indication should be given, however, to show the position of slab steel and column bars relative to beam bars or other intersecting reinforcement. Sections through beams and columns showing the detailed arrangement of the steel should be placed as closely as possible to the position where the section is taken.

For reinforced concrete details it is best to indicate the outline of the concrete by a bold line and the reinforcement by a thinner full line representing the centre line of the bar, and wherever clearness is not otherwise sacrificed, the line representing the bar should be placed in the exact position intended for the centre line of the bar, due regard being given to the appropriate cover. Thus the reinforcement as shown on the drawing should represent as nearly as possible the appearance of the reinforcement as fixed on the site, all hooks, etc., being drawn to scale.

The dimensions given on the drawing should be planned so that the primary dimensions connect column and beam centres or other leading setting-out lines and so that secondary dimensions give the detailed sizes with reference to the main setting-out lines. The dimensions on working drawings should also be given in such a way that the carpenters making the shuttering have as little arithmetic as possible to do. Thus, generally, the distances between breaks in any surface should be figured. Disjointed dimensions should be avoided by combining as much information as possible in a single string of dimensions.

Marks indicating where cross sections are taken should be bold and, unless other considerations such as appropriateness control, the sections should be drawn as viewed in the same two directions throughout the drawing ; for example, they may be drawn as viewed looking towards the left and as viewed looking from the bottom of the drawing. Consistency in this makes it easier to understand complicated details.

Any notes on general or detailed drawings should be concise and free from superfluity in wording or meaning. Notes which apply to all working drawings

can be reasonably given on the general arrangement with a reference to the latter on the detailed drawings. Descriptions of concrete mixes, cover, etc., are usually included in the specification or bill of quantities, and only special mixes or covers peculiar to parts of the work shown on the detailed drawings should be described on these. There should, however, be a general reference to the specification on the general drawing If the bar-bending schedule does not appear on the detailed drawing, a reference should be made to the page numbers of the bending list relating to details on the particular drawing.

Notes that apply to one view or detail only should be placed as closely as possible to the appropriate view, and only those notes that apply to the drawing as a whole should be collected together. If a group of notes becomes lengthy there is a danger that they will be only cursorily read and an important requirement thereby overlooked.

CHAPTER II

LOADS

1.—Dead Loads.

THE judicious assessment of the loading on a complete structure or on any particular member is as important in the production of an efficient design as the general planning or the determination of the dimensions of the members. Loading can be divided into (1) dead or permanent loads and (2) superimposed or live loads.

Weight of Concrete.—The primary dead load is usually the weight of the reinforced concrete structure or member itself, which for design purposes is conveniently taken as 144 lb. per cubic foot, that is 1 lb. per lineal foot for each square inch of cross-sectional area. Actually the weight of reinforced concrete is rarely less than 150 lb. per cubic foot and varies with the weight of aggregate and percentage of reinforcement. Some typical weights for plain and reinforced concrete are given on *Table* No. 1, together with the design weights of solid concrete slabs and hollow clay tile floors.

Structural Materials.—Other dead loads include such permanent weights as those of wall finishes; floor, stair, ceiling and roof finishes; asphalt and other waterproofing; partitions; doors, windows, roof lights, pavement lights, etc.; superstructures of concrete, steelwork, masonry, brickwork, and timber; concrete bases for machinery and tanks; fillings of earth, sand, puddled clay, plain concrete, hardcore, cork, etc.; rail tracks; firebricks and other linings; road surfaces, etc. Suitable basic values of structural materials are given on *Table* No. 1, together with the weights of glazed, sheeted and slated roofs and the average equivalent loads per square foot of horizontal projection for steel trusses of various spans. These values are applicable when estimating the loads imposed on a concrete substructure with a pitched steel roof. For structural steelwork made up of joists, channels, angles, or compound sections, it is generally sufficient if the total weight is estimated by adding to the nominal weight of the section an allowance of 10 per cent. for cleats, connections, rivets, and bolts, although an extra allowance should be made for stanchion caps, bases, and grillages.

Loads on Lintels.—Where concrete lintels support brick walls it is not necessary to consider the lintel as carrying the whole wall above it; it is sufficient to allow only for the triangular areas indicated on the diagrams on *Table* No. 1.

Partitions.—The weights of partitions should be included in the dead loads on floors and it is convenient to consider such weight as an equivalent uniformly distributed load. The London By-laws specify a minimum allowance for partitions of 20 lb. per square foot for office floors, but usually this value will

only allow for timber or glazed partitions. The material of which the partition is constructed and the story height will control the weight of the partition, and for the design of floor slabs in any given case the equivalent distributed dead load (w_p lb. per square foot) can be calculated fairly accurately from the following values:

(*a*) When the partition is at right-angles to the direction of the span of the slab and is placed approximately in the centre of the span:

$$w_p = 2p$$

$$\text{where } p = \frac{\text{weight of partition in lb. per foot run}}{\text{span of slab (ft.)}}$$

(*b*) When the partition is in the direction of the span of the slab:

$$w_p = \frac{\text{weight of partition in lb. per foot run}}{2}$$

In the case of brick or similarly bonded partitions some relief of loading on the slab can occur due to the arching effect of the partition if it is continuous over two or more beams, but the presence of doorways or other openings will destroy this relieving action. For the design of beams, the uniformly distributed load due to partitions can be considered as the proportion of the total weight of partitions carried by the beam under consideration suitably adjusted to allow for non-uniform incidence. This value should not, however, be less than 20 lb. per square foot of floor area.

2.—Superimposed Loads on Buildings.

Live or superimposed loadings include any external loads that may be imposed when the structure is serving its normal purpose, and include stored materials, furniture and movable equipment, cranes, vehicles, snow, wind, and people. The actual value of these loads depends on the requirements of each part of the structure.

Floor Loads.—For buildings in most cities, the superimposed loads to be taken by floors, staircases, and roofs are laid down in regulations, and the values given on *Table* No. 2 are based on the London By-laws (1938) and the Code of Practice. For floors and roofs with low loading intensities, it is usual to design the beams for a smaller superimposed load than the corresponding slabs, since the likelihood of the whole floor being simultaneously loaded is remote. It is common practice to design slabs and beams for a minimum total superimposed load irrespective of the span; the values of these loads are given on *Table* No. 2 together with the limiting span for slabs below which the minimum total superimposed load will control the design of the slab. For spans in excess of these limiting values, the normal intensity of superimposed loading will be the controlling factor. In the case of slabs, the minimum total superimposed load is expressed in tons per foot width of slab, whereas for beams the corresponding load is the total load in tons on the beam.

Floors of industrial buildings where machinery and plant are installed should not only be designed for the possible loading when the plant is in running order, but the probable loading during erection and testing of the plant should be

investigated, since in many cases this may be more severe than the working load.

The support of heavy safes in buildings requires special consideration, and the floors should be designed not only to carry the safe in its permanent position but also when the safe is being moved into position, unless steps are taken to provide temporary props or other relief during installation.

The superimposed loading peculiar to floors of warehouses and garages is dealt with later. In all cases of floors in buildings it is advisable, and in some cities it is compulsory, to erect a tablet indicating the superimposed load for which the floor has been designed.

Loadings on Stairs, Landings, and Corridors.—The superimposed loads on stairs, landings, and corridors should generally be the same as the loads on the floors served by the staircases, except in warehouses or buildings with similarly heavily loaded floors, where there is little possibility of heavy goods being carried up the stairs. *Table* No. 2 gives the normal superimposed loads for stairs, landings, and corridors in residences and other buildings, together with the values of the minimum total superimposed loads on the corresponding slabs and supporting beams.

Roof Loads.—The superimposed loads for roofs given on *Table* No. 2 are additional to all surfacing materials and include for wind, ice, snow, and other incidental loads. Freshly fallen snow weighs about 5 lb. per cubic foot, but compact snow attains 20 lb. per cubic foot, which should be considered in districts subject to heavy snowfalls. For pitched roofs the snow load decreases with an increase in the slope, but the wind load increases, and allowance must be made for the suction on the leeward side in addition to the pressure on the windward side. Further consideration of wind pressures is given later.

If a flat roof is used for any purpose such as a café or roof garden, the appropriate floor loading must be allowed. The possibility of converting a flat roof into a floor in the future should also be anticipated.

Loads on Columns, Walls, and Foundations.—Columns, walls, and foundations of buildings should be designed for the same superimposed loads as the floors they support. When the superimposed load for the floor beams is less than for the slab, the beam load should be used in assessing the column load. In the case of buildings of more than two stories, and which are not warehouses or stores and for which the superimposed floor load is less than 100 lb. per square foot, the column and foundation loads can be reduced in accordance with the following rule.

Consider the roof and top floor as being fully loaded; reduce the superimposed load on the second floor from the top by 10 per cent., the superimposed load on the third floor from the top by 20 per cent., and so on in 10-per cent. increments down to the sixth floor from the top, where the superimposed load is reduced by 50 per cent. The superimposed load on all floors below the sixth from the top can also be reduced by 50 per cent.

3.—Storage Loads.

Warehouses.—Floor slabs, beams, columns, and foundations of buildings of the warehouse class should be designed for a minimum superimposed load of

200 lb. per square foot, but the purpose for which the building is to be used should be considered and, if necessary, a higher value should be taken. For example, paper stores and printing works are usually designed for a floor load of 3 cwt. per square foot.

Stored Materials.—Wherever possible the weights of the material to be stored should be ascertained and used in the design. This applies particularly to storage structures such as tanks, reservoirs, silos, and bunkers. The weights of some of the materials more commonly stored in concrete containers are given on *Table* No. 4. Where material is floating or submerged in water the loads or pressures carried by the bottom of the container are calculated in accordance with the expressions given in Chapter III, where also are considered the horizontal pressures due to liquids, earth, and other contained and retained materials.

4.—Moving Loads.

Structures such as bridges, gantries, and garages must be designed for loads due to moving vehicles, trains, or cranes, while in buildings consideration has to be given to the dynamic effect of lifts and vibration due to machinery.

Bridge Loading.—Most main road bridges in this country are designed for the loading prescribed by the Ministry of Transport and to simplify calculations the Ministry issues a memorandum reproduced in *Fig.* 1. This indicates for various spans, an equivalent uniformly distributed load with which has to be combined a knife-edge load of 2,700-lb. per foot run, as described in *Fig.* 1. For major road bridges outside official jurisdiction, either the Ministry of Transport loading or the loading specified by the British Standards Institution should be adopted. The B.S.S. loading train is illustrated on *Table* No. 3. It should be observed that, whereas the Ministry of Transport's load includes a 50-per cent. allowance for impact effect, the B.S.S. loading excludes impact.

The British Standard Specification for girder bridges gives the following:

IMPACT FACTORS FOR ROAD BRIDGES.

$$I = N - 0{\cdot}002\,L$$

Bridges with one traffic line : $N = 0{\cdot}75$; $I = 0{\cdot}60$ for $L =$ or $\not> 75$ ft.
Bridges with two or more traffic lines : $N = 0{\cdot}65$; $I = 0{\cdot}50$ ditto
L = effective span of longitudinal beams, or
= twice spacing of intermediate transverse beams, or
= distance between end and penultimate beams for end transverse beams.

These impact factors are for use with road loading in B.S. No. 153 (Part 3), 1937.

For impact on railway bridges, see B.S. No. 153.

In addition to the standard loadings mentioned or where this loading would be out of all proportion to the requirements of the traffic using the bridge (having regard to possible increases in the future), the loading from normal and special types of vehicles using the structure should be considered in the design. The effect of the occasional passage of steam rollers, heavy lorries, and abnormal

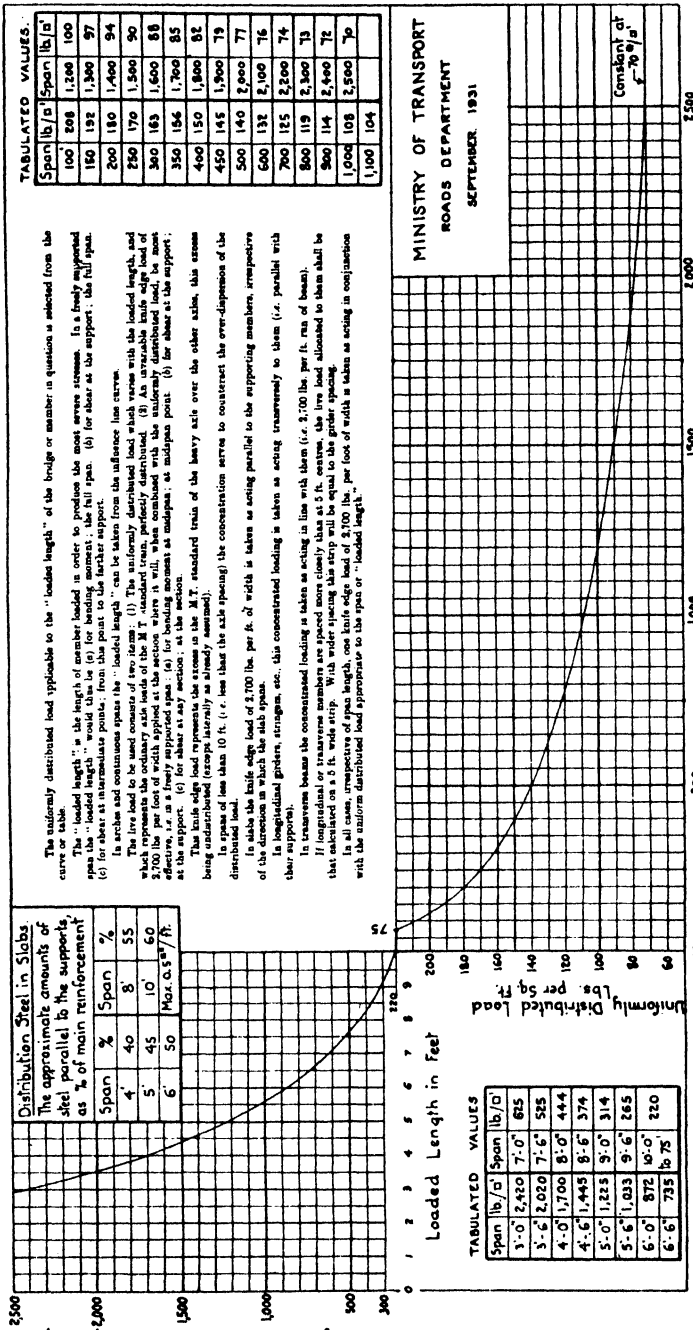


Fig. 1.—Ministry of Transport Loading for Road Bridges.

loads (boilers, heavy machinery, ships' propellers, transformers, etc.) should be studied, and in the case of railway bridges allowance should be made for the rolling loads indicated on the diagrams supplied by the railway company. Axle loads (without impact) and other data for various types of road and rail vehicles are given on *Table* No. 3. It will be appreciated that the wheel loads and dimensions of vehicles vary with different types and manufacturers, and the tabulated values are indications of magnitude rather than of precise values. Many of the tabulated weights and dimensions are those specified by the Ministry of Transport in regulations issued in connection with the Road Traffic Act. There are, however, many variations in this data on account of the use to which the vehicle may be put, the date of manufacture, the type of wheel, etc., and where exact figures are required reference should be made to the relevant regulations. Generally the tabulated values are the greatest under any condition. A useful guide is the regulation whereby the weight transmitted to a strip of road surface 2 ft. wide normal to the longitudinal axis of the vehicle, must not exceed 10 tons; impact is excluded from this figure.

The weights of buses, trolley-buses, and eight-wheeled tramcars given on *Table* No. 3 relate to vehicles used in London and exclude impact.

The raised footpaths of road bridges should be designed for crowding and for the possibility of a vehicle running on to the path. A load of $\frac{3}{4}$ cwt. to 1 cwt. per square foot is sufficient to allow for pedestrian traffic, and a single wheel load of 3 tons to 5 tons including the impact should be adequate for accidental loading. If there is any possibility of the footpath being converted into a roadway in the future, the pavement slab and supporting beams should be designed for the same loads as the original roadway.

The impact effect of rolling loads is usually allowed for by increasing the static load by 10 per cent. to 75 per cent., and for general cases the allowance should be decided after consideration of the type of vehicle, the nature of the road or rail surface, the type of wheel (whether rubber or steel-tyred), and the speed and frequency of crossing the structure. An allowance of 25 per cent. is sufficient for road bridges if the maximum load for which the structure is designed occurs infrequently. Road structures not designed for the maximum loads that are common in any district should be protected by a permanent notification of the maximum loads permitted to use the structure, and a limitation in both weight and speed should be enforced on all concrete bridges during the first few months after completion of the concrete work. For concrete bridges over roads or railways a speed limitation should be enforced on all traffic passing under the bridge during the period of construction and for a few weeks afterwards.

Loading from Cranes.—Overhead travelling cranes are commonly supported on reinforced concrete columns and gantries and the maximum wheel loads for cranes of normal design are as given in the table on the following page.

To allow for vibration and impact, the wheel loads should be increased by 25 per cent. Braking induces a horizontal thrust in the railbeam which is transferred to the supporting brackets, and the magnitude of this thrust can be taken as 20 per cent. of the maximum wheel load. The traversing of the crane and load produces a horizontal thrust transversely to the rail beam equal to about 10 per cent. of the total weight of the crane.

In designing gantries for jib cranes running on rails loading problems arise

which depend on the disposition of the weights of the crane. In general, the appropriate wheel loads would be obtained from the manufacturer and would allow for the static and dynamic effects of lifting, discharging, slewing, travelling, etc. The maximum wheel load under practical conditions may occur when the crane is stationary and hoisting the load at the maximum radius with the line of the jib diagonally over one wheel.

For special types of travelling machines, such as the ram engines, coke-cars, charging-cars, etc., in coke-oven plant, the wheel loads and thrusts should be obtained from the manufacturers and suitable impact allowances added. Conveyors such as are used in coal, cement, and grain-storage plants usually weigh 2 to 3 cwt. per lineal foot complete, while a conveyor gantry with sheeted sides and roof would weigh 5 cwt. to 6 cwt. per lineal foot.

OVERHEAD TRAVELLING CRANES :—

MAX. LOADS (TONS) ON PAIR OF WHEELS.

CAPACITY OF CRANE.	SPAN				WHEEL BASE	TOTAL WEIGHT APPROX.
	30 FT.	40 FT.	50 FT.	60 FT.		
2 TONS	5¼	6	6½	7½	8'-6"	10 TONS
5 "	9¾	10½	11½	12¼	8'-6"	17 "
10 "	15½	16¾	18¼	19¼	10'-0"	25 "
20 "	-	28¾	30¼	31½	10'-6"	40 "
30 "	-	40½	43	45	12'-0"	55 "
50 "	-	64	67½	70¼	13'-0"	84 "

Vibration.—For floors subjected to vibration from such causes as dancing, drilling, gymnastics, etc., the superimposed load of 100 lb. per square foot specified, on *Table* No. 2 is adequate to allow for the dynamic effect. For structural members subjected to continuous vibration due to machinery, crushing plant centrifugal driers, etc., allowance for dynamic effect is usually made by reducing the working stresses by 15 per cent. to 20 per cent. or by increasing the total dead and live load by this amount. The advantage of the latter method is that normal stresses can be used in the design worked to, thus making the standard tables and curves applicable.

Garage Floors.—Garage floors are usually divided into two classes, those accommodating vehicles not exceeding 2¼ tons in dead weight and those for heavier vehicles. Floors in the former class can be designed for the uniformly distributed load given on *Table* No. 4, but for floors in the heavier class consideration should be given to the magnitude and disposition of probable wheel loads. The actual wheel loads should be increased by 50 per cent. to allow for dynamic effect, and the bending moment and shear forces should not be less than those given by the prescribed minimum uniformly distributed load. Values for these minimum loadings are given on *Table* No. 4, together with the equivalent uniformly distributed load for bending moment calculation due to a load of one ton distributed over an area 2 ft. 6 in. square. These equivalent loads are tabulated for various conditions of continuity for both slabs and beams, and should be multiplied by one-and-a-half times the actual load (in tons). With a wheel

load directly over a beam and spread on a 2 ft. 6-in. square, the load will be partially taken by adjacent beams. To allow for this effect reduction factors are introduced on *Table* No. 4; the value of the reduction factor approaches unity for wide beams and for widely spaced beams. Since the thickness of the slab also enters into an accurate deduction for such factors, the tabulated values are approximate.

Lifts.—The effect of acceleration must be considered in addition to the static loads when calculating the loading due to lifts and similar machinery. If a net static load of W_D is subject to an acceleration of a feet per second per second the load on the supporting structure is given approximately by $W_M = W_D\ (1 + 0{\cdot}03\ a)$. The average acceleration of a passenger lift may be about 2 ft. per second per second, but the maximum acceleration will be considerably greater. An equivalent load of $1{\cdot}25\ W_D$ should be taken as the minimum to allow for dynamic effects. The load for which lift supports, pit-head gears, and similar structures should be designed, should bear a relation to the load on the ropes. If the latter is W_M and the ropes have a factor of safety of ten, the load on the supports should not be taken at less than $2\frac{1}{2}\ W_M$ to ensure that the structure is equally as strong as the ropes.

5.—Dispersion of Point Loads.

A wheel or similar load is usually considered as bearing on a definite area of the supporting surface, and can then be further dispersed over an area dependent upon the combined thickness of the road surfacing, filling, concrete slab and other constructional materials. Upon this latter area is based the estimation of the breadth of slab that assists in carrying the wheel load.

Contact Area.—The width of the contact area of the wheel on the slab is equal to the width of the tyre, which can be taken as 1 ft. 6 in. for the heaviest loaded wheels of the tractor in the standard loadings, and 9 in. for the trailer wheels. For other vehicles a minimum of 6 in. is usually taken. The length of the contact area depends on the type of tyre and the nature of the road surface, and would be zero for iron tyres on a steel-plate or concrete surface. The maximum contact would probably be obtained with an iron wheel on loose metalling or a pneumatic tyre on a tar-macadam surface. A maximum figure of 12 in. is suggested.

Dispersal through Slabs.—The further dispersal of the load through the total thickness of the road formation and concrete slab is usually considered as acting at an angle of 45 deg. from the edge of the contact area to the centre of the bottom reinforcement, as is shown in the diagram on *Table* No. 4. If "a" is the length or breadth of the contact area and D is the depth from the road surface to the centre of the reinforcement, the corresponding length or breadth of the dispersal plane is given by

$$A = a + 2D$$

Mr. W. L. Scott, in his treatment of the wheel loads in accordance with Pigeaud's theory, employs the expression

$$A = \sqrt{(a + 2d)^2 + H^2}$$

where H = the slab thickness and d = the thickness of metalling. This formula gives smaller values than dispersion at 45 deg.

In the case of a pair of wheel loads on two rails, if the latter are carried on sleepers the load in any position can be considered as distributed longitudinally over two effective sleepers and transversely over the length of a sleeper. The dispersal is then taken at 45 deg. through the ballast and decking, as indicated on *Table* No. 4. When a rail bears directly on a concrete beam the longitudinal dispersion of the load can be taken at from four to six times the depth of the rail.

Width of Slab Supporting Point Load.—When the concrete slab spans between longitudinal beams, the span of the slab being less than half the span of the beams (as in normal slab and beam bridge construction), authorities differ concerning what width of slab S should be considered as effective in supporting a point load and transmitting this to the beams. All authorities allow dispersion as described to be taken down to the steel, and the most conservative present-day practice is to consider S equal to A. In the United States the practice is to disperse the load horizontally at an angle the tangent of which is $\frac{2}{3}$ that is, $S = 0{\cdot}67(L + B) + A$ (as indicated on *Table* No. 4). Mr. E. A. Scott advocates an expression of the form $S = \frac{L}{4} + A$, and Mr. W. L. Scott considers the moment in the slab directly by the Pigeaud method, without computing the equivalent width of slab ; this method is further discussed for uniform loads in Chapter IV, and is the basis of the Ministry of Transport's stipulation for the amount of distribution steel given in *Fig.* 1. Sir E. O. Williams has given as an approximate formula $S = 0{\cdot}67L + 6$ in. with a maximum value of 7 ft. These various formulæ give rather differing results, and, when applied to a wheel contact area of 12 in. by 9 in., an effective depth $D = 14$ in. and a slab span $L = 7$ ft., the values of S vary from a minimum width of 3 ft. 1 in. to 5 ft. 6 in. when calculated in accordance with the United States method. For general purposes, the latter gives economical results and is recommended if ample longitudinal or distribution reinforcement, say, up to 50 per cent. of the area of the principal transverse reinforcement is provided.

For the design of garage floors as already described each wheel load can be considered as bearing on an area 2 ft. 6 in. square without further dispersion down to the reinforcement.

6.—Wind Pressure.

Buildings.—The magnitude of the wind pressure on a structure depends on the position and height of the structure. For buildings in Great Britain a pressure of 15 lb. per square foot on the upper two-thirds of the structure is specified both in the Code of Practice and the London By-laws, with an additional 10 lb. per square foot on any projection above the general roof level. If the height of the building does not exceed twice the width, wind pressure can be neglected. It is sometimes recommended that each wall panel should be designed for a pressure of 25 lb. per square foot acting from either side of the panel.

Wind Velocity and Pressures.—When special structures are to be built in exposed positions where the probable wind velocities are known from records,

it is preferable to design for the pressures calculated from the maximum velocity. The pressure of the wind varies as the square of the velocity and various relationships have been derived by different investigators. If P is the pressure in lb. per square foot and V is the velocity in miles per hour, a reasonable value would be $P = 0{\cdot}003V^2$, and pressures calculated in accordance with this expression are given on *Table* No. 2 for various velocities of the wind. For a steady wind with a velocity of 100 miles per hour, as occurs at rare intervals in this country, the pressure would be 30 lb. per square foot but the impact from gusts of high wind may exert greater pressures.

Chimneys and Exposed Structures.—Since a primary factor in the design of chimneys and similarly exposed isolated structures is wind pressure, every problem should be given careful consideration to avoid either underestimating or making an unduly high assessment. Where records of winds in the locality are available an estimate of the probable wind pressures can be made. Due account should be taken of the susceptibility of narrow shafts to the impacts of gusts, and a pressure of 40 lb. per square foot is mentioned in the Memorandum of the London By-laws. This value, which allows for pressure on the windward side, for suction on the leeward side, and for impact, is primarily applicable to brick chimneys, but it should be remembered that the margin of safety is greater for concrete structures than for brickwork or masonry owing to the ability of reinforced concrete to resist tensile stresses.

For chimneys and other structures of considerable height, a variable intensity of pressure is often recommended, since the wind velocity, and therefore the pressure, increases with increased height. Suitable values are given on *Table* No. 2.

For circular shafts the pressure can be reduced by 40 per cent. owing to the relieving effect of the curved surface. This reduction can also be applied to the wind pressure on circular tanks of water towers if the diameter is 20 ft. or less, but for larger diameters and for tanks polygonal in plan a smaller reduction should be made. A reduction of 33⅓ per cent. can be assumed when the wind blows normally to a diagonal of a square tower or shaft and 20 per cent. for other shapes. For circular chimneys with fluted surfaces a reduction of only 20 per cent. should be made.

Bridges.—Wind pressures on steel bridges are dealt with in British Standard Specification No. 153, and for reinforced concrete bridges a similar consideration should be applicable. The pressures given on *Table* No. 2 apply per square foot of exposed surface, which for a loaded bridge includes the vertical area of the structure and any portions of the load projecting above the parapet. For an unloaded bridge where the width between the parapets exceeds twice the combined depths of the parapet and outer girder, the exposed area should be calculated as twice the area of the parapet and outer girder. If the width is less than this the exposed area should be reckoned as three-quarters of the value for a wide bridge.

Note: The reader is referred to British Standard Code of Practice CP4: 1944 "Functional Requirements of Buildings: Chapter V: Loading" wherein are given the latest recommendations for floor and roof loadings. In particular this code deals more thoroughly with wind pressure on buildings than hitherto.

CHAPTER III

PRESSURES DUE TO RETAINED AND CONTAINED MATERIALS

1.—Active Pressures.

Liquids.—The calculation of the exact value of the horizontal pressure exerted by contained materials on the sides and bottoms of a container is an uncertain operation except when the material is a liquid. In the latter case the intensity of pressure (p) normal to the surface of the wall of the container is equal to the intensity of vertical pressure at any depth (h) from the free surface of the liquid, and is given by the simple hydrostatic expression :

$$p = wh \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where w = unit weight of the liquid. The units of w and h determine those of p, that is, with w in lb. per cubic foot and h in feet, p would be in lb. per square foot.

Granular Materials.—When the contained material is a "semi-fluid" and granular in form, for example, dry sand, grain, small coal, etc., the value of p would be in accordance with the general expression :

$$p = kwh \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where k is a factor depending principally on the fluidity of the contained or retained material and varying from unity for perfect fluids to zero for materials that will stand with a vertical face. The actual value of k depends not only on the physical characteristics, water content, and angle of repose of the earth, grain, or other material, but also on the slope of the wall, on the material of which the wall is made, and on the amount of surcharge on the contained material.

The value of k is determined either graphically or by calculations that are usually based on the wedge theories or the developments of Rankine or Cain. The total pressure normal to the back of a wall of height H, *Fig.* 2, is given by

$$P = \frac{KwH^2}{2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$K = \left[\frac{\sin(\beta - \theta)}{(n+1)\sin\beta}\right]^2 \frac{\cos\mu}{\sin(\mu+\beta)} \quad . \quad . \quad . \quad (4)$$

in which

$$n = \sqrt{\frac{\sin(\theta+\mu)\sin(\theta-\phi)}{\sin(\mu+\beta)\sin(\beta-\phi)}}.$$

Referring to *Fig.* 2 the notation is as follows :

θ = angle of repose of the material
μ = angle of friction between the wall and the contained material
ϕ = angle of surcharge of the surface of the material
β = angle between wall and horizontal.

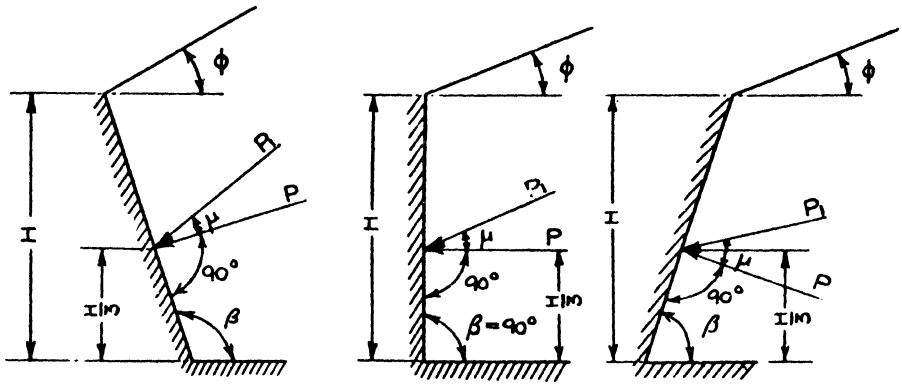
The total pressure on the back of the wall is given by

$$P_1 = \frac{P}{\cos \mu} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the line of application of P_1 makes an angle of (90 deg. $- \mu$) with the back of the wall. The corresponding force parallel to the wall is

$$F = P_1 \sin \mu = P \tan \mu \quad . \quad . \quad . \quad . \quad (6)$$

These general formulæ can be modified to suit the various conditions met with in practice, when usually the friction between the wall and the material is neglected. This modification results in a higher normal pressure and is therefore on the safe side. It is advocated that for general cases the effect of friction should be neglected, and imperatively so in all cases where the material in contact



(a) WALL INCLINED OUTWARDS. (b) VERTICAL WALL. (c) WALL INCLINED INWARDS.

Fig. 2.—Pressure on Walls.

with the wall can become saturated and thus reduce the friction practically to zero or to an uncertain figure. Only in the cases where dry coal, dry sand, grain, cement, or other materials of well-known properties, are being stored, should friction be accounted as relieving the wall of pressure.

When friction is neglected (i.e. μ = zero) formula (4) reduces to

$$K = \left[\frac{\sin(\beta - \theta)}{(n+1)\sin\beta}\right]^2 \cdot \frac{1}{\sin\beta} \quad . \quad . \quad . \quad . \quad (4a)$$

where $$n = \sqrt{\frac{\sin\theta \sin(\theta - \phi)}{\sin\beta \sin(\beta - \phi)}}.$$

Similarly $$P_1 = P \text{ and } F = 0 \quad . \quad . \quad . \quad . \quad (5a) \text{ and } (6a)$$

For sloping walls the pressures are determined by substituting the known factors in either formula (4) or (4a) depending on whether friction is included or not.

2.—Pressures on Vertical Walls.

Friction Neglected.—Generally in the case of earth-retaining walls and bunker walls the wall is vertical and the introduction of $\beta = 90$ deg. into formula (4*a*) in which friction is neglected results in the expressions (7), (8), (9), and (10) which give the factors for determining the intensity of horizontal pressure at any depth h for the limiting conditions of maximum positive surcharge ($\phi = \theta$), level fill ($\phi =$ zero), and maximum negative surcharge ($\phi = -\theta$), and for the general condition of any surcharge ϕ.

Maximum positive surcharge

$$k_1 = \cos^2\theta \qquad (7)$$

Level fill

$$k_2 = \frac{1 - \sin\theta}{1 + \sin\theta} \qquad (8)$$

Maximum negative surcharge

$$k_3 = \left[\frac{\cos\theta}{1 + \sqrt{2}.\sin\theta}\right]^2 \qquad (9)$$

Any surcharge

$$k = \left(\frac{\cos\theta}{n + 1}\right)^2 \qquad (10)$$

where $$n = \sqrt{\sin^2\theta - \tfrac{1}{2}\tan\phi\sin 2\theta}.$$

Values of k_1, k_2, and k_3 for various angles of repose (expressed in degrees and as gradients) are given on *Table* No. 5, together with the various soils, grains, coals, etc., that correspond to these angles. For ordinary earth retaining wall problems with level fill, the value of k_2 is usually taken as 0·27, and, considering the weight of earth as 100 lb. per cubic foot, the corresponding intensity of horizontal pressure is 27 lb. per square foot per foot of height. Reference should be made also to Chapter VII for further particulars on the properties of coal, etc.

Friction Included.—When friction between the contained material and the back of a vertical wall is allowed for, the formulæ for the factors for determining the pressure intensity normal to a vertical wall are as follows:

Maximum positive surcharge

$$k_1 = \cos^2\theta \qquad (7)$$

Level fill

$$k_2 = \left[\frac{\cos\theta}{n + 1}\right]^2 \qquad (8a)$$

where $$n = \sqrt{\sin\theta\,(\sin\theta + \cos\theta\tan\mu)}.$$

Maximum negative surcharge

$$k_3 = \left[\frac{\cos\theta}{n\sqrt{2} + 1}\right]^2 \qquad (9a)$$

where n is as in (8*a*).

Any surcharge

$$k = \left(\frac{\cos\theta}{n+1}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

where

$$n = \sqrt{\frac{\sin(\theta+\mu)\sin(\theta-\phi)}{\cos\mu.\cos\phi}}.$$

Generally accepted values of μ, θ, and w for such dry materials as finely-ground cement, coal, grain, sand, and ashes are given on *Table* No. 5, and for these materials the values of K_1, K_2, and K_3 ($= k_1w$, etc.) are tabulated, having been calculated in accordance with formulæ (7), (8*a*) and (9*a*). From these values of K can be computed the intensity of horizontal pressure: $p = Kh$. If H is the total height of the wall, the total horizontal pressure is given by $P = p\frac{H}{2}$, and the total thrust on the wall acting at an angle μ to the horizontal is given by $P_1 = \frac{P}{\cos\mu}$.

3.—Surcharge.

Non-Level Fill.—As will be seen from the tabulated values of the pressure factors, the amount of surcharge of the surface of the filling behind the wall has a marked effect on the magnitude of the pressure. For all practical purposes any alteration of the profile of the fill beyond the point B, *Fig.* 3(*a*), or any additional loading beyond this point, has little or no effect upon the pressure on the wall. The general formula takes account of the surcharge due to any fill profile lying between the limits DB and DB_1.

When the surface of the filling is not uniformly sloped, special treatment is required. In the common cases depicted in *Figs.* 3(*b*) and 3(*c*) the magnitude of the pressure on the wall is somewhere between that due to level fill and maximum positive natural surcharge. On *Table* No. 5 empirical expressions are suggested that allow for the increase of pressure due to these types of intermediate surcharges. The total pressure on the wall is augmented above that for a level fill by an amount proportional to the mean increase in head of material. Such surcharges give zero pressure at the top of the wall, and the centre of total pressure will be at one-third of the total height.

Superimposed Load on Filling.—When the fill behind a wall is level but liable to subjection to self-retaining superloads, such as stacked materials, traffic, or buildings, the total superload should be converted into an equivalent head of the same material as that retained by the wall, and the pressure intensity on the back of the wall augmented uniformly throughout the depth of the wall. Thus there will be a definite intensity of pressure at the top of the wall; these conditions are illustrated in *Fig.* 3(*d*) and indicated on *Table* No. 5.

Surcharge beyond Natural Angle of Repose.—A type of surcharge not covered by the foregoing observations and formulæ is that shown in *Fig.* 3(*e*). In this case the angle of surcharge exceeds the natural angle of repose, as may occur by artificially protecting a bank of earth by turfing or stone pitching. For such a case it has been suggested that the weight of earth W above the

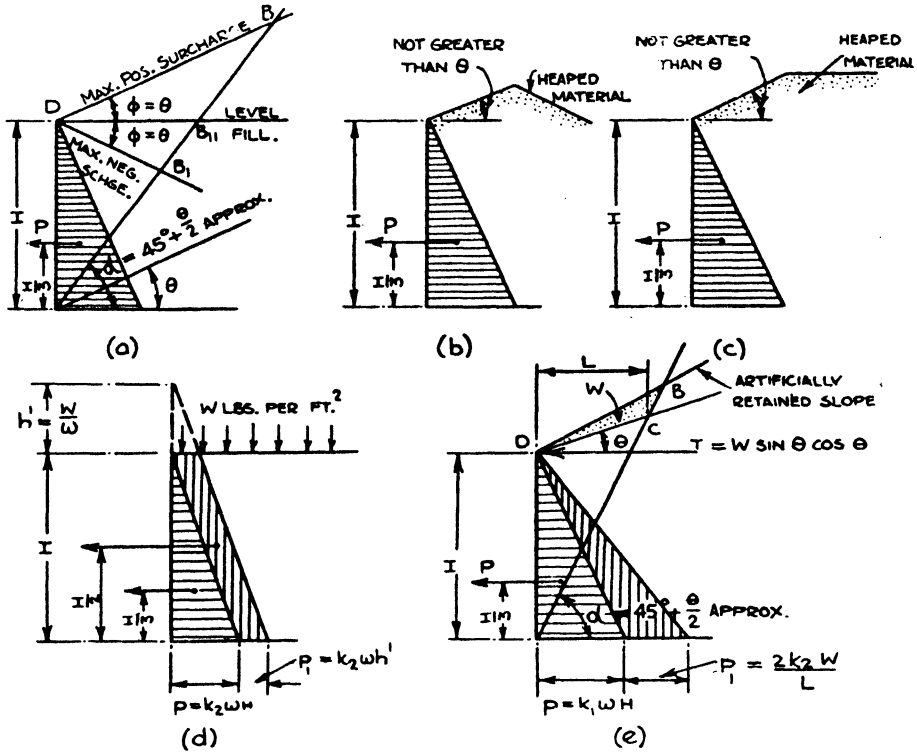


Fig. 3.—Surcharges on Walls.

angle of repose, represented by the triangle *BCD*, should be considered as a point load operating at the top of the wall. The magnitude of the resultant horizontal thrust *T* and increase in pressure would be as shown in *Fig.* 3(*e*), and each section of the wall should be designed for the extra moment due to *T*.

Point Load.—A single point superload on the filling behind a wall is best dealt with as a problem in dispersion, and, considering 45 deg. as a reasonable angle of dispersion, the pressure intensity additional to the pressure due to the filling alone is indicated in *Fig.* 4.

4.—Experimental Data.

Active Pressures.—In general practice in Great Britain, earth pressures are determined by the purely theoretical formulæ of Rankine, Cain, and Coulomb. Many investigators, working with a sand filling, have experimented on model walls in order to determine what relation actual pressures bear to the theoretical pressures. The unanimous conclusion is that the Rankine formula gives too great a value for the pressure exerted, and thus retaining walls designed on this formula err on the side of safety. The theoretical deduction assumes that the angle of internal friction and the surface angle of repose are identical, whereas Crosthwaite and other investigators have found that the friction angle is less

than the angle of repose and depends on the consolidation of the material. The ratio between the internal friction angle and the natural angle is approximately 0·9 to 1 according to experiments carried out at the University of Cincinnati.

For level fills the horizontal pressure is given by Coulomb's formula (a special case of the general formula) in the form

$p = wh \,.\, \tan^2\left(45^\circ - \frac{\theta}{2}\right)$. This is formula No. 8 in another form and agrees very closely with actual pressures if the angle of internal friction is substituted for the natural angle of repose. The maximum pressure seems to occur immediately after back-filling has been completed, and the pressure decreases as settling proceeds.

The measured vertical component of the pressure on the back of the wall appears to conform to the theoretical relationship $V = P \tan \mu$, where $P =$

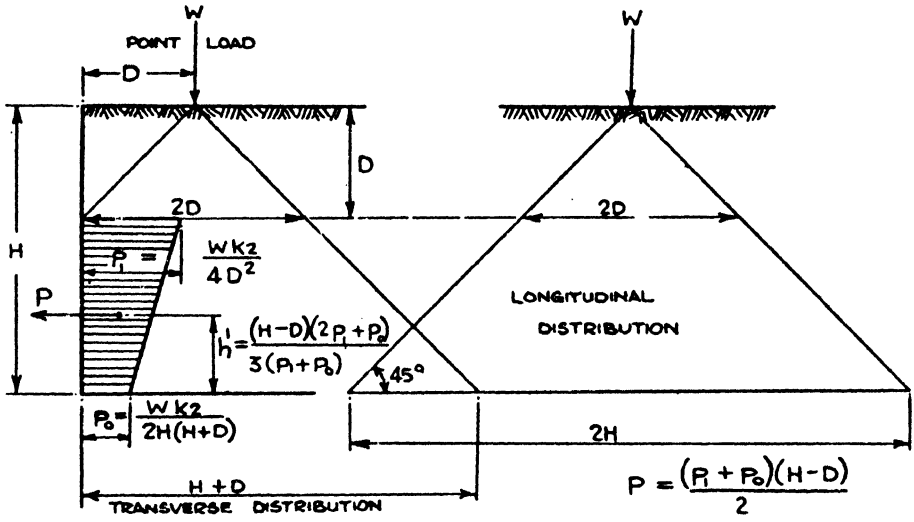


Fig. 4.—Dispersion of Point Load behind Retaining Wall.

horizontal pressure and μ is the independently determined angle of friction between the wall and filling. A rise in temperature produces measurable increases in pressure, the increase being of the order of 1 per cent. per 10 deg. F.

The point of application of the resultant thrust on walls with level fills appears to be at the theoretical height of $\frac{\text{total height}}{3}$ for shallow fills, and according to the Cincinnati tests rises with time and with increased depth of fill. According to the Feld-Moncrieff investigations the height of application is at the third-point for negative surcharge and level fills, but rises with increase of angle of surcharge.

Some investigators report that superloading beyond the plane of rupture increases the pressure on the wall, and others come to the conclusion that loads outside the wedge ordinarily considered can be neglected. The increase of pressure due to incidental superloading remains after the load is removed. For

walls surcharged by the slope of earth behind them the wedge theory seems to give results almost 30 per cent. in excess of measured values.

The discrepancies between theoretical and observed results relating to earth pressures are explained in part by soil mechanics principles. In particular, clays do not conform to theories evolved for non-cohesive soils. Bell's theories for clays are given below and on page 218.

Active Pressure from Clay.—Mr. A. L. Bell has shown that Rankine's formula is inadmissible for pressures due to clay, and gives the following expression for the intensity of pressure at any depth h:

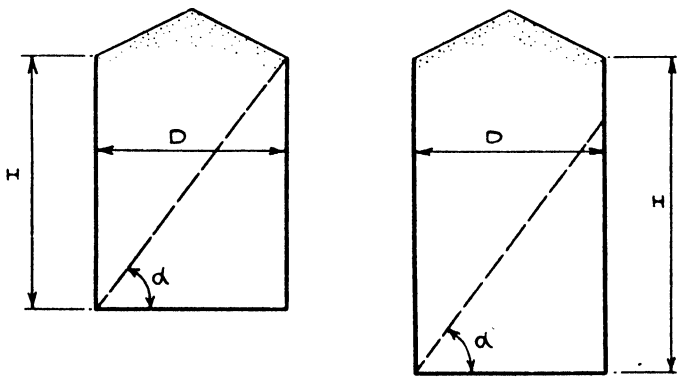
$$p = wh \tan^2\left(\frac{\pi}{4} - \frac{a}{2}\right) - 2k \tan\left(\frac{\pi}{4} - \frac{a}{2}\right)$$

where the factors k and a have the following values:

Very soft puddled clay:	k = 450	lb. per	square	foot	a = zero	
Soft puddled clay:	670	,,	,,	,,	3	degrees
Moderately firm clay:	1120	,,	,,	,,	5	,,
Stiff clay:	1570	,,	,,	,,	7	,,
Very stiff boulder clay:	3600	,,	,,	,,	16	,,

5.—Deep Containers.

Limiting Depths.—The foregoing observations and formulæ for the pressures on walls of containers refer only to retaining walls and shallow containers. In



(a) SHALLOW CONTAINER (b) DEEP CONTAINER

Fig. 5.—Silos.

deep containers such as tall grain silos, the arching effect of the contained material considerably reduces the horizontal pressure, so that the principle that the pressure is proportional to the depth of filling no longer holds good. The limiting condition that determines whether any particular container shall be considered as "shallow" or "deep" (that is, as a "bunker" or as a "silo") is that if the plane of rupture strikes the opposite wall before reaching the free surface, as in *Fig.* 5(*b*),

the container can be treated as " deep." If otherwise, as in *Fig.* 5(*a*), the container must be considered as " shallow."

The theoretical minimum depth of a " deep " container is given by

$$H = D\left(\tan\theta + \sqrt{\tan\theta\frac{1+\tan^2\theta}{\tan\theta+\tan\mu}}\right)$$

where D is the least breadth of the container.

For grain the least depth is theoretically equal to approximately one-and-a-quarter times the breadth of the container, and for cement equal to approximately one-and-a-half times the breadth. During the emptying process the arching action on which pressure reduction depends is completely destroyed, and pressures appreciably in excess of the theoretically determined pressures are obtained. It would seem therefore that for absolute security no container that is not at least twice as deep as it is wide should be treated as a " silo."

Pressures in Silos.—The value and variation of value of the pressure in silos is usually computed by either Janssen's formula or Airy's formula. The former gives results slightly less than the latter and in its general form is as follows :

$$p = \frac{wQ}{\tan\mu}\left(1 - \frac{1}{N}\right)$$

where

p = horizontal pressure in lb. per square foot

w = weight in lb. per cubic foot of material

Q = ratio of plan area in sq. ft. to perimeter in ft. of bin $= \frac{A}{L}$

μ = angle of friction between wall and material, and

N = the number whose common log. is $\frac{K\tan\mu\,.\,H}{2{\cdot}303Q}$

where

K = ratio of horizontal to vertical pressures, and

H = depth of filling in feet at point considered.

The vertical pressure (lb. square foot) on any horizontal section of the material is given by

$$V = \frac{p}{K}.$$

Therefore the total pressure on any horizontal plane is $\frac{pA}{K}$, and the load transferred to the walls of the container by friction is $\left(wH - \frac{p}{K}\right)Q$ lb. per foot run of wall.

For granular materials such as grain and cement a value of $K = 0{\cdot}5$ is usually adopted, and a value of $\tan\mu = 0{\cdot}444$. *Table* No. 6 gives the magnitude of $C = \frac{p}{w}$ for various values of Q and H, incorporating these values of K and $\tan\mu$.

For circular, square, or regular polygonal containers, $Q = \frac{D}{4}$ where $D =$

diameter or distance between opposite faces of the container, and the following expressions apply for grain at 48 lb. per cubic foot:

$$N = \text{the number whose common log. is } \frac{0{\cdot}385H}{D}$$

$$p = 27D\left(1 - \frac{1}{N}\right) \text{ lb. per square foot}$$

$$V = 2p \text{ lb. per square foot,}$$

and the load carried by the walls at any depth H

$$= (12H - 0{\cdot}5p)D \text{ lb. per foot run.}$$

From an inspection of the data on *Table* No. 6 it will be seen that the pressure increases very little below a depth of three times the least width of the container. Tests have indicated that the maximum pressures occur in silos during filling, and are somewhat greater for a bin filled rapidly than one filled slowly.

6.—Passive Resistance.

The whole of the foregoing remarks on the horizontal pressure exerted by contained or retained filling concern the active pressures due to such filling. If a horizontal pressure in excess of this active pressure is applied to the vertical face of a retained bulk of granular material, the passive resistance of the material is brought into action. Up to a limit, determined by the characteristics of the particular material, the passive resistance will be equal to the applied pressure; the maximum value the resistance can attain for a granular material with a level surface is given theoretically by the reciprocal of the Rankine active pressure coefficient, that is,

$$p_r = \left(\frac{1 + \sin\theta}{1 - \sin\theta}\right)wh = \frac{wh}{k_2}$$

where p_r = maximum passive resistance at any depth
w = weight per cubic foot of material
θ = angle of repose
k_2 = the coefficient given on *Table* No. 5.

It is not easy to assess a practicable working value for the passive resistance when the surface of the material is other than level, and it is advisable never to assume a value exceeding the value above and to use discretion when the surface has a negative surcharge, in which case the passive resistance may be very small.

The passive resistance of earth has to be taken into account when considering the resistance to sliding in retaining wall design, when dealing with the forces acting on sheet piling, and when designing earth anchorages, but in these cases careful consideration must be given to those factors that may reduce the probable passive resistance to a value approaching more nearly the active pressure. The chief of these factors is wetness. Abnormal dryness may cause clayey soils to shrink away from the surface of the structure, thus necessitating a small but most undesirable movement of the structure before the passive resistance is brought into play.

7.—Pressures due to Materials in Water.

Horizontal Pressures.—When a material such as coal or broken stone is stored fully immersed in liquid the intensity of horizontal pressure on the side of the container is given by the expression

$$p = w_L h\left[1 + k\left(\frac{w_m}{w_L} - 1\right)(1 - V)\right]$$

where w_L = weight per cubic foot of the liquid (lb.)
w_m = weight per cubic foot of the solid material (lb.)
V = ratio of volume of voids in any given volume of the dry material
k = horizontal pressure factor depending on the profile of the fill and the angle of repose of dry material (= factors k_1, k_2, or k_3 on *Table* No. 5)
h = depth (ft.) from top of submerged material to level at which pressure is being calculated.

If the surface of the liquid is at a distance h_0 ft. above the top of the submerged material, there will be an additional pressure at any depth equal to $w_L h_0$. Thus, for a material submerged in water, the total intensity of pressure at any depth $h_0 + h$ below the surface of the liquid will be

$$p_t = 62{\cdot}4(h_0 + hF)$$

$$\text{where } F = \left[1 + k\left(\frac{w_m}{62{\cdot}4} - 1\right)(1 - V)\right],$$

which is dependent on the characteristics of the submerged material.

The foregoing expressions only apply to substances whose angle of repose is not materially affected by submergence. When dealing with materials such as sand, of which the angle of repose has a definite value when it is dry but becomes zero when it is saturated, the intensity of horizontal pressure becomes

$$p = w_L h\left[1 + \left(\frac{w_m}{w_L} - 1\right)(1 - V)\right],$$

and when immersed in water

$$F = \left[1 + \left(\frac{w_m}{62{\cdot}4} - 1\right)(1 - V)\right].$$

The values of p (lb. per square foot) for unit value of h for characteristic materials immersed in water and with level fill are as follows:

Material	Weight solid (lb. per cu. ft.)	Angle of repose (deg.)	Percentage of voids (= 100 V)					
			25	30	35	40	45	50
Crushed Coal	80	35	66	65¾	65½	65¼	65	64½
Broken Stone	160	35	82	81	79½	78½	77	76
Sand	160	—	135	130	125	122	116	111

When materials lighter than water are stored in water, the horizontal pressure on the walls of the container is equal to the simple hydrostatic pressure $w_L h$.

Vertical Pressures.—When materials heavier than a given liquid are submerged, the vertical pressure on the bottom of the container is the weight of material in bulk together with the weight of liquid in the voids and the weight of liquid above the top of submerged material, that is,

$$p_v = h_m[w_m(1 - V) + w_L V] + w_L h_0$$

where h_m = depth (ft.) of submerged material.

When material stored in a given liquid floats in the liquid, the vertical pressure on the bottom of the container is given by the product of the weight per cubic foot of the liquid and the total depth of the liquid.

CHAPTER IV

BENDING MOMENT AND SHEARING FORCE

1.—Bending Moment, Shear, and Deflection.

Definitions.—The moment at any section in a member subject to flexure is defined as the algebraic sum of the moments of all loads and reactions to the left (or right) of the section considered, and the shear force at any section is the algebraic sum of all the loads and reactions to the left (or right) of the section. The shear force at any section can thus be expressed as the rate of change of bending moment at the section ; expressed mathematically, bending moment is the first integral of shear, which in turn is the first integral of the load. It follows from the definition of shear that the point of zero shear is the point of maximum bending moment.

From considerations of the theory of elastic flexural deformations it is shown that the rate of change of the slope at any point in a beam is expressed by the value of the $\frac{M}{EI}$ factor for that point, where M = bending moment, I = moment of inertia, and E = elastic modulus. Thus slope is the first integral of $\frac{M}{EI}$, and the first integral of the expression for the slope at any point in the beam represents the expression for the elastic deflection.

Cantilevers and Single Span Beams.—For a cantilever or for a single-span beam simply supported (that is without restraint) at its two extremities, the practical interpretation of the foregoing mathematical definitions of bending moment and shear force at any section is a simple matter. On *Table* No. 7 are tabulated the maximum bending moments and end shears (that is, reactions on the supports) produced by various commonly occurring loads carried on cantilevers and single-span freely-supported beams.

Propped Cantilevers.—A beam with one end fixed and one end freely supported can be considered as a cantilever subject to two distinct systems of loading :

(i) the imposed load

(ii) a point load (the reaction on the prop)

the magnitude of which is such as would give the same end deflection upwards on a cantilever as the downward deflection produced by the imposed load. For the loaded cantilevers tabulated on *Table* No. 7 the maximum deflection co-efficients are given, as are also the moments and reactions produced by similar loads on single-span beams fixed at one end and simply supported at the other. For cases of loading other than those tabulated, the deflections can be found from the expressions given on *Table* No. 7A (p. 225).

The values of I and E that should be adopted are discussed in Chapters V and IX, but when comparative deflections are required the exact numerical values of these factors do not enter into the calculation. The method of applying these deflection formulæ is illustrated in the following example.

To find the reaction in the prop at B and the bending moment at A for a beam loaded as shown in *Fig.* 6.

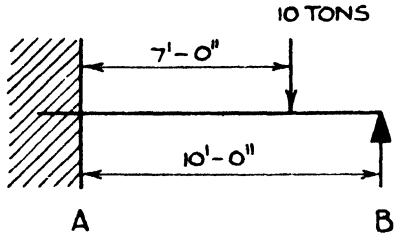


Fig. 6.—Propped Cantilever.

The expression (a) given on *Table* No. 7A is applicable for determining the end deflection of a 10-ft. cantilever supporting a load placed 7 ft. from the support, that is, $a = \frac{7}{10} = 0{\cdot}7$

$$\therefore d = \frac{0{\cdot}7^2(3 - 0{\cdot}7)WL^3}{6EI} = 0{\cdot}188\frac{WL^3}{EI}.$$

From *Table* No. 7 the deflection at the end of a cantilever subjected to a point load R located at its extremity is given by $\frac{RL^3}{3EI}$, and since this deflection must equal d

$$0{\cdot}188\frac{WL^3}{EI} = \frac{RL^3}{3EI}; \text{ therefore } R = 0{\cdot}188 \times 3W = 0{\cdot}564W$$

hence $R = 5{\cdot}64$ tons, which is the reaction on the prop at B and the shear at this end of the beam. The shear at A will be $10 - 5{\cdot}64 = 4{\cdot}36$ tons.

The bending moment at A will be the algebraic sum of the moments of R and W about A, that is

$$\text{B.M.} = (5{\cdot}64 \times 10) - (10 \times 7) = -13{\cdot}6 \text{ foot-tons.}$$

From these values the complete bending moment and shear diagram can be drawn similarly to those on *Fig.* 7 for uniformly distributed loads.

Fixed Beams.—The bending moment diagram for an encastré beam, that is, one fixed at both ends, is derived from the principle that the area of the bending moment diagram due to the same imposed load on a freely-supported beam of equal span (the " free-moment " diagram) is equal to the area of the restraint moment diagram ; the centroids of the two diagrams should be vertically one above the other. The shape of the free-moment diagram depends upon the characteristics of the imposed load, but the restraint-moment diagram is a trapezium. For loads symmetrically disposed on the beam the centroid of the free-moment diagram will be above the mid-point of the span, and thus the restraint-moment diagram will be a rectangle, giving at each support identical

restraint moments each equal to the mean height of the free B.M. diagram. Typical bending moment and shear force diagrams for cantilevers, propped cantilevers, freely-supported beams, and encastré beams are illustrated in *Fig.* 7 together

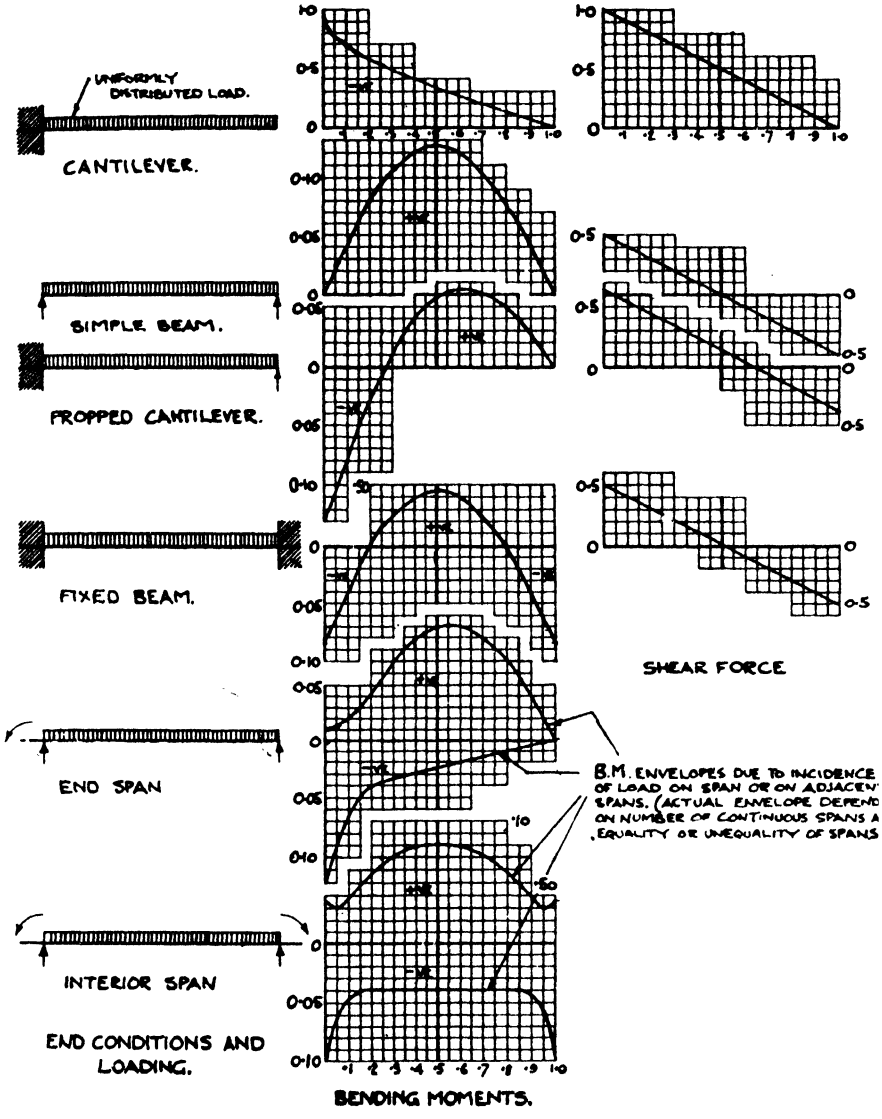


Fig. 7.—Typical Bending Moment and Shear Force Diagrams.

with bending moment envelopes for end and interior spans of a series of continuous spans. These diagrams are plotted with the horizontal ordinates divided into ten parts, thus facilitating the reading of intermediate values.

Shear Force for Restrained Beams.—The shear force diagram for a beam with one or both ends fixed can be derived from consideration of the variation

in bending moment. The shear due to the restraint moment alone is constant throughout the length of the beam, and is equal to the difference between the two end moments divided by the span. This shear should be algebraically added to the shear due to the imposed load, the latter shear being determined as if the beam were simply supported; that is, the resultant reactions at either support will be the sum or difference of the restraint-moment shear and the free-moment shear.

With a symmetrically loaded beam with both ends fixed, the restraint moments at each end being the same, the shear due to these moments considered alone is zero; therefore the resultant shear force variation is identical with that for the same beam freely supported, and the end reactions are both equal to half the total load on the beam. The reactions and maximum bending moments in encastré beams carrying ordinary types of loading are given on *Table* No. 7, while the same particulars for triangular loading, such as is met with in the design of tanks and other containers, are given on *Table* No. 7B.

2.—Continuous Beams.

Theoretical Consideration.—The bending moment in a beam continuous over two or more spans is ordinarily calculated by the Three Moment Theorem (or can be derived from the general formulæ for members subject to flexure given in Chapter V). In its general form, Clapeyron's Theorem of Three Moments for any two successive continuous spans is expressed by

$$M_A\frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\frac{L_2}{I_2}$$
$$= 6E\left[\frac{d_B - d_A}{L_1} - \frac{d_C - d_B}{L_2}\right] - 6\left[\frac{A_1z_1}{I_1L_1} + \frac{A_2(L_2 - z_2)}{I_2L_2}\right]$$

where, referring to *Fig.* 8,

M_A, M_B, and M_C = bending moments at supports A, B, and C

I_1 and I_2 = moment of inertia of spans L_1 and L_2 respectively

d_A, d_B, and d_C = deflection of each support from original position of beam

A_1 and A_2 = area of free moment diagrams for loads on span L_1 and L_2 respectively

z_1 and z_2 = position of centroid of A_1 and A_2 respectively.

For the usual consideration of level supports this general expression becomes

$$M_A\frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\frac{L_2}{I_2} = -6\left[\frac{A_1z_1}{I_1L_1} + \frac{A_2(L_2 - z_2)}{I_2L_2}\right].$$

If the moment of inertia of the beam throughout the two spans is assumed to be constant, the formula reduces to

$$M_AL_1 + 2M_B(L_1 + L_2) + M_CL_2$$
$$= 6EI\left[\frac{d_B - d_A}{L_1} - \frac{d_C - d_B}{L_2}\right] - 6\left[\frac{A_1z_1}{L_1} + \frac{A_2(L_2 - z_2)}{L_2}\right]$$

and with level supports and constant moment of inertia (the conditions usually approximated to in normal design) the Three Moment Theorem becomes

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = -6\left[\frac{A_1 z_1}{L_1} + \frac{A_2(L_2 - z_2)}{L_2}\right].$$

If the load is uniformly distributed and equal throughout the two spans considered, and assuming level supports and constant moment of inertia, the following formula applies:

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = -\frac{w}{4}(L_1^3 + L_2^3)$$

where w = load per foot run of beam.

The shear force variation can be determined by first considering each span

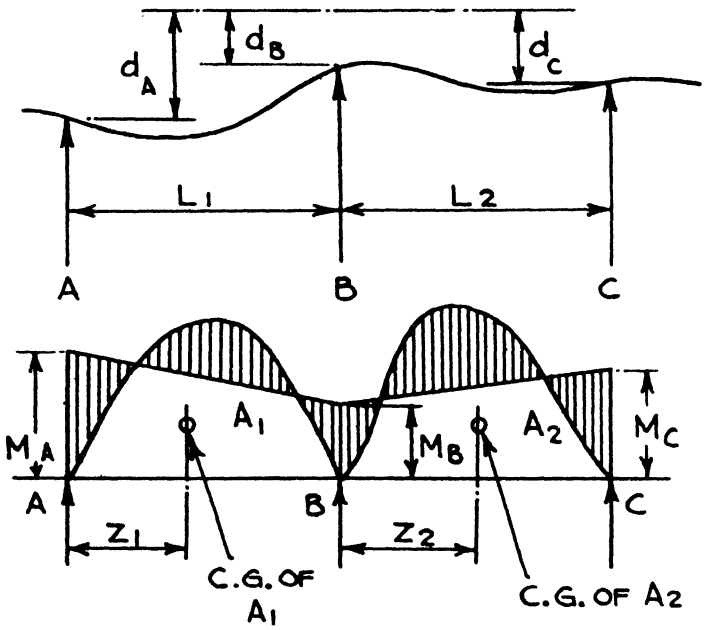


Fig. 8.—Continuous Beams.

as freely supported and algebraically adding the rate of change of restraint moment for the span considered. For example, if W_1 is the total load on span AB, the "free beam" shear at A is $\frac{W_1(L_1 - z_0)}{L_1}$ and at B is $-\frac{W_1 z_0}{L_1}$, where z_0 = distance from A to centre of gravity of the load. The restraint moment shear is constant throughout the span and $= -\frac{M_A - M_B}{L_1}$. The resultant shear at A is given by

$$V_A = \frac{W_1(L_1 - z_0)}{L_1} - \frac{M_A - M_B}{L_1}$$

and at B is $V_B = -\frac{W_1 z_0}{L_1} - \frac{M_A - M_B}{L_1}$.

When the known factors relative to load, span, moment of inertia, and relative levels of the supports are substituted in the appropriate general moment formula, an expression for three unknown moments is obtained for any pair of spans. That is, for n spans, $n - 1$ equations can be obtained containing $n + 1$ unknowns (the moments at $n + 1$ supports). The two excess unknowns represent the moments at the end supports, and if these moments are known or can be assumed the moments at the intermediate supports can be determined. Thus for a freely-supported end the moment would be zero, and the unknown representing the moment at a free support would automatically disappear. For a perfectly fixed end the moments can be determined if an additional span is considered continuous at the fixed end. This additional span should be identical in length, loading, etc., with the original end span except that the loading is so arranged as to produce symmetry about the original end support with the loading on the original end span. The moment at the new end support should be considered equal to that at the original penultimate support, and thus an additional equation is obtained without introducing a further unknown.

A numerical example will illustrate the application of the Three Moment

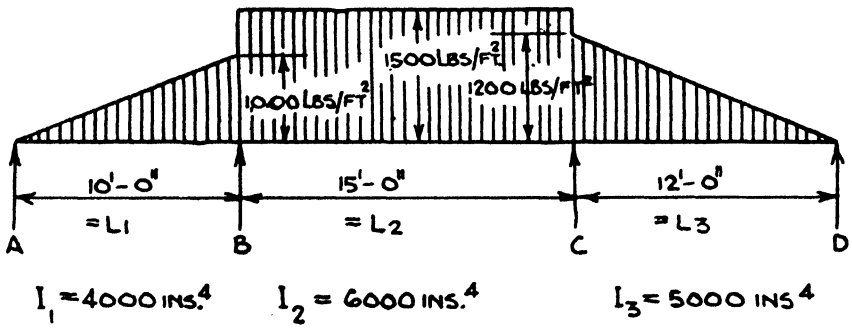


Fig. 9.—Continuous Beam Example.

Theorem in determining the support moments for the beam system indicated in *Fig.* 9. Assuming level supports, the appropriate formula for spans AB and BC is

$$M_A\frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_C L_2}{I_2} = -\left[\frac{A_1 z_1}{I_1 L_1} + \frac{A_2(L_2 - z_2)}{I_2 L_2}\right]6. \qquad (1)$$

and for spans BC and CD

$$M_B\frac{L_2}{I_2} + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + \frac{M_D L_3}{I_3} = -\left[\frac{A_2 z_2}{I_2 L_2} + \frac{A_3(L_3 - z_3)}{I_3 L_3}\right]6. \qquad (2)$$

For span AB, $\dfrac{L_1}{I_1} = \dfrac{10}{4000} = 0{\cdot}0025 \left(\dfrac{\text{ft.}}{\text{ins.}^4}\right)$

$$\frac{A_1}{L_1} = \frac{w_1 L_1^2}{24} = \frac{1000 \times 10^2}{24} = 4{,}170 \text{ ft.-lb.}$$

$$z_1 = \frac{8}{15}L_1; \text{ thus } \frac{z_1}{I_1} = \frac{8}{15} \times 0{\cdot}0025 = 0{\cdot}00133.$$

$$\text{Hence } \frac{A_1 z_1}{I_1 L_1} = 4170 \times 0{\cdot}00133 = 5{\cdot}55.$$

For span BC, $\frac{L_2}{I_2} = \frac{15}{6000} = 0{\cdot}0025$

$$\frac{A_2}{L_2} = \frac{w_2 L_2^2}{12} \text{ (from } \textit{Table} \text{ No. 7}D\text{)} = \frac{1500 \times 15^2}{12} = 28{,}125 \text{ ft.-lb.}$$

$$\frac{z_2}{I_2} = \frac{L_2}{2I_2} = \frac{L_2 - z_2}{I_2} = 0{\cdot}5 \times 0{\cdot}0025 = 0{\cdot}00125.$$

$$\text{Hence } \frac{A_2 z_2}{I_2 L_2} = \frac{A_2(L_2 - z_2)}{I_2 L_2} = 28{,}125 \times 0{\cdot}00125 = 35{\cdot}156.$$

For span CD, $\frac{L_3}{I_3} = \frac{12}{5000} = 0{\cdot}0024$

$$\frac{A_3}{L_3} = \frac{w_3 L_3^2}{24} = \frac{1200 \times 12^2}{24} = 7{,}200 \text{ ft.-lb.}$$

$$z_3 = \frac{7}{15} L_3; \text{ thus } \frac{L_3 - z_3}{I_3} = \frac{8}{15} \times 0{\cdot}0024 = 0{\cdot}00128.$$

$$\text{Hence } \frac{A_3(L_3 - z_3)}{I_3 L_3} = 7{,}200 \times 0{\cdot}00128 = 9{\cdot}21.$$

If the beam is simply supported at A and D, $M_A = M_D =$ zero, and substituting known values in (1) and (2), we obtain

$$2M_B(0{\cdot}0025 + 0{\cdot}0025) + 0{\cdot}0025M_C = -(5{\cdot}55 + 35{\cdot}16)6 \quad . \quad . \quad (3)$$
$$0{\cdot}0025M_B + 2M_C(0{\cdot}0025 + 0{\cdot}0024) = -(35{\cdot}16 + 9{\cdot}21)6 \quad . \quad . \quad (4)$$

Thus

$$0{\cdot}0100M_B + 0{\cdot}0025M_C = -244 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$0{\cdot}0025M_B + 0{\cdot}0098M_C = -266 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Multiplying (6) by $\frac{0{\cdot}0100M_B}{0{\cdot}0025M_B} = 4$

$$0{\cdot}0100M_B + 0{\cdot}0392M_C = -1064 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Subtracting (5) from (7)

$$0{\cdot}0367M_C = -820$$

Hence $M_C = -22{,}400$ ft.-lb.

By substituting in (5)

$$0{\cdot}0100M_B = -244 - 55 = -189.$$

Hence $M_B = -18{,}900$ ft.-lb.

When the negative (or positive) support moments have been calculated, the diagram of the continuity moments is combined with the diagram of "free moments" and the resulting bending moments on the beam system are obtained.

Bending Moment and Shear Force Coefficients—Equal Spans.—The value of the bending moments at the support and in the spans depends upon the incidence of live loading, and for equal spans or with spans approximately equal the disposition of live loading given in *Fig.* 10(*a*) gives the maximum positive bending moment at midspan, and that given in *Fig.* 10(*b*) results in the maximum negative bending moment at a support. On *Table* No. 8 are given

the values of the coefficients $\left(\frac{\text{moment}}{\text{total load and span}}\right)$ for the bending moments at the middle of each span and at each support for two, three, four, and five or more continuous equal spans carrying identical loading on each span, which is the usual disposition of the dead loading on a beam system. The coefficients for the maximum bending moments at midspan and support for any incidence of loading are also tabulated, the types of loading considered being a uniformly distributed load, a single point load at mid-span, and point loads at the two third-points of the span. These theoretical bending moments are sometimes adjusted to reduce the peak negative bending moments. The latter can be reduced by 15 per cent. if the numerical value of the reduction is added to the positive bending moments in the two adjacent spans. This adjustment is of great convenience when it is desired to reduce the inequality between the negative and positive bending moments, this being specially desirable in slab design where the same section is used at midspan and the supports.

The London By-laws and the Code of Practice allow incidences of live loadings slightly less severe than those given in *Fig.* 10, as when calculating the

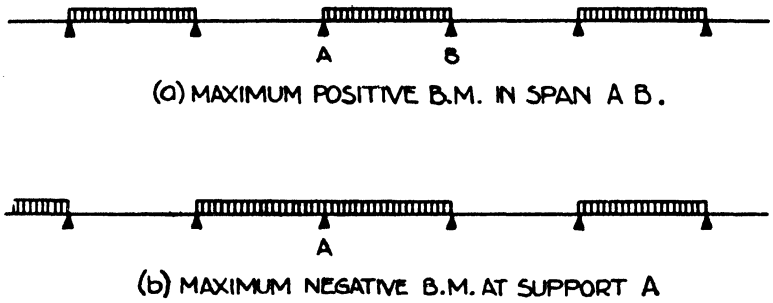


Fig. 10.—Incidence of Loading to Produce Maximum Moments.

maximum negative bending moments at any support only the spans immediately on either side of the support under consideration need be loaded. This only affects the coefficients for four or more continuous spans and the reduction is usually much less than 5 per cent. For a fully comprehensive series of tables giving the bending moment and shear force coefficients for dead and live loads and various ratios of dead and live loads, and allowing for various degrees of negative moment adjustment, the reader is referred to the author's " Practical Examples of Reinforced Concrete Design."

The shear forces produced by a uniformly distributed loading when all spans are loaded, together with the maximum shears due to any incidence of such loading, are also tabulated on *Table* No. 8.

Bending Moment Coefficients—Unequal Spans.—The factors on *Table* No. 9 have been prepared to facilitate the calculation of the support moments in beams with constant moment of inertia and continuous over two, three, or four equal or unequal spans, and carrying almost any type or incidence of live and dead loads. The basis of the method is that the loading on any span of

a given system can be divided into one or more of the types shown on *Table* No. 9. Other types of loading can be allowed for thus: Partially distributed load at one end of span gives coefficients between those for (*a*) and (*c*); Three or more equal point loads give coefficients between those for (*a*) and (*e*).

A given beam system can be divided into a series of identical systems each with only one span loaded with one type of load. Each of these loads produces a certain bending moment at each support, the total bending moment at any support being the algebraic sum of the moments due to each type of load. When the support moments are known, the mid-span moments can be calculated or determined graphically therefrom.

For equal spans the bending moment at any support due to any one span loaded with any one type of load is given by

$$M_S = \text{load factor } (F) \times \text{support moment coefficient } (Q) \times \text{total load } (W) \times \text{span } (L).$$

The total moment at any support $= \Sigma M_S$.

For unequal spans the method is similar to that for equal spans, except for the introduction of the "moment-multiplier," U, which varies with the support being treated and for the particular loaded span, and depends on the ratio that each span bears to the base span. Thus for unequal spans

$$M_S = UFQ \,.\, WL.$$

If one type of load extends equally over all spans of a system of equal spans, the loading need not be divided into separate span loads, as the values of Q tabulated for this condition apply.

Moment-Distribution applied to Continuous Beams.—The bending moments at supports of beams continuous over any number of equal or unequal spans, with any type of loading, and with different moment of inertia in each span, can be calculated directly by moment-distribution methods. Formulæ for particular supports in systems of two spans and upwards are given on *Table* No. 11A. The basic formula is that for any interior support C in four or more spans. This formula is based on two distributing and carry-over operations, and the results are within 5 per cent. of those obtained by more tedious solutions. All terms substituted in the tabulated formulæ are positive numerical values. If the resultant sign of the support bending moment is negative, tension in the top edge of the beam is indicated. The formulæ are applicable to all spans loaded, or to one or more spans only loaded.

Two conditions of end support are allowed for, namely, complete fixity and free support. For any other conditions, such as cantilevered beyond end support, or continuity with an exterior column, the support moments due to the loaded spans are first calculated assuming free support at the end support, and an adjustment made to the penultimate support bending moment (as indicated on *Table* No. 11A) for the externally-applied bending moment at the end support. The externally-applied bending moment is the cantilever bending moment in the case of a beam overhanging beyond an end support. If the beam is monolithic with an interior column, the externally-applied bending moment can be calculated from the appropriate formulæ given on *Table* No. 15 (p. 247). The bending moment at the end support is, of course, the externally-applied bending moment. The

general principles of moment-distribution are given on *Table* No. 16A (p. 248) but the difference in sign convention should be observed.

Continuous Beams with Varying Moments of Inertia.—The foregoing considerations have particular application to beams that have a constant moment of inertia throughout the span and, although strictly speaking the moment of inertia of a reinforced concrete beam of constant depth may vary not inconsiderably at various sections, it is usual to neglect these variations in practice both for beams having equal depth throughout and for beams having normally proportioned haunches at the supports.

Where the depth of a beam system varies beyond these limits, neglect of the variation of moment of inertia in the bending moment calculations leads to results that differ considerably from the probable moments. When the moment of inertia is practically constant throughout each span of a system of continuous beams, but differs in one span relative to another, the general formula given on pages 34 and 36 and on *Table* No. 11A for these conditions is applicable, but when the moment of inertia varies within the length of each span the semi-graphical method given on *Table* No. 10 should be employed.

If the moment of inertia varies throughout the span in such a way that it can be represented by a simple equation, the general Three Moment Theorem can be used if $\frac{M}{I}$ is substituted for M and if the area of the $\frac{M}{I}$ diagram is taken instead of the area of the free moment diagram. The solution of the derived simultaneous equations will then give values of $\frac{\text{support moment}}{I}$ enabling a complete $\frac{M}{I}$ diagram for the beam system to be constructed, for which the moment at any section can be readily obtained by multiplying the appropriate ordinate of the $\frac{M}{I}$ diagram by the moment of inertia at the section.

When time or other circumstances do not permit these methods to be followed, and the bending moments are calculated upon the assumption of constant moment of inertia, some adjustment can be made for the effect of the neglected factor if it is borne in mind that an increase in the moment of inertia in the vicinity of the support will cause an increase in the bending moment at that support and a consequent decrease in the positive bending moments in the adjacent spans, and vice versa. As a guide in making the adjustment the approximate factors given on *Table* No. 10, representing the percentage addition to or deduction from the calculated moments, can be employed.

Approximate Bending Moment Coefficients.—The precise determination of the theoretical moments in continuous beams involves much mathematical labour, except in certain cases that occur often enough to warrant tabulation. Having regard to the assumptions of unyielding knife-edge supports and probably constant moment of inertia that are involved in the theoretical elastic analysis, the probability of the mathematically obtained moments being greater or less than those actually realised should be considered. The effect of variation of moment of inertia has been discussed in the preceding paragraph, and of equal importance is the consideration of the characteristics of the support.

The following factors cause a decrease in the negative moment at a support :

(*a*) Yielding of the support considered relative to adjacent supports, and consequently an increase in the positive moment in the adjacent spans.

(*b*) Supports of considerable width.

(*c*) Support and beam monolithic.

Likewise sinking of one or both of the adjacent supports can cause an increase in the negative moment at a support and consequently a decrease in the positive moment in the adjacent span. Yielding of one support relative to the neighbouring ones may be sufficient to convert the moment at that support into a positive moment.

When the moments have been calculated with spans equal to the distance between centres of supports, the critical moment can be considered as that occurring at the edge of the support. When the supports are of considerable width the span can be considered as the clear distance between the supports plus the depth of beam, and an additional span introduced equal to the width of the support minus the depth of the beam. The loading on this additional intermediate span can be considered as the reaction on the support spread uniformly along the part of the beam over the support. When a beam is constructed monolithic with a very wide and massive support the continuity effect of the span or spans beyond the support may be negligible, in which case the beam should be treated as fixed at the support.

Other cases of beams being constructed monolithic with a support of moderate size, such as in normal beam and column construction, are dealt with in the following chapter, but in these cases the moment at the support section of the beam is never more than the moment calculated by the Three Moment Theorem.

The indeterminate nature of the actual moments produced leads in practice to the adoption of approximate bending moment coefficients for continuous beams of approximately equal spans and with uniformly distributed loads, and the commonly accepted coefficients are given on *Table* No. 10. These coefficients can be applied to the total load on the span, or can be used to determine the moments due to dead and live loading separately. In the table the coefficients have been extended to give the values for normal ratios of dead to live uniformly distributed loads. The use of the lower coefficients given in the column headed "Total Load" conforms to the requirements of the London By-laws.

Graphical Determination of Bending Moments.—A commonly adopted method of finding the distribution of bending moments on a continuous beam is by a graphical method based on the determination of "fixed points." The basis of the method, which is fully described on *Table* No. 11, is that there is a point (termed the "fixed point") adjacent to the left-hand support of any span of a continuous system at which the moment is unaffected by any alteration in the bending moment at the right-hand support. A similar point occurs near the right-hand support, the moment at this point being unaffected by alteration in the moment at the left-hand support. When a beam is rigidly fixed at a support the "fixed point" is one-third of the span from that support, and when freely supported the fixed point coincides with the support. For intermediate conditions of fixity the "fixed point" will be located between these extremes,

and the relative position of the left-hand (or right-hand) fixed points in two adjacent continuous spans (*Fig.* 11) is given by the expression

$$l_2 = \frac{L_2^2(L_1 - l_1)}{3(L_1 + L_2)(L_1 - l_1) - L_1^2}$$

or l_2 can be found from l_1 by a graphic construction.

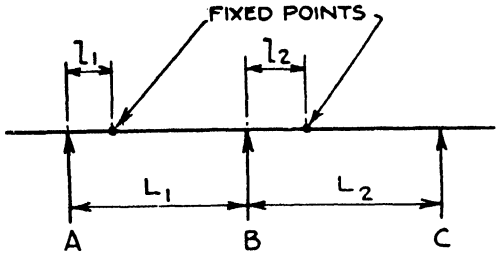


Fig. 11.—Fixed Points.

Upon combining the free moment diagram with the position of the fixed points for any given span, as described on *Table* No. 11, the resultant negative and positive moments throughout the beam system, due to this loaded span, can be determined, and by treating each span separately the envelopes of the maximum possible moments throughout the system can be readily developed.

Influence Lines.—Moving (or rolling) loads, such as vehicular loads, that can traverse a system of continuous spans and can be in any position thereon, are best treated by influence line methods. An influence line is usually a curve with the beam span as a base, the ordinates of the curve at any selected point giving the value of the bending moment produced at some particular section of the beam when a unit load is acting at the selected point. Influence lines can be drawn for any section of the beam, and sufficient data are given on *Tables* Nos. 12 and 13 to enable the influence lines for the critical sections of beams continuous over two, three, four, or more spans to be constructed. By plotting the relative position of the load on the beam (drawn to scale) the moments at the section being considered are derived as explained in the example given on the page facing *Table* No. 13. The supplementary *Tables* Nos. 12A, 13A, and 13B give curves which are more particularly directly applicable to equal spans, but upon which the curves for unequal spans are also plotted. The bending moment due to a load at any point is the ordinate of the influence line at that point multiplied by the product of the load and the short span. The influence lines are drawn for symmetrical inequality of spans and coefficients for span ratios not plotted can be interpolated. The symbols on each curve indicate the point and span ratios for which the curve is drawn.

End Restraint.—In the preceding discussion on continuous beams it has been assumed that the beam is freely supported on the end supports. If, as is common in column and beam construction, the end of the beam is subject to a restraining moment due to monolithic construction, the positive and negative moment and shear will be affected throughout the beam, but principally in the first two spans. *Table* No. 10A (p. 230) has been compiled to give coefficients

by which the bending moments and shears can be adjusted when unit bending moment is applied to one or both ends of the beam.

Mid-span Negative Bending Moments.—Rectangular and tee beams, and slabs such as bridge decks, where the ratio of live to dead load is high, should be designed for a possible negative bending moment at midspan equal to

$$M_{neg.} = \frac{L^2}{24} (kW_L - W_D) \text{ ft. lb.}$$

where W_L and W_D = intensity of live and dead load respectively (lb. per sq. ft.),

L = span (ft.), and

k = 0·67 for beams and 0·50 for slabs.

The limit of application of this formula is when adjacent spans vary by more than 20 per cent.

Slabs Spanning in One Direction.—The bending moments in slabs supported on two opposite sides are usually calculated in the same way as are beam moments, due account being taken of continuity where necessary. For slabs carrying uniformly distributed loads and continuous over approximately equal spans, the coefficients given on *Table* No. 10B conform to the London By-laws. Other coefficients, allowing for the effect of haunches on the bending moments, are also tabulated. The coefficients given on *Table* Nc. 10 for beams continuous over approximately equal spans with various ratios of live to dead load, can also be applied to slabs.

Spans can be considered equal if the difference between adjacent spans does not exceed 15 per cent. of the longer. Generally, the coefficients on *Table* No. 10B can be adopted for any symmetrical loading by making the adjustment indicated on the table.

Even when a slab is nominally freely supported at a support, it is advisable to provide resistance to a negative bending moment at the support of not less than $\frac{wL^2}{24}$. Although designed as spanning in one direction, a slab is also reinforced in a direction normal to the main reinforcement with distribution steel as described on page 159.

The critical design section for a slab built monolithic with its support is usually at the face of the support, where the negative bending moment is slightly less than the calculated bending moment at the centre of the support. The amount by which the bending moment (in. lb.) can be reduced is $0{\cdot}33sb$

where s = shear force (lb.) at edge of support, and

b = width of support (in.).

3.—Slabs Spanning in Two Directions.

Freely Supported on Four Edges.—When a slab panel is supported otherwise than on two opposite sides only, the precise amount and distribution of load taken by each support (and consequently the bending moment in the slab) is not readily determinable with assumptions resembling practical conditions. Various empirical formulæ and approximate theories have therefore been put forward with the object of calculating the bending moments induced in rectangular slabs.

The most commonly used in this country are the Grashof and Rankine formula and the Pigeaud theory, or coefficients closely resembling those obtained by application of these methods. For slabs uniformly loaded and freely supported on all four sides, the Grashof and Rankine formula is applicable, thus

if $K_B W = W_B$ = proportion of load carried in direction of short span (L_B); W = total load on slab.

$K_L W = W_L$ = proportion of load carried in direction of long span (L_L),
k = ratio of long span to short span.

$$K_B = \frac{k^4}{k^4 + 1} \text{ and } K_L = 1 - K_B.$$

The usual limit of application of this formula is when $k = 2$, i.e. when the length of the panel equals twice the width; beyond this limit the slab is considered as spanning across the short span only. In cases near the limit of k care should be taken to ensure that the amount of reinforcement in the long direction of the panel is not less than that required for the minimum amount of distribution steel (see p. 159).

The Pigeaud theory, which is given in English texts by Mr. W. L. Scott, is based on consideration of thin plates with Poisson's ratio as zero; it is unrestricted by the value of k, and is applicable to slabs nominally freely supported on all four sides if the corners are prevented from lifting.

On *Table* No. 14 the values of K_B and K_L for k varying from 1·0 to 3·0 are tabulated approximately in accordance with the two methods mentioned. In normal practice it is recommended that the Grashof and Rankine coefficients should be used for slabs freely supported on four sides, and the Pigeaud coefficients when the corners are prevented from lifting or if the slab is continuous over the supports on all four sides. This practice conforms to the rules and coefficients given in the Memorandum relating to the London By-laws.

Recommendations for the provision of corner reinforcement to resist the torsion occurring in the corners of the panel are given on *Table* No. 14, and such steel should be provided (normally by the reinforcement from adjacent panels) if the Pigeaud coefficients are used for calculating the principal bending moments.

The Pigeaud method is also applicable to nominal point loading and partially distributed loading, as explained on *Table* No 14.

Ratio of Spans for other Conditions of Support.—In deriving the methods just discussed, consideration is only given to slabs freely supported on four edges, and in practice the same subdivision of the loading is assumed when the slab is continuous or fixed along all four supports. The simplest way to allow for conditions of free support on some sides with continuity on other sides

is by substituting for $k = \frac{\text{long span}}{\text{short span}}$ the expression

$$k = \frac{\text{longer equivalent span}}{\text{shorter equivalent span}}$$

where the equivalent span is f times the actual span. Suggested values of f for various conditions of supports are given on *Table* No. 14 and vary from 0·67

for a span continuous over both supports to 3·0 for a cantilever, with unity for a span freely supported at both ends. This method approximates very closely to attempts to calculate theoretically the effect of continuity over some of the supports, and in its adoption it should be observed that the shorter actual span constitute "free" support and which can be considered as "continuous" is discussed on p. 46.

Bending Moment for any Conditions of Support.—When a slab carrying a uniformly distributed load and spanning in two directions is freely supported on all four sides the bending moment across the short span will be $\frac{W_B L_B}{8}$, and across the long span $\frac{W_L L_L}{8}$, where W_B and W_L are determined from the Grashof and Rankine formulæ and where L_B = short span and L_L = long span.

For a slab continuous over all four edges the lower coefficients given by the Pigeaud method are applicable, and the negative bending moment over the supports is then equal to $\frac{W_B L_B}{8}$ for the support normal to the short span and equal to $\frac{W_L L_L}{8}$ for the support normal to the long span. The midspan positive bending moment can be taken as 80 per cent. of these values. Thus the short span midspan moment is $\frac{W_B L_B}{10}$ and the long span midspan moment is $\frac{W_L L_L}{10}$, the loads W_B and W_L being proportioned from the Pigeaud coefficients.

When a slab is freely supported along one edge and fixed along the remaining three edges, the Pigeaud coefficients can be used combined with bending moment factors of $\frac{1}{8}$ for all negative moments over the supports, $\frac{1}{10}$ for the positive midspan moment for the span that is continuous at both ends, and $\frac{1}{9}$ for the positive midspan moment for the span that is continuous at one end and free at the other. The values of W_B and W_L would in this case depend on the ratio of the "equivalent" spans as described above.

When a slab is freely supported along two opposite edges and continuous along the remaining two opposite edges, the Pigeaud coefficients apply based on the appropriate "equivalent" span ratio. The bending moment factors in this case would be $\frac{1}{8}$ for the negative moments and for the positive moment in the span that is freely supported at both ends, and $\frac{1}{10}$ for the positive moment in the span that is continuous at both ends.

For all other cases the Grashof and Rankine coefficients should be adopted with the appropriate "equivalent" span ratio. With continuity along two adjacent sides and free support along the remaining two, the bending moment

factors can then be taken as $\frac{1}{10}$ for the positive and negative bending moments. With continuity along one side only and free support along the remaining three sides, the bending moment factors should be $\frac{1}{10}$ for the negative bending moment and for the positive midspan moment in the span continuous at one end and free at the other, and $\frac{1}{8}$ for the positive bending moment in the span that is free at both ends.

The curves on *Table* No. 14A (p. 243) have been prepared to enable the bending moment coefficient for any combination of supporting conditions to be read off conveniently. These coefficients include the combined effect of the equivalent span ratio, the appropriate series of load reduction factors, and the bending moment factors already described.

For the purpose of adopting the coefficients on *Table* No. 14A, a slab can be considered as continuous along any particular edge if it is continuous with another panel beyond the edge considered, or if it is monolithic with a heavy beam or other construction, as long as negative bending moment reinforcement is provided. Conditions of "free" support obtain when the edge is non-continuous and bears on a brick wall or on structural steelwork. If the edge of the slab is built well into a brick wall or is monolithic with concrete encasing steelwork, or a small beam, some restraint is obtained and can be allowed for by adopting the United States series of coefficients for these conditions as given on *Table* No. 14C.

Load on Supports.—The foregoing discussion on rectangular panels is concerned with the sub-division of the load for determining the bending moments in the slab. With the exception of the Grashof and Rankine method, W_B and W_L do not represent the loads carried by the supports parallel to the long and short axis respectively. In those cases where it is permissible for the Pigeaud coefficients to be adopted for the bending moment, the load carried by the various supports must be calculated independently. This can be done by taking for this purpose only Grashof and Rankine coefficients for the proportions in question, or more logically the total load should be divided into two parts, W_b, the load carried on the supports parallel to the equivalent longer sides, and W_l, that carried on the supports parallel to the equivalent shorter sides; and $W_l + W_b = W$; $W_b = \frac{W_B}{W_L + W_B} \cdot W$ and $W_l = \frac{W_L}{W_L + W_B} \cdot W$, where W_L and W_B are determined by the method adopted for the moment calculation. It is usually sufficient to consider that each parallel support takes an equal share of the load.

An alternative method is to consider that each of the beams across the short span carries the load from a 45 deg. isosceles triangle having the span of the beam as a base and a height equal to half the span. Half the remaining load (that is on a trapezoidal area) is carried by each of the longitudinal beams. In the case of a square slab each beam will carry one-quarter of the total load of the panel.

In the case of continuity along one edge and free support along the opposite

edge, the beam along the former will carry a greater load (up to five-eighths of the sum of the loads carried by the two edges considered) than the beam along the freely supported edge.

It should be noted that the loading on beams supporting panels spanning in two directions may not be uniform, but increase from zero at the ends to a maximum in the centre. Some allowance should be made for this variation in assessing the bending moment coefficients for the beam.

Triangularly Distributed Loading.—In the design of the walls of rectangular tanks, bunkers, and similar structures, triangularly distributed loadings on panels spanning in two directions often occur. In such cases the intensity of loading on the horizontal span is uniform at any given level, but on the vertical span the loading will vary from zero at the top support to a maximum at the bottom support. The formulæ at the bottom of *Table* No. 14B have been evolved to determine rational values for the maximum bending moments on a unit width vertical strip and on a unit width horizontal strip. Various conditions of end supports are allowed for; the factor K takes these conditions into account, together with the ratio of the horizontal to vertical spans of the panel. Values of k_1 and k_2 for use in the bending moment formulæ are also tabulated.

Other cases of loading, such as a load varying uniformly from a maximum at one support to a minimum at the other, can be dealt with by adding to the bending moment due to a triangularly distributed load and adding the bending moment for a uniformly distributed load.

Non-Rectangular Panels.—When a non-rectangular slab is supported on four edges and is of such proportions that reinforcement in two directions is suggested, the moments can be determined by the following approximate considerations.

For a trapezoidal slab, the size of an equivalent rectangle having the same area and ratio of sides as the mean axes of the given slab, should be computed and the bending moments calculated for this rectangle.

If the slab is triangular (or very nearly so) the inscribed circle should be drawn touching the three sides. If D = diameter of this circle, the bending moment in each of two directions at right angles to each other, assuming the slab is freely supported on all sides, can be taken as

$$M = \frac{wD^2}{16} \text{ per strip of unit width}$$

where w = the intensity of loading.

It is important that the reinforcement calculated by this expression should extend over the whole area of the triangle.

If the slab is fixed at one or more edges, suitable allowance should be made for the reduction of the positive moments and production of negative moments.

Polygonal Slabs.—When a slab is in the form of a regular polygon or approximates to a square slab, the size of a square slab equal in area to the given slab should be determined and the moments determined for such a square slab. Alternatively a regular polygonal panel can be treated as a circular slab, the diameter of which is the mean of the diameter of the inscribed and circumscribed circles.

Circular Slabs.—For a circular slab of diameter D ft. freely supported along the circumference and carrying a concentrated load W lb. symmetrically disposed about the centre, the total bending moment across a diameter that should be provided for in each of two directions at right-angles is given by

$$M = \frac{WD}{2\pi}\left(1 - \frac{2d}{3D}\right) \text{ ft.-lb.}$$

where d = the diameter (ft.) of the loaded area.

If the slab is fixed around the circumference the positive moment can be reduced to about two-thirds of this value if the load approximates to a point load, or to 80 per cent. if the load covers practically the whole of the slab. The negative moment provided for along the edges should be approximately equal to the positive bending moment at midspan.

The bending moment on a circular slab carrying a uniformly distributed load can be deduced from one or two considerations (see the author's " Practical Examples of Reinforced Concrete Design "). If the slab is simply supported around the circumference, the total bending moment across a diamater D can be conservatively taken as $\frac{wD^3}{24}$ ft.-lb., where w is the load per square foot and D is the diameter in feet. The average bending moment is therefore $\frac{wD^2}{24}$ ft.-lb. per foot width, but actually the moment varies from zero at the extremities of the diameter to a maximum in the centre, which can be safely assessed at double the average bending moment, that is $\frac{wD^2}{12}$ ft.-lb. per foot width. On this same basis, if the slab is " fixed " along the circumference, the maximum midspan bending moment can be taken as $\frac{wD^2}{24}$ and the support bending moment as $\frac{wD^2}{18}$ ft.-lb. per foot width. If partial fixity only occurs, a maximum bending moment of $\frac{wD^2}{20}$ at midspan and support can be taken.

The foregoing bending moments ignore the full effect of spanning in two directions and over-rate the ratio between the maximum and mean values. A mathematical analysis applied to a reinforced concrete slab leads to the following alternative methods of design for uniformly loaded circular slabs.

(1) Design for a maximum central midspan bending moment of $\frac{wD^2}{24}$ ft.-lb. per foot width if freely supported at the edges, and reduce the reinforcement or slab thickness towards the circumference. If partly fixed, design for a maximum midspan positive bending moment and support negative bending moment of $\frac{wD^2}{30}$ ft.-lb. per foot width ; the positive moment steel can be reduced towards the circumference.

(2) Design for an average midspan bending moment of $\frac{wD^2}{36}$ ft.-lb. per foot width if freely supported, and retain the same slab section and reinforcement throughout. If partly fixed at the edge, the support negative bending moment

and the midspan positive bending moment can be taken as $\frac{wD^2}{45}$ ft.-lb. per foot width.

The reinforcement required by these bending moments should be provided in two directions mutually at right angles.

4.—Flat Slabs.

Notation and Dimensions.—The design of beamless-slabs (" flat slabs " or " mushroom floors ") is based on empirical considerations. The formulæ and principles which follow, and are summarised on *Table* No. 42, are generally in accordance with the London By-laws. The design of this type of floor can either incorporate drop panels at the column heads (as in *Fig.* 12) or the slab can be of uniform thickness throughout as in *Fig.* 13.

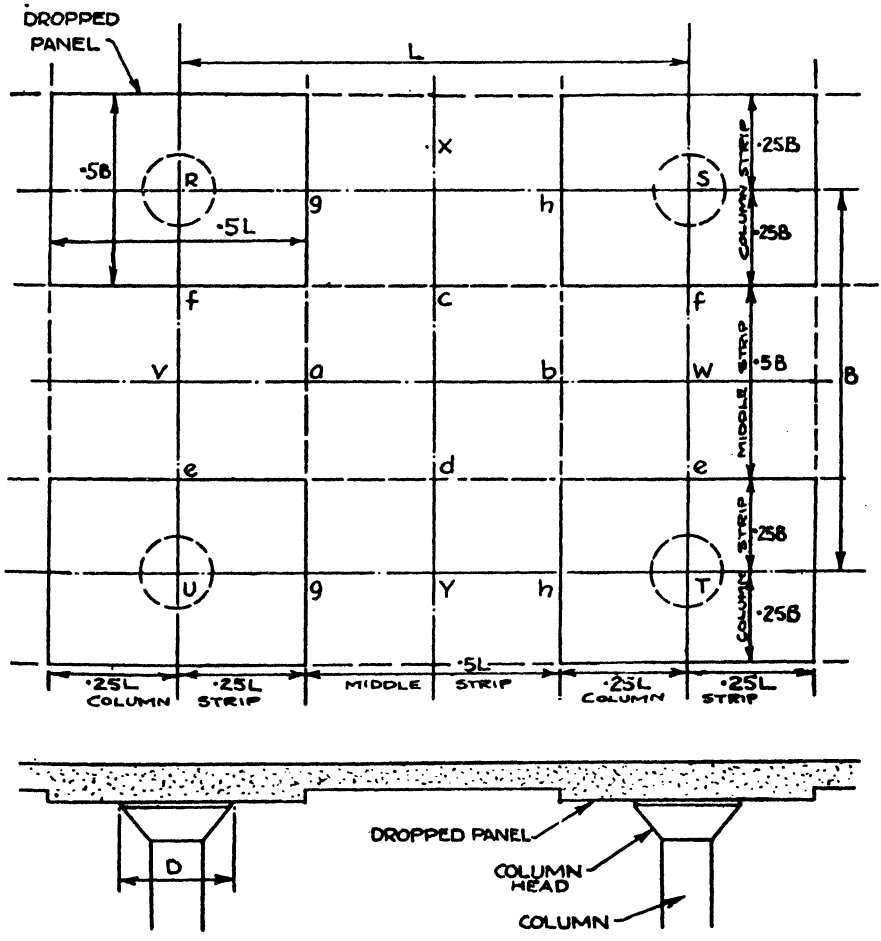


Fig 12.—Flat Slab with Dropped Panel.

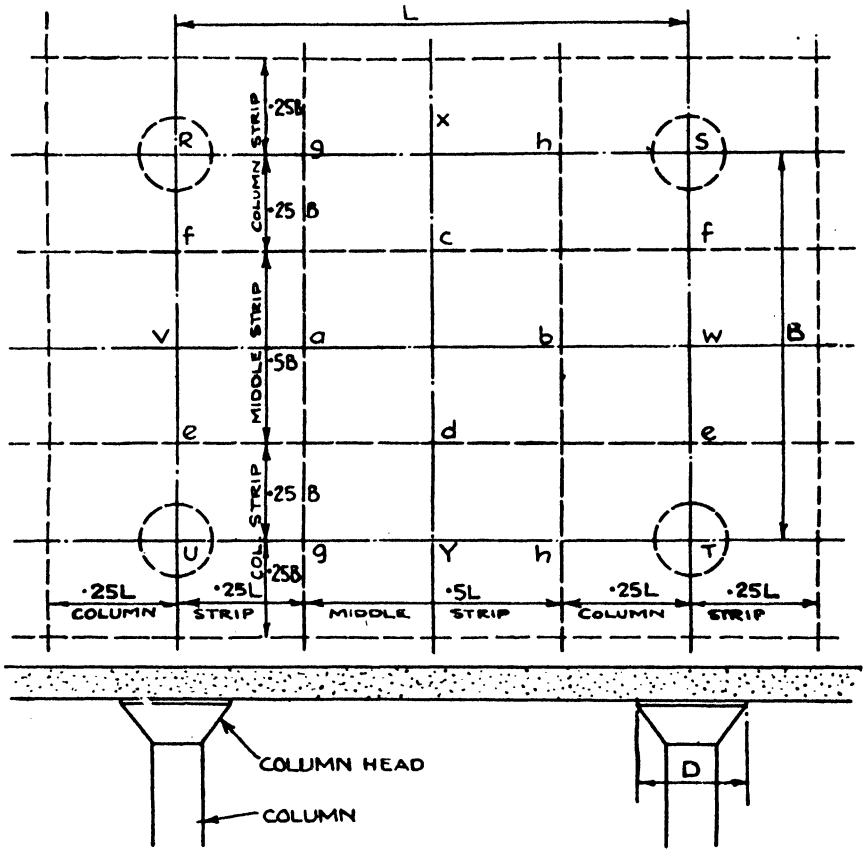


Fig. 13.—Flat Slab without Dropped Panel.

In the following formulæ it is assumed that all dimensions are in feet, and the notation is

L = length of panel side
B = breadth of panel side
w = intensity of loading (lb. per square foot)
D = diameter of column head
S_L = effective span in direction of L
S_B = effective span in direction of B.

The lengths (or widths) of adjacent panels should not differ by more than 10 per cent. of the greater length (or width). The ratio of $\frac{L}{B}$ should not exceed $1\frac{1}{3}$.

The length in any direction of the drop should not be greater than half (nor less than one-third) of the length of the panel in the same direction. The drops can, however, be continuous (that is, in the form of wide shallow beams).

For the purpose of determining the bending moments, the panel is divided

into "middle strips" and "column strips" (as shown in *Figs.* 12 and 13), the width of each strip being taken as equal to half the panel length or width. If drops of less width than half the panel length or width are adopted, the width of the column strip can be reduced to the width of the drop and the middle strip increased accordingly.

The thicknesses of the slab and drops should be such as are required by the shear force and bending moments.

Some regulations specify minimum thicknesses and ratios between the slab thickness and panel size, but no such limitations are given in the London By-laws.

Bending Moments.—For the purpose of moment determination the effective spans are given by

$$S_L = L - 0{\cdot}67D$$
$$S_B = B - 0{\cdot}67D.$$

The total bending moments in ft.-lb. to be provided for along the principal lines of the slabs are:

	Without drops.	With drops.
Along XY	$= M_X = +\ 0{\cdot}040wBS_L^2$	$+\ 0{\cdot}038wBS_L^2$
,, VW	$= M_V = +\ 0{\cdot}040wLS_B^2$	$+\ 0{\cdot}038wLS_B^2$
,, RS	$= M_S = -\ 0{\cdot}060wLS_B^2$	$-\ 0{\cdot}062wLS_B^2$
,, UT	As for RS.	
,, RU	$= M_U = -\ 0{\cdot}060wBS_L^2$	$-\ 0{\cdot}062wBS_L^2$
,, ST	As for RU.	

These moments should be provided for as follows:

Section.	With dropped panels.	Without dropped panels.
cd	42 per cent. of M_X	45 per cent. of M_X
Xc and *dY*	29 per cent. of M_X in each	27½ per cent. of M_X in each
ab	42 per cent. of M_V	45 per cent. of M_V
Va and *bW*	29 per cent. of M_V in each	27½ per cent. of M_V in each
fe	26 per cent. of M_U in each	30 per cent. of M_U in each
eU, *fR*, *Sf* and *eT*	37 per cent. of M_U in each	35 per cent. of M_U in each
gh	26 per cent. of M_S in each	30 per cent. of M_S in each
Rg, *hS*, *Ug*, and *hT*	37 per cent. of M_S in each	35 per cent. of M_S in each

If the width of the drop is less than half the panel width, the total bending moment in the middle strip should be increased in proportion to the increased width of the middle strip and the additional moment can be deducted from the specified total for the corresponding column strips. This adjustment ensures that the specified total bending moments along the principal axes are unaltered.

In end spans the positive moments should be increased by 25 per cent. All walls and other concentrated loads must be carried on beams, and beams should be provided to trim around openings other than small openings; most regulations limit the size of openings permissible in the column and middle strips.

Reinforcement.—It is usually most convenient for the reinforcement to be arranged in bands in two directions, one parallel to each of the spans L and B.

Alternatively the reinforcement can be arranged in four bands including two bands of diagonal bars. The area value of these diagonal bars is their area multiplied by the sine of the angle between the diagonal and section considered, that is, area of bars $\times \frac{B}{\sqrt{L^2 + B^2}}$ for a section parallel to L, and area of bars $\times \frac{L}{\sqrt{L^2 + B^2}}$ for a section parallel to B. Various regulations, which should be referred to, control the detail thoughout of bars in a four-way system. The amount of tensile reinforcement in any section should not exceed 1 per cent. of the product of the width of the section and the effective depth. Generally the slab thickness should be sufficient to render compression reinforcement unnecessary, except perhaps in end panels.

With two-way reinforcement, 40 per cent. of the bars in the positive moment reinforcement should remain in the bottom of the slab and extend over at least the middle three-quarters of the span, and no reduction of the positive moment steel should be made within the middle-half of the span. The negative moment reinforcement should extend to the quarter points in adjacent panels, and if the bars are staggered the shortest should extend to the fifth-point and the longest be arranged so that the average reaches the quarter point.

Shear.—The shearing stresses, which should not exceed the values given on *Table* No. 23 for conditions when no shear reinforcement is provided, should be investigated at two sections, namely, (1) around the edge of the drop and (2) around the head of the column. In the latter case the perimeter of the critical shear section can be taken as that of a circle of diameter equal to $D + (2 \times$ effective depth of slab or drop).

Columns.—The diameter of the column head, D, (see *Figs.* 12 and 13), should be between $\frac{L}{5}$ and $\frac{L}{4}$ for internal columns. The angle between the slope of the column head and the horizontal should be not less than 45 deg. Should the diameter of the column shaft be greater than $\frac{L}{5}$, no special head is required. External columns should be provided with as much as possible of the head prescribed for interior columns.

Columns should be designed for bending moments equal to 50 per cent., in the case of internal columns, (and 90 per cent. in the case of external columns) of the negative bending moment specified for the column strip. The bending moment should be divided between the upper and lower columns in proportion to their stiffnesses. If the floor panels are cantilevered beyond the outer row of columns, the column bending moment can be reduced by the cantilever bending moment due to the dead load.

An example of the design of a panel of a flat-slab floor is given on the pages following *Table* No. 40. When considering the economy of this type of construction for heavily loaded large span floors, note should be taken of the saving in overall height of a building when compared with beam and slab construction.

5.—Curved Beams.

Bow girders and beams that are not rectilinear in plan are subjected

to twisting moments in addition to the normal bending moments and shear forces. Coefficients for determining the maximum negative bending moments over the support of beams in the form of circular arcs, together with coefficients for the maximum twisting moment, are given on *Table* No. 7c (p. 224). This table is applicable to beams that form part of a complete circular system supported on columns, etc. that are equally spaced around the circumference of the beams. Such conditions occur in water towers, silos, and similar cylindrical structures, but the equivalent of these conditions also occurs if the circle is incomplete as long as the appropriate negative moment can be developed at the end supports. This type of circular beam occurs in such structures as semicircular balconies.

The maximum twisting moment occurs at a section intermediate between the support and the centre of the arc, and the position of this section, expressed in terms of the angular distance from the support is also given on *Table* No. 7c. The method of designing for twisting moments is given in Chapter XII and *Table* No. 34. Since the point of maximum twisting moment almost corresponds to the point of contraflexure, the design of the section in torsion can be based on the full section, thus assuming no cracking of the concrete due to tensile stresses caused by bending or shear.

The maximum shear, equal in value to half the total load on the beam span, occurs at the edge of the support.

Midway between the supports occurs the maximum positive bending moment, and it is preferable to design for positive moment in excess of the theoretical value, say, half that of the negative moment at the support.

The critical design sections are therefore: (*a*) Midspan—maximum positive bending moment and zero shear and twisting moment; (*b*) Point of contraflexure—zero bending moment, maximum negative twisting moment combined with shear force; (*c*) Support—maximum negative bending moment, maximum shear, and maximum positive twisting moment.

The comparatively small positive twisting moment at the support is ignored in some analyses, and when this is so the moment coefficients given for Case I (*Table* 7c) apply and are derived from the fundamental equations for any point X:

$$\text{Twisting moment} = WR\left[\sin\theta_X\left(\frac{1}{\theta} - \frac{1}{2}\cot\frac{\theta}{2}\right) - \frac{1}{2}(1 - \cos\theta_X) + \frac{1}{\theta}(\theta_X - \sin\theta_X)\right]$$

$$\text{Bending moment} = WR\left[\frac{\sin\theta_X}{2} - \cos\theta_X\left(\frac{1}{\theta} - \frac{1}{2}\cot\frac{\theta}{2}\right) - \left(\frac{2}{\theta}\cdot\sin^2\frac{\theta}{2}\right)\right].$$

The more rigid analysis of curved beams ignores neither the twisting moment at the support nor the torsional rigidity of the section, and results in

$$\text{Twisting moment} = -M_S\sin\theta_X + T_S\cos\theta_X + WR\left(\frac{1 - \cos\theta_X}{2} - \frac{\theta_X - \sin\theta_X}{\theta}\right),$$

$$\text{Bending moment} = M_S\cos\theta_X + T_S\sin\theta_X - WR\left(\frac{\sin\theta_X}{2} - \frac{1 - \cos\theta_X}{\theta}\right),$$

where M_S and T_S are the support bending and twisting moments respectively, values of which, together with other essential coefficients, are given in Case II (*Table* No. 7c).

CHAPTER V

FRAMED STRUCTURES

1.—Theoretical Considerations.

General Formulæ for Bending Moments.—The process of solving problems dealing with indeterminate framed structures is generally based on one of two fundamental principles:

(*a*) The Principle of Least Work, and
(*b*) The Slope-Deflection Theory.

The consideration that will here be given to this aspect of structural design will be based on the latter of these two principles.

The two axioms in the slope deflection method are:

(*a*) That the difference in slope between any two points in the length of a member subject to bending is equal to the area of the $\frac{M}{EI}$ diagram between these two points.

(*b*) That the distance of any point from a line drawn tangential to the elastic curve at any other point, the distance being measured normal to the initial position of the member, is equal to the moment (taken about the first point) of the $\frac{M}{EI}$ diagram between these two points.

In these considerations M represents the bending moment, E the elastic modulus of the material, and I the moment of inertia of the member. Combining these axioms leads to the following general expressions for the moments at the ends of a member subject to bending as shown in *Fig.* 14.

$$M_{AB} = 2EK\left(2\theta_A + \theta_B - \frac{3d}{L}\right) - P \quad . \quad . \quad . \quad . \quad (1)$$

$$M_{BA} = 2EK\left(2\theta_B + \theta_A - \frac{3d}{L}\right) + Q \quad . \quad . \quad . \quad . \quad (2)$$

where M_{AB} = bending moment at A towards B
M_{BA} = bending moment at B towards A
L, θ_A, θ_B, and d are shown in *Fig.* 14

$$K = \text{the stiffness factor} = \frac{I}{L}$$

$$P = \frac{2A}{L^2}(2L - 3Z) \text{ and}$$

$$Q = \frac{2A}{L^2}(3Z - L),$$

in which A = area of free bending moment diagram due to the external loads acting between A and B.

Z = distance of centre of area of the free moment diagram measured from A.

Formulæ for Special Cases.—When there is no external load the factors P and Q become zero, and when the load is symmetrically disposed on the member $Z = \frac{L}{2}$ and $P = Q = \frac{A}{L}$.

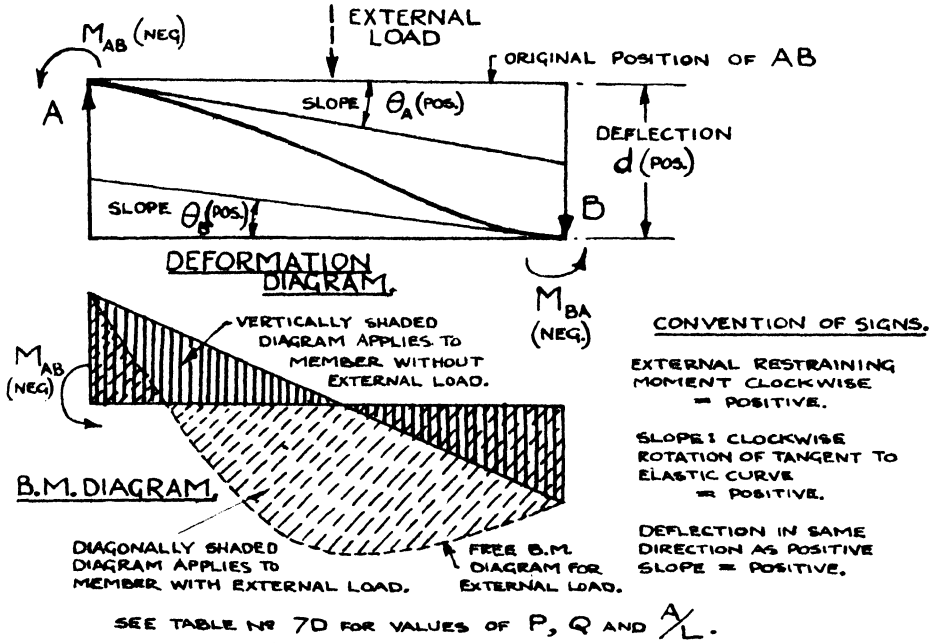


Fig. 14.—Frame Notation.

When there is no elastic deflection of one end of the member relative to the other (for example, when non-elastic supports are assumed), that is, when $d = o$, the general expressions (1) and (2) become

$$M_{AB} = 2EK(2\theta_A + \theta_B) - P \quad . \quad . \quad . \quad . \quad (1a)$$

$$M_{BA} = -2EK(2\theta_B + \theta_A) - Q \quad . \quad . \quad . \quad . \quad (2a)$$

Further simplifications are introduced when the end conditions of the member are known. Thus when the end B of the member AB is hinged, $M_{BA} = 0$, and the general expression becomes

$$M_{AB} = EK\left(3\theta_A - \frac{3d}{L}\right) - \left(P + \frac{Q}{2}\right). \quad . \quad . \quad . \quad (1b)$$

Similarly if the end A were hinged and not B, $M_{AB} = 0$, and the general expression becomes

$$M_{BA} = -EK\left(3\theta_B - \frac{3d}{L}\right) - \left(Q + \frac{P}{2}\right) \quad . \quad . \quad . \quad (2b)$$

Hence for conditions of one end hinged, level supports, and symmetrical loading, the formulæ become

For end B hinged, $M_{AB} = 3EK\theta_A - 1{\cdot}5\frac{A}{L}$ . . . . . (1c)

For end A hinged, $M_{BA} = 3EK\theta_B + 1{\cdot}5\frac{A}{L}$ . . . . . (2c)

When the restraint at end A is equivalent to perfect fixity, $\theta_A = 0$, or when the end B is fixed, $\theta_B = 0$, then

For end A fixed, $M_{AB} = 2EK\left(\theta_B - \frac{3d}{L}\right) - P$ . . . . (1d)

$$M_{BA} = 2EK\left(2\theta_B - \frac{3d}{L}\right) + Q \quad . \quad . \quad . \quad (2d)$$

Hence for conditions of one end fixed, level supports, and symmetrical loading, the formulæ become

For end A fixed, $M_{AB} = 2EK\theta_B - \frac{A}{L}$ . . . . . (1e)

$$M_{BA} = 4EK\theta_B + \frac{A}{L} \quad . \quad . \quad . \quad . \quad . \quad (2e)$$

For end B fixed, $M_{AB} = 4EK\theta_A - \frac{A}{L}$ . . . . . (1f)

$$M_{BA} = 2EK\theta_A + \frac{A}{L} \quad . \quad . \quad . \quad . \quad . \quad (2f)$$

The modified general formulæ for members having various conditions of end restraint are given on *Table* No. 15.

The general problem of determining the moments in a framed structure is solved by applying the appropriate general formulæ to each member successively. The algebraic sum of the moments at any joint is equal to zero.

As a simple illustration of the application of the formulæ, consider the problem of determining the bending moment at A when the beam AB, indicated in *Fig.* 16(*a*), is loaded.

From formula (1c), $M_{AB} = 3EK_B\theta_A - 1{\cdot}5\frac{A}{L}$

From formula (1f), $M_{AC} = 4EK_C\theta_A$

From formula (1f), $M_{AD} = 4EK_D\theta_A$.

By addition, $$0 = E\theta_A(3K_B + 4K_C + 4K_D) - 1{\cdot}5\frac{A}{L}$$

hence $$E\theta_A = \frac{1{\cdot}5\frac{A}{L}}{3K_B + 4K_C + 4K_D}.$$

By substitution, $$M_{AB} = \frac{6(K_D + K_C)}{3K_B + 4K_C + 4K_D} \cdot \frac{A}{L} \text{ (negative).}$$

The other moments could be found in a similar manner.

Apart from the consideration of continuity between columns and beams in a building skeleton, simple framed structures most commonly occur in the form of portal frames. On *Table* No. 16 the formulæ for the bending moments at the ends of the columns are given for such frames having any type of vertical loading on the top beam or any type of horizontal loading on the columns.

Shear Force.—The shear forces on any member forming part of a frame can be readily determined when the moments have been found by consideration of the rate of change of bending moment. The shear or reaction at the ends of a member AB that carries no external load is given by

$$F_{AB} = \frac{M_{AB} + M_{BA}}{L}.$$

For a member with end B hinged

$$F_{BA} = \frac{M_{AB}}{L}.$$

With external loads, the static shear forces due to these loads should be added algebraically.

2.—Bending on Columns.

External Columns.—The importance of computing and designing for the bending moments that are produced in monolithic beam and column construction due to the rigidity of the joints is now recognised by all modern authorities on reinforced concrete design, and most regulations to-day treat this aspect of design as an essential factor rather than as a refinement.

Exterior columns in a framed structure are subject to a greater bending moment than interior columns (other conditions being equal) and the magnitude of the moment depends on the relative stiffness of the column and beam and on the end conditions of the members. There are two principal cases to consider for exterior columns:

Case (i). Beam supported on topmost point of the column, as in *Fig.* 15.

Case (ii). Beam fixed to column at some intermediate point as in *Fig.* 16.

Since either end of the column or beam can be considered as hinged, fixed, or subject to some intermediate degree of restraint, it follows that there are nine individual conditions of Case (i) and twenty-seven conditions of Case (ii).

For Case (i) the maximum reverse moment at point A, the junction of the beam and column, occurs when the end B of the beam is hinged and the foot of the column is fixed at C, as in *Fig.* 15(*a*). The minimum reverse moments at A occur when the beam is rigidly fixed at B and the column is hinged at C as shown in *Fig.* 15(*b*). Conditions met with in practice lie between these two extremes, and with any condition of fixity of the column foot C the moment at A decreases as the degree of fixity at B increases. With any degree of fixity at the end B of the beam, the moment at A increases very slightly as the degree of fixity at C increases.

The maximum reverse moment in the beam at the junction with the column for Case (ii) is experienced when the beam is hinged at B and the column is perfectly fixed at both C and D as indicated in *Fig.* 16(*a*). With perfect fixity

at B and hinges at C and D, as in *Fig.* 16(*b*), the reverse moment in the beam at A is a minimum.

Intermediate cases of fixity follow the following rules:

Increase in fixity at B decreases the beam moment at A.

Decrease in fixity at either C or D decreases the beam moment at A and vice versa.

Values of the maximum and minimum bending moment in terms of the $\frac{A}{L}$

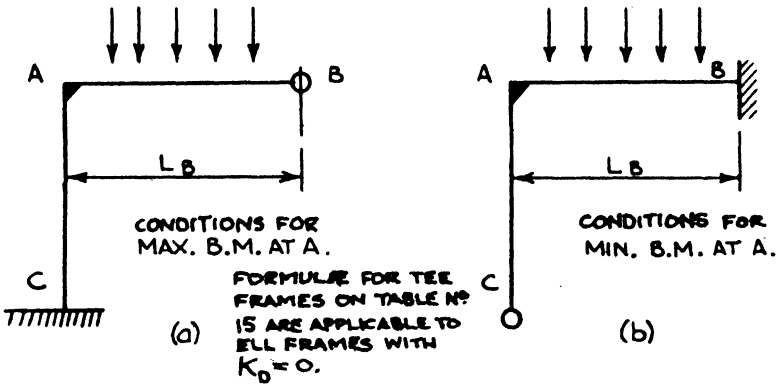


Fig. 15.—Ell Frames.

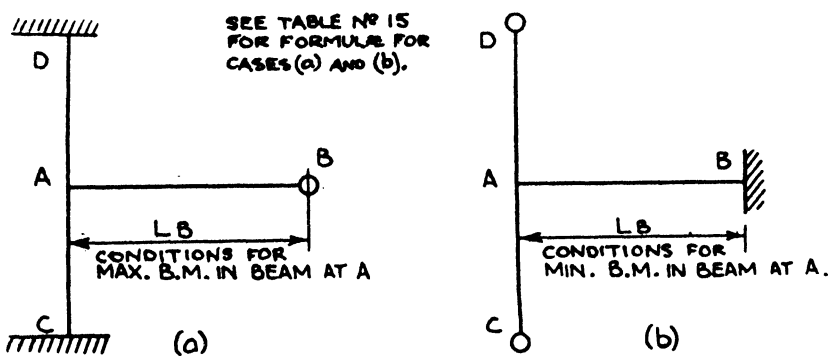


Fig. 16.—Tee Frames.

The beam moment at A is divided between the upper and lower columns in the ratio of their stiffness factors when their end conditions are identical. When one column is hinged and the other fixed the ratio of the two moments is in accordance with the expression

$$\frac{\text{Moment in hinged portion}}{\text{Moment in fixed portion}} = \frac{0{\cdot}75K \text{ for hinged portion}}{K \text{ for fixed portion}}.$$

External Columns in Building Frames.—For normal building frames the bending moments in exterior columns can be computed from the series of formulæ given on p. 247. These expressions, which conform to those given in the London By-laws Memorandum and the Code of Practice, neglect any non-uniformity in the end conditions of either the beams or columns, but are suitable for use in ordinary building structures.

When there is no upper column (that is, the structure is only one story high or the top story is being considered) as in *Fig.* 15, the factor K_D in these formulæ is zero.

Corner Columns.—An external corner column in a building is usually subject to bending moments from beams in two directions at right angles. These bending moments can be calculated by considering two frames (also at right angles) independently, but practical methods of computing the stresses depend upon the relative magnitude of the bending moments and the direct load (see Chapter XIV).

Interior Columns.—There is less variation in the moments due to continuity between beams and columns in the case of the interior columns of a building frame than in the case of exterior columns. End fixing conditions of the various members do not affect the moments so seriously, and it should be remembered that the moment in the beams at the column junction is in all cases less than the moment at this support when computed by the Three Moment Theorem neglecting fixity with the column. On *Table* No. 15 are tabulated bending moment coefficients from which the moment in the upper or lower column can be calculated by multiplying by the $\frac{A}{L}$ factor for the "live" load only on one of the adjacent beam spans, when the latter are equal. When unequal, the bending moment in the column can be calculated by considering two separate frames in each of which the column under consideration is an external column. The maximum difference between the bending moment on the column calculated for the effect of the beam on the left-hand side and that on the right-hand side separately, is the resultant bending moment on the column. This maximum difference is usually obtained when the longer beam is loaded with live and dead load, while the shorter beam is loaded with dead load only.

Approximate Treatment.—The methods hitherto enumerated for evaluating the bending moments in column and beam construction with rigid joints involve a fair amount of calculation, including that of the moment of inertia of the members concerned. In general practice, especially in preliminary schemes, it is not always possible to spare the time required to make these calculations, and therefore approximate methods are of value.

Since the top lift of all columns should be designed for bending in addition to direct load, and even when the beam has been designed as if freely supported at the column or as continuous over and independent of the column (as is usual), some moment should be taken on the column. Certain reinforced concrete regulations give reasonable values for these bending moments. When the base of the column is considered hinged, the moment to be taken at the top should be one-third of the maximum positive moment in the adjacent beam span. When the base of the column is fixed, the moment at the top should be taken as one-

third of the maximum positive moment in the adjacent beam span, and the moment at the base as one-sixth of this positive moment.

When the midspan moment is taken as $\frac{WL^2}{10}$ the bending moment at the top of the column would thus be $\frac{WL^2}{30}$ and $\frac{WL^2}{60}$ at the base of fixed columns. For midspan moments of $\frac{WL^2}{8}$ the corresponding moments would be $\frac{WL^2}{24}$ and $\frac{WL^2}{48}$.

For heavy columns and light beams, the effect of the bending moment will not be large, and in such cases the margin between the permissible compressive stress for direct compression and for bending combined with direct compression (see Chapter IX) is usually sufficient to enable the preliminary design of the column to be based on the direct load only.

In cases where this margin does not occur and in order to allow a margin for the bending stresses on the column, the column can be designed provisionally for the static reaction increased by the amounts shown on *Table* No. 15 for the appropriate arrangement of beams supported by the column.

Wind Moments on Columns.—Where conditions do not warrant a close analysis of the bending moments to which a column in a framework are subjected due to wind forces or similar horizontal thrusts, the following treatment is sufficiently accurate. In such structures as water towers, elevated bunkers, and silos, and the frames of tall buildings, the calculation of wind moments in the columns is necessary. For braced columns of equal cross section, the expressions shown on *Table* No. 15 and *Fig.* 17*a* give the bending moments, shear forces, and additional direct loads on both the columns and the braces due to the effect of a horizontal force at the head of the columns.

In general, the bending moment in the column is the shear on the column multiplied by half the distance between braced points, but if the columns are not continuous or braced at one end, the bending moment will have twice this value.

The bending moment in the brace at an external column will be the sum of the bending moments in the columns at the intersection with the brace. The shear on the brace is equal to the change of bending moment from one end of the brace to the other divided by the length of the brace. These shears and moments are additional to those created by the dead weight of the brace itself and any external loadings to which it may be subjected.

The general overturning moment on the frame will induce an additional direct load in the leeward column and a corresponding relief of load in the windward column, the maximum value of this direct load being approached at the foot of the column and being equal to the overturning moment (PH) divided by the distance between the columns.

The case illustrated in *Fig.* 17(*b*) is common in the design of bunkers and silos where a superstructure of considerable rigidity is carried on a series of comparatively short columns. If the columns are fixed at the base, the bending moment on a single column is given by

$$\text{B.M.} = \frac{Ph}{2N},$$

where N = the number of columns if all are of equal size.

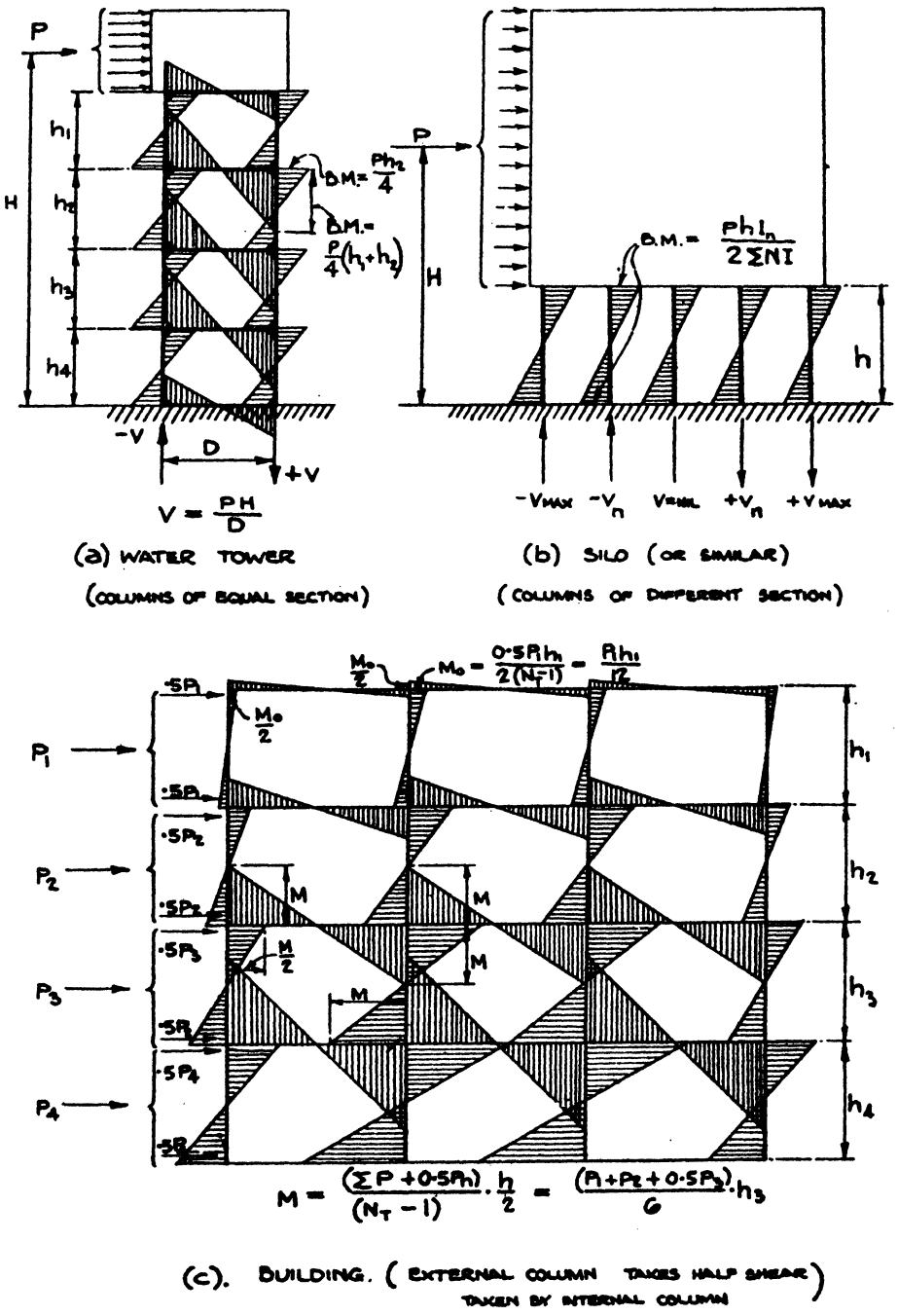


Fig. 17.—Wind Moments on Columns.

If the columns are of different cross sections, since all deform about equal amounts, the total shear should be divided among the columns in any one line in proportion to their separate moments of inertia. If N_1 = the number of columns with moment of inertia I_1, N_2 = number of columns with moment of inertia I_2, etc., the total moment of inertia $= N_1I_1 + N_2I_2 + \text{etc} = \Sigma NI$. On any column with moment of inertia $= I_n$, the bending moment will be given by

$$\text{B.M.}_n = \frac{PhI_n}{2\Sigma NI}.$$

The total horizontal shear is sometimes divided among the columns in the ratio of their cross-sectional areas (thus giving uniformity of shear stress) and with this method the formula for the bending moment in any column with cross-sectional area A_n becomes

$$\text{B.M.}_n = \frac{PhA_n}{2\Sigma NA_1},$$

where ΣNA_1 = total cross-sectional area of all the columns resisting the total shear P.

More often in building frames, where the wind effect is small compared with other stress-producing factors, the horizontal shear is simply divided up on the basis that external columns take half the shear on internal columns. If N_T is the total number of columns in one frame, $N = N_T - 1$ for the purpose of finding the bending moment on the interior columns, thus assuming that the two outer columns are equivalent to a single interior column.

On a normal building frame subject to wind pressure, the pressure on each panel (or story height) P_1, P_2, P_3, etc. (*Fig.* 17(*c*)) is usually subdivided into equal shears at the head and base of each lift of columns. The bending moment at the base of any interior column, n stories from the top, can be found from the expression

$$\text{B.M.} = \frac{\left(\Sigma P + \frac{P_n}{2}\right)h_n}{2(N_T - 1)},$$

where $\Sigma P = P_1 + P_2 + P_3 + \ldots + P_{n-1}$.

A bending moment and corresponding shears are induced in the floor beams in the same way as in the braces of a water tower, except that at an internal column the sum of the moments in the two beams meeting at the column is equal to the sum of the moments at the base of the upper column and at the head of the lower column.

3.—Frame Construction.

Earthquake-Resisting Structures.—The design of structures to withstand the disruptive forces induced by earth tremors and quakes presents a problem in frame design. Authoritative opinions differ on the question whether such structures should be designed as rigid or flexible structures. A semi-flexible type has been advocated in the United States, but generally engineers seem to favour the rigid-frame construction. The effect of earth tremors is considered as being equivalent to a horizontal thrust that is additional to the loads and wind pressures

for which the building would ordinarily be designed. The usual dead and live loads should be increased by 20 per cent. to allow for vertical movement. The magnitude of the horizontal thrust would depend on the probable acceleration of the earth tremor to which the building is likely to be subjected, and the value of this acceleration varies from less than 3 feet per second per second in firm compact ground to over 12 feet per second per second in alluvial soils and filling. A horizontal thrust equal to about one-tenth the mass of the building would seem to be adequate for all but major shocks when the building does not exceed 20 ft. in height, increasing to one-eighth of the mass when the building is of greater height. The horizontal shear on the building at any level should be considered as one-eighth or one-tenth of the total weight of the structure (including live loads) above this level. The analysis of the bending moments and shears in the columns and floor beams is similar to that described for wind pressure on building frames.

In order that the structure shall act as a unit, all parts of the building should be effectively bonded together. Panel walls, finishes, and ornaments should be permanently attached to the framework, so that in the event of a shock they will not collapse independently of the main structure. Isolated column footings should be connected by ties designed to take a thrust or a pull equal to one-tenth of the load on the heavier of the two footings connected.

End Conditions.—Since the results given by the more accurate frame formulæ vary considerably with different degrees of fixity of the ends of the component members, it is essential that the end conditions assumed should be reasonably obtained in the actual construction. Absolute fixity is difficult to attain unless the beam or column is embedded monolithically in a comparatively huge mass of concrete. Embedment in a brick or masonry wall is more nearly of the nature of a hinge, and should be assumed as such. The standard type of isolated footing, designed for the limiting uniform ground pressure under the direct load only, should also be considered as a hinge at the foot of the column. A continuous beam supported on a beam or column is only partly fixed, and where an end span is supported at the outer end on another beam hinged conditions should be assumed. A column built out of a pile cap supported by two, three, or four piles is not absolutely fixed but a bending moment can be developed if the resulting reactions (thrust and pull) can be taken on the piles. A column can be considered as fixed if it is built out of a thick raft whether the raft is supported on piles or not, as long as the necessary restraining moment can be developed in the raft slab or beams.

Hinges.—In special structures such as arches, large-capacity bunkers, etc., where it is necessary that the assumption of a hinged joint should be fully maintained, it is generally worth while and always safer to make a definite hinge in the construction. This can be done either by inserting an appropriately designed all-steel hinge, or by forming a hinge monolithic with the column as shown in *Fig.* 18. The hinge bars " a " should be calculated to take the whole of the horizontal shear at this section, and the area of concrete at " D " should be sufficient to transfer all the compressive force from the upper to the lower part of the column. The hinge bars should be well bound together by suitable binders " d " and the main column bars " e " should terminate on each side of the slots B and C. These slots should be filled with felt, paper, bitumen or a

similar separating layer, but sometimes the slots are omitted and the concrete is allowed to crack when moments producing stresses in excess of the ultimate tensile strength of the concrete occur.

Moments of Inertia of Reinforced Concrete Sections.—The moment of inertia of a reinforced concrete section is theoretically the equivalent moment of inertia of the stressed section about the neutral axis expressed in concrete or in steel units. The concrete in tension is neglected and the steel is considered as being equivalent to m times its area of concrete (see notes on the modular ratio in Chapter IX). Until the complete section is designed (or assumed) and the percentage of steel known, such a calculation cannot be made with any precision. Moreover, the moment of inertia of an ordinary beam changes con-

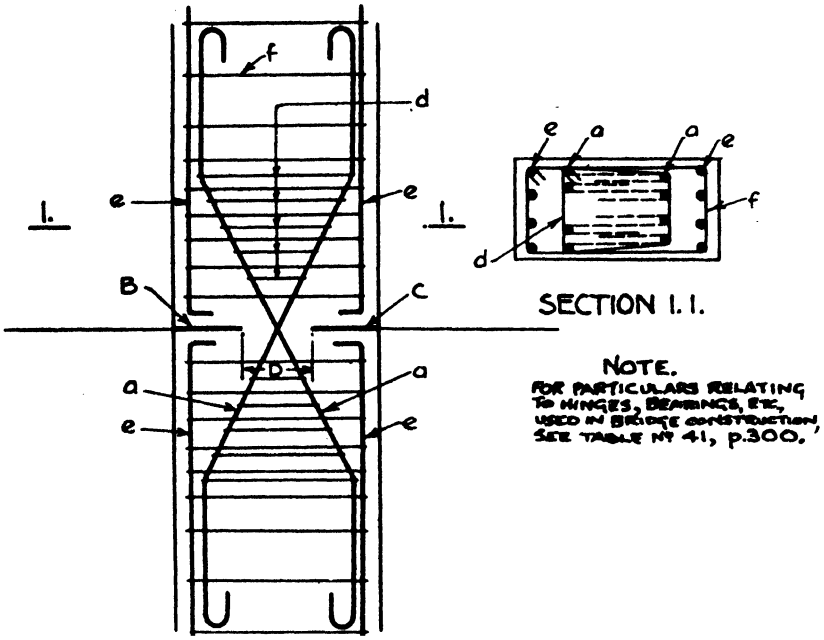


Fig. 18.—Typical Detail of Hinge.

siderably throughout its length, especially with a flanged beam which acts as a tee beam at midspan but has to be treated as a simple rectangular beam at its ends where reverse moments occur. Consideration should also be given to the question whether the probable stresses are sufficiently high to cause complete cracking. This applies particularly to tee and ell-beams when the flanges are in tension. Although the beam will be designed on the assumption that the concrete has cracked and that the reinforcement takes all the tension due to bending, cracking may not take place owing to the comparatively large area of concrete in the flange.

It is important that the method of assessing the moment of inertia should be the same for all members in a single calculation.

Since only comparisons of moments of inertia are required in frame design,

the errors due to approximations are of minor importance; it is therefore usually sufficient to compare the moments of inertia of the concrete sections alone for members that have somewhat similar percentages of steel. Thus for comparing a rectangular column with a rectangular beam the ratio of the moments of inertia of the whole concrete section only is

$$\frac{I_C}{I_B} = \frac{b_c d_c^3}{b_b d_b^3},$$

where I_c = moment of inertia of the column
b_c and d_c = breadth and width of column
I_B = moment of inertia of the beam, and
b_b and d_b = breadth and depth of beam.

On *Table* No. 39 are given values of I for octagonal, diamond-shape, and other non-rectangular sections, together with factors and formulæ for determining the moment of inertia of tee-beams (see example on p. 296, facing *Table* No. 39). The breadth of the flange assumed for the purpose of calculating the moment of inertia should not exceed the maximum permissible width given on *Table* No. 32.

The effect of the alternative methods can be seen by considering the following examples applicable to the sections illustrated in *Fig.* 19. The modular ratio is assumed to be 15; i.e., $m - 1 = 14$.

(*a*) To compare the moments of inertia of the rectangular beam shown in *Fig.* 19(*a*) with the square column in *Fig.* 19(*b*):

(i) Concrete section only:

$$\frac{I_C}{I_B} = \frac{15^4}{8 \times 18^3} = 1{\cdot}08.$$

(ii) Stressed Area:

Column:	Concrete	$= 0{\cdot}083 \times 15^4$	$= 4{,}210$	concrete units
	Steel	$= 14 \times 3{\cdot}14 \times 5{\cdot}5^2$	$= 1{,}330$	,, ,,
		$I_C =$	$5{,}540$	,, ,,
Beam:	Concrete	$= 0{\cdot}33 \times 8 \times 8^3$	$= 1{,}360$	concrete units
	Compression steel	$= 14 \times 1{\cdot}33 \times 6{\cdot}5^2$	$= 790$	,, ,,
	Tension ,,	$= 15 \times 2{\cdot}36 \times 8{\cdot}5^2$	$= 2{,}550$	,, ,,
		$I_B =$	$4{,}700$	,, ,,

$$\frac{I_C}{I_B} = \frac{5{,}540}{4{,}700} = 1{\cdot}18 \text{ as compared with } 1{\cdot}08.$$

(*b*) To compare the rectangular beam in *Fig.* 19(*a*) with the octagonal column in *Fig.* 19(*c*):

(i) Concrete Section only:

From *Table* No. 39 the M. of I. of an octagonal section $= 0{\cdot}055D^4$.

$$\therefore \frac{I_C}{I_B} = \frac{0{\cdot}055 \times 18^4}{0{\cdot}083 \times 8 \times 18^3} = 1{\cdot}49.$$

(ii) Stressed Area:

I_C by calculation similar to (*a*) = 6,780 concrete units

I_B as already calculated = 4,700 ,, ,,

$$\frac{I_C}{I_B} = \frac{6{,}780}{4{,}700} = 1{\cdot}44 \text{ as compared with } 1{\cdot}49.$$

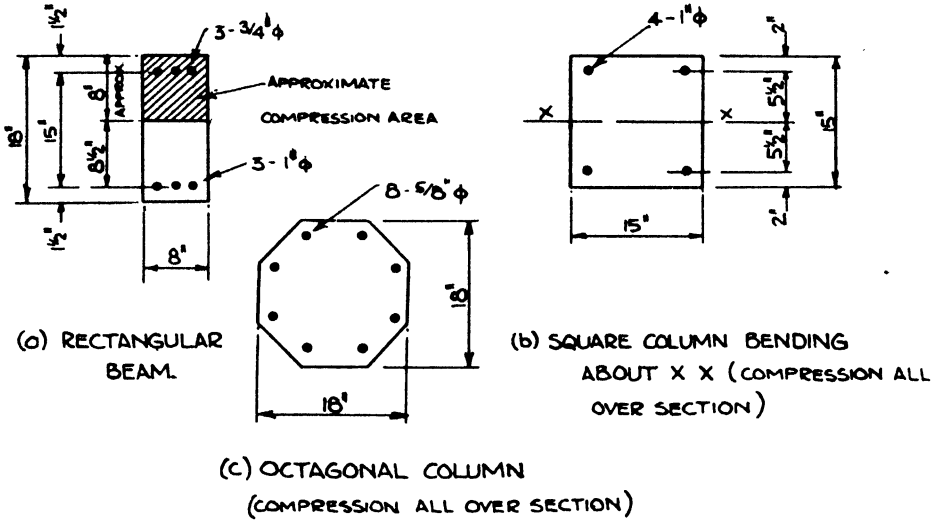


Fig. 19.—Typical Sections for Comparison of Moments of Inertia.

From these examples it will be seen that the approximate method readily gives comparative values that are accurate enough for trial calculations and close enough for general design. Another useful approximation that can be employed when the position of the neutral axis is known or can be estimated, is that the moment of inertia of a section subject to tensile and compressive stresses simultaneously is equal to twice the second moment of the tension steel area about the neutral axis. Applying this to the beam in *Fig.* 19(*a*),

$$I_B = 2 \times 15 \times 2{\cdot}36 \times 8{\cdot}5^2 = 5{,}100 \text{ concrete units,}$$

compared with 4,700 concrete units by the alternative calculation.

4.—Frame Analysis by Moment-Distribution.

The procedure of frame analysis by moment-distribution methods is described, with a worked example, on *Table* No. 16A (p. 248). Similar principles are applied to the solution of continuous beams on *Table* No. 11A (p. 232), but the difference in the convention of signs adopted in the two cases should be observed.

CHAPTER VI

FOUNDATIONS

1.—Loading on Ground.

Design.—The complete design of a foundation involves a three-stage procedure. The first stage is to determine from an inspection of the site the nature of the ground, and, having chosen which of the available strata it is proposed to subject to loading, the maximum safe bearing pressure must be decided. The second stage is to select a suitable type of foundation, whether it shall be an isolated footing, a raft, or piles, and consideration may have to be given to alternative types. To design the selected type adequately to transfer and distribute the loads from the superstructure on to the ground constitutes the third stage.

Site Inspection.—Obviously the first step in a site investigation is to undertake a surface inspection and to determine from the appearance of the site itself or from the immediate vicinity whether the subsoil is rock, chalk, clay, gravel, sand, alluvial soil, or filling. Such a superficial inspection tells the engineer nothing about the thickness of the apparent strata, although in this first instance he can perhaps draw on his own or others' local experience, and with extreme caution can base his deduction on probabilities.

For more precise determination of the type of ground underlying the site he must resort to digging trial holes, sinking bores, or driving test piles. A trial hole can only be carried down to a moderate depth, but affords an excellent opportunity of studying the nature of the soil in its undisturbed form, of determining the difficulty or otherwise of excavation, and the need or otherwise of timbering and pumping. The chief objection to a trial hole is the localised nature of the inspection, but this can be overcome by sinking a number of holes at salient points.

A bore can be carried down very much deeper than a trial hole, but still has the objection that it is a localised investigation. It is a very good method of determining the geological formation below a site, especially when the successive strata are of greatly differing nature giving a sharply contrasted " throw-up." With alluvia, clays, and other recent sedimentary depositions, however, care must be taken with the deductions made from the material brought up by the bore, since some such materials may be quite compact in bulk and firm when buried, thus affording an excellent foundation material, but when brought up may be churned into mud and become friable upon exposure to air and sun.

A test pile does not definitely indicate the kind of soil it has been driven through, but the driving data combined with local information may give a clue. A test pile is particularly valuable in determining the thickness of top crusts or the depth below poorer soil at which good bearing strata lie.

Determination of the strata underlying a site is succeeded by consideration of their suitability for foundations. The development of soil mechanics theories has resulted in a more satisfactory (although more complex) approach to ground loading problems, in contrast with the conventional (and sometimes erroneous) methods. In all but the most straightforward cases, a correct assessment and solution of a given foundation problem should be entrusted to engineers having the experience and experimental means to conduct the necessary preliminary investigations. Detailed consideration of soil mechanics methods cannot adequately be dealt with in the scope of the present volume, and the reader is referred to the extensive available literature on this subject.

Among the matters relating to foundations that the theories of soil mechanics explain and anticipate (and therefore guard against) are settlement of footings, rafts, and groups of piles, consolidation of clays, and settlements due to withdrawal of ground water from sand strata underlying clays. The principal factors to be investigated regarding a particular problem are the distribution of vertical pressures and shear stresses. One of the conclusions of these theories is that the depth to which knowledge of sub-strata should be obtained by bores (or otherwise) should be not less than 1½ times the width of an isolated base nor less than 1½ times the width of a structure with closely spaced footings. This rather extensive departure from the conventional methods is due to the fact that at these depths the intensity of vertical pressure may still be some 20 per cent. of the pressure applied at the foundation level. Thus if within this depth is found a stratum softer than that upon which the structure is founded, the pressure on such stratum may be excessive and result in settlement although the bearing pressure on the overlying, but superior, ground at foundation level may be considered reasonable.

The more important factors to consider relative to different classes of ground are enumerated below.

Igneous rocks such as granite, sedimentary rocks such as sandstone, limestone, and chalk, none of which suffers volumetric change with variation in saturation, obviously constitute the best foundation media provided that they are in solid formation of ample thickness and not partially decomposed. Inspection should be made for faults, splits, inclined strata with intervening thin clay bands, and the incidence of pockets of soft material.

For sands and gravels, information concerning the thickness of the deposit, the level of the water-table, the degree of clay impregnation, and the nature and thickness of underlying strata is essential. If the deposit is waterlogged, investigation must be made into the question of whether the proposed works or probable adjacent construction is likely to drain away some of the contained water, carrying with it the finer sand particles and leading to settlement due to consolidation. If such action is likely, precautions, such as isolating the site with sheet piling, are advisable, especially in the case of sands.

Clays, shales, and particularly boulder clay, constitute grounds that require special consideration of each case. When below overburden of considerable thickness, and where permanently waterlogged, satisfactory foundation strata, especially for piles, are provided. Clay strata near the surface are liable to periodical drying and wetting and precautions must be taken to ensure stable foundations. The thickness and profile of the strata, the physical nature of the

material, not only of the selected foundation stratum but of those immediately below, are among the essential data required for clay and clayey soils.

Alluvial soils and artificial filling also present a serious problem. The former should be investigated as fully as clay. The primary factors concerning an artificial filling include age, compactness, and depth. Normally, passage of time renders a fill more compact, although the time factor may also make it less secure, since the decay of buried timber, paper, or vegetation may leave cavities or soft patches. The discovery of such isolated weaknesses is not assured by bores or trial holes, as a trial at each foundation cannot usually be entertained. More confidence can be gained if a pointed steel bar is driven into the soil at every base, or in several places under a large base. Any variation in the resistance to driving will provide valuable information relating to the hidden stable.

Without any externally imposed load, clay fillings, or fillings in which the clay content preponderates, exhibit progressive settlements over a number of years. Similarly chalk fills subject to periodical changes of water-table will continue to settle over a long period. Foundations proposed on such fillings must therefore be designed for such movements, and usually piles are necessary to transfer the loading to a lower and more stable stratum.

Safe Bearing Pressures.—The maximum bearing pressures that can be imposed on normal soils are in some districts subject to regulation. *Table* No. 17 gives generally accepted values, together with those recommended as a guide in the Code of Practice and the London By-laws. Such bearing pressures must be considered as maxima and should be treated with reserve since the numerous primary factors already described may necessitate using lower values. For soils of uncertain bearing resistance, a study of existing buildings on identical soils may be instructive. Bearing tests may also be of restricted value.

Permissible pressures can be exceeded by an amount equal to the weight of earth between the foundation level and adjacent ground level. Any earth carried on the base must be included in the foundation load.

Eccentric Loading.—When a foundation is subjected to a concentric load —when the centre of gravity of the superimposed load coincides with the centroid of the base—the intensity of bearing pressure to which the ground is subjected is equal to the total applied load divided by the total area. When the loading is eccentrically placed on the base the pressure will not be uniformly distributed but will vary uniformly from a maximum at one edge to a minimum at the opposite edge, or to zero at some intermediate point. This variation of pressure depends on the magnitude of the eccentricity, and values of the maximum and minimum pressures are given by the formulæ on *Table* No. 17.

Depth of Foundations.—To prevent the ground spewing out from under the foundation when pressure is exerted, it is essential that the lateral pressure due to the head of earth outside the foundation should be more than the lateral pressure due to the bearing pressure. This principle will dictate the minimum depth below the surface to which the foundations should be taken, and a formula for ascertaining this depth is given on *Table* No. 17.

Unless foundations are on rock, footings should be founded at a level at least 2 ft. below the ground surface, since apart from considerations of spewing it is very seldom that virgin soil or a satisfactory consolidation is reached in less depth. In clayey soils depths of 5 ft. and upwards are necessary to ensure

protection of the bearing stratum from the effects of weathering. In the case of footings in which there is no mass concrete at the base forming an integral part of the base, the bottom of the excavation should be covered with a lean concrete in order to provide a clean surface upon which to place the reinforcement. The thickness of this layer depends upon the compactness and wetness of the bottom of the excavation, and would generally be from 1 in. to 3 in. thick.

2.—Types of Foundations.

Independent Bases.—The type of foundation most suitable for any given structure depends primarily upon the relation between the load and the safe bearing pressure. The most simple type of column foundation is an independent base in one of the forms (*a*), (*b*), or (*c*) illustrated on *Table* No. 18. Such types are most suitable for ground having upwards of 1½ tons per square foot safe bearing value, and for bases of small area the form (*a*) with a mass concrete sub-base is the most suitable. This type of base is also best adopted for foundations on rock, the mass concrete sub-base being formed on a level bed roughly hewn out of the rock. In designing independent bases for the columns of a building founded on relatively compressible soils, to avoid in part the risk of local settlement the relative sizes of the bases should be strictly in proportion to the dead load carried by the corresponding columns, and obviously the ground pressure under any base due to the combined dead and live load, together with any bending on the base of the superimposed column, should not exceed the safe bearing pressure of the ground.

When the columns in a structure are at fairly close centres in any one direction it is usual to link up the bases to form a continuous strip footing, a similar form of footing being used for wall foundations.

Rafts.—When the columns or other supports of a superstructure are at close centres in all directions, or when the column loads are so high and the safe ground pressure so low that the extent of independent bases almost or totally covers the space between the columns, the bases are linked up one with another in both directions and form a single raft foundation. The magnitude of the loading and the spacing of the columns determine the shear and bending moments that will in turn determine the thickness of the raft. If this thickness does not exceed 12 in. a solid slab (*Fig.* 20(*a*)) is usually the most convenient and economical form ; if this thickness must be exceeded a beam and slab construction designed as an inverted floor (*Fig.* 20(*b*)) is more satisfactory. In cases where the total depth is of the order of 5 ft. or more, a cellular construction consisting of a top and bottom slab with intermediate ribs gives the most economical design for rafts of large extent (*Fig.* 20(*c*)).

Care should be taken in the preparation of the ground upon which a raft is to be built, to ensure uniformity of bearing capacity. Weak places and hollows should be filled in with hardcore or with the same material as the ground itself, this filling being placed in thin layers well watered and rolled. Any patches of gound harder than the general nature of the site, such as rock outcrops, old walls, etc., should be cut away for a depth of 2 ft. or more below the foundation level and the hollows treated as described. A layer of hardcore well rolled in

all over the site thus prepared is also beneficial in producing a uniform quality of surface.

Basements.—The design of a basement, a typical cross section of which is shown in *Fig.* 20(*d*), is partly a problem of raft design, since the loads of the ground floor over the basement, the walls and other superstructure above ground-floor level, and the weight of the basement itself, are carried on the ground under the floor of the basement. For water-tightness it is usual to construct

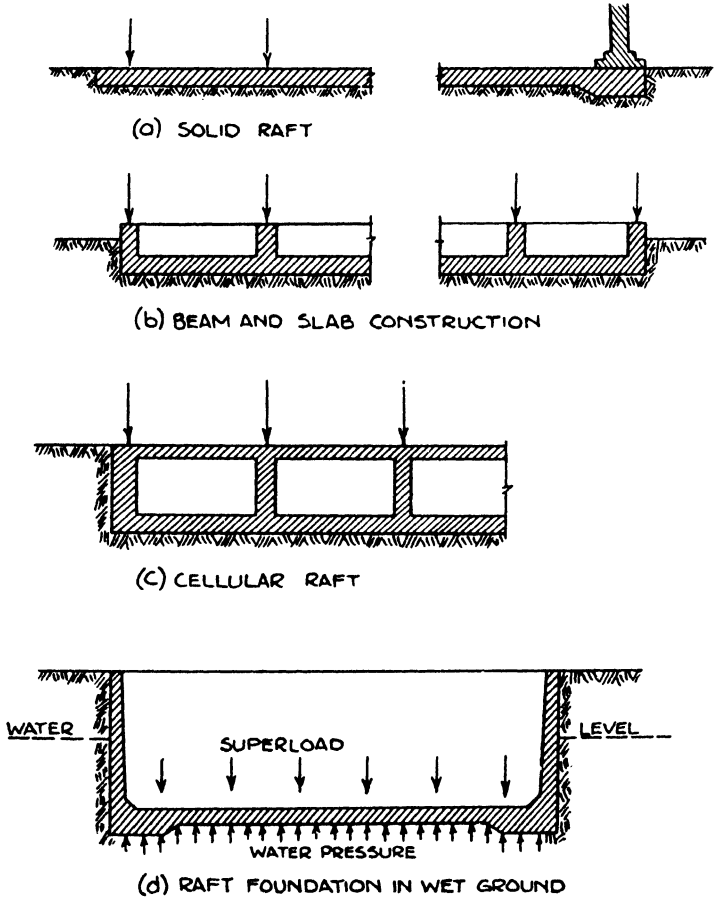


Fig. 20.—Raft Foundations.

the wall and floor of the basement monolithically, thus tending to spread the total load over the whole area of the floor. In average cases this would produce a low ground pressure which, however, owing to the large span involved would result in large bending moments and consequently a thick floor. Since the bulk of the load is transmitted through the walls of the basement, it is more economical to consider the load to be spread on a strip immediately under the wall and use a ground pressure up to the maximum allowed. The bending moment at the edge of the wall due to the cantilever action of this strip will

determine the thickness of the strip, but the remainder of the floor can be reduced in thickness to 5 in. or 6 in.

Where basements are in waterbearing soils, consideration must be given to the effect of water pressure. The walls must be designed to resist the horizontal pressure of the water-logged ground, and the bottom must be strong enough to resist the net upward pressure due to the head of water in the surrounding ground. It is also equally important to investigate the basement for flotation, of which there are two critical stages. When the structure is complete the total weight of the basement and all superimposed dead loading should exceed the maximum upward pressure of the water by a substantial margin. When the basement alone is complete, there should also be an excess of downward load; if not, one of the following steps is required:

(1) Pumping or similar measures should be taken to control the ground-water level in the vicinity of the basement.

(2) Temporary vent holes should be formed in the floor (or at the base of the walls) to enable water freely to enter the basement, thereby equalising the external and internal pressures; the vents should be sealed up when sufficient dead load from the superstructure is erected.

(3) The basement should be temporarily flooded to a depth such that the weight of water in the basement together with the dead load exceeds the total upward force on the structure.

During the construction of the basement, method (1) is usually the most convenient, but when the basement is complete method (3) is preferable on account of its simplicity.

The designing engineer should specify the depth of water required, and a suitable rule for ascertaining this depth in a large basement is 1 ft. for every foot head of ground water less 1 ft. for every 5-in. thickness of concrete in either the basement floor above the waterproof layer or in ground floor above. The omission of the weight of the basement walls provides a margin of safety.

Foundation Piers.—When a satisfactory foundation material is found at a depth of 5 ft. to 15 ft. below the natural ground level a suitable foundation can be made by building up piers from the low level to ground level, and constructing independent bases to the columns or other supports at ground level. The piers are generally in the form shown on *Table* No. 18, and can be constructed in brick, masonry, plain or reinforced concrete, whichever is most convenient. The maximum bearing pressure of the base on the top of the pier depends on the material of which the latter is constructed; safe values of this pressure for various materials are given on *Table* No. 17. If the top strata are such that the excavation will not stand up without timbering, or if water is present in such quantities that continuous pumping is required, piers of this type are not usually economical. If neither of these disadvantages exists, however, the minimum economic size of the pier is when the load it carries is great enough to require a pier base equal in area to the smallest hole in which men can conveniently work, otherwise unnecessary excavation has to be taken out and refilled. Thus, assuming a man can conveniently work in a hole one yard square at a depth of 12 to 15 ft., the total load would be 36 tons on a stratum capable of sustaining 4 tons per square foot. Generally it is better to provide as few piers as possible and to collect as much superload as practicable on each pier, thus making each

pier of generous proportions. Often it is not necessary to take out a bigger hole than is required for the stem of the pier, as the ground may be firm enough to be undercut for the widening at the base. In the case of concrete piers this procedure eliminates shuttering.

Reinforced concrete columns can sometimes be economically carried down to moderate depths, but in order to avoid slender columns it is usually necessary to provide an adequate lateral support at ground level.

When piers are impracticable, either by reason of the depth at which a firm bearing stratum occurs or otherwise, piles have to be adopted.

Selection of Type.—In selecting a type of foundation suitable for any particular purpose the type of structure should be considered, and sometimes it has to be decided whether any injury to the superstructure may be risked as the result of a little local settlement in preference to incurring the expense of putting in a more elaborate foundation. In the case of silo foundations and the abutments of fixed-end arches, all risk of unequal settlement must be eliminated, but for gantries and bases for large steel tanks a simple foundation can be provided and probable settlement allowed for in the superstructure design. In mining districts, where settlement can be reasonably anticipated, raft foundations should be provided for all major structures in order that the structure as a whole may settle equally in all parts.

3.—Design of Foundations.

Independent Bases.—Column footings of the form (*a*) shown on *Table* No. 18 should be so proportioned that the bending stresses are negligible, and therefore the thickness of both the upper reinforced concrete footing and the lower mass concrete footing should be determined so that the load can be distributed on the mass concrete and ground by dispersion. When the angle of dispersion is assumed to be not less than 45 deg. to the horizontal the proportions are given by the formulæ on *Table* No. 18. The thickness of each part of the footing must also be sufficient to keep the punching shear stresses within the permissible limits specified on *Table* No. 23; expressions giving these thicknesses are also incorporated on *Table* No. 18.

The minimum thickness of reinforced concrete footings of the usual splayed type illustrated by (*b*) and (*c*) should also be determined from considerations of punching shear, and reinforcement provided to resist the bending moments specified on *Table* No. 18, where expressions for the moment at the edge of the column are given for square and rectangular bases. The resistance moment of splayed footings cannot be determined with precision, and the formulæ given on p. 252 allow the calculation to be made on a rational and conservative basis. When the size of the base relative to its thickness is such that the column load can be spread by dispersion over the whole area of the base, no bending moment need be considered and only nominal reinforcement need be provided.

When piers are provided to convey the column loads down to a suitable bearing stratum, the proportions of the various parts of the pier should be in accordance with the data given on *Table* No. 18.

Combined Footings.—When more than one column or load is carried on a single base the centre of gravity of the several loads should, if possible,

coincide with the centre of the area of the base, in which case the pressure under the base will be uniformly distributed. The base should be symmetrically disposed about the line of the loads and should be either trapezium-shape as in *Fig.* 21(*a*), or be made up of a series of rectangles as in *Fig.* 21(*b*). In the latter case each rectangle should be so proportioned that the load upon it acts at its

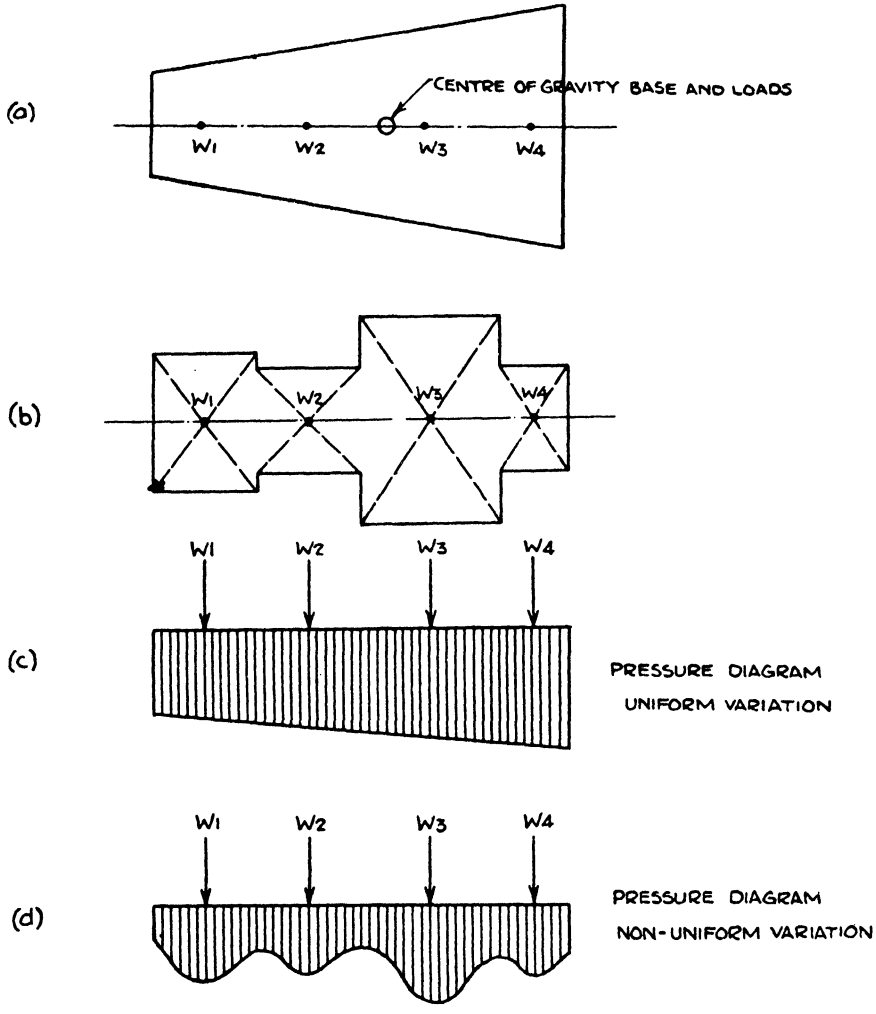


Fig. 21.—Combined Bases.

centre of area, and the area of each rectangle should be equal to the corresponding applied load divided by a safe bearing pressure, the value adopted for this pressure being the same for all the rectangles. If all the loads are equal and the columns placed at equal intervals the width of the base would be constant.

If it is not possible or practicable to proportion the bases as described the resultant load will be eccentric, and thus the centre of pressure of the upward

ground pressure will have the same eccentricity relative to the centre of area of the base. If the base is thick enough to be considered as acting as a single rigid member, the ground pressure will vary according to the formulæ for eccentric loading and will give a pressure distribution diagram as in *Fig.* 21(*c*). If the base is comparatively thin this theoretical distribution may not be realised, and owing to the flexibility of the base the ground pressure may be greater immediately under the loads than at intermediate points, giving a pressure diagram somewhat as in *Fig.* 21(*d*).

In the case of uniform distribution, or uniform variation of distribution, the longitudinal bending moment on the base at any section is calculable, being equal to the sum of anti-clockwise moments of each load to the left of the section minus the clockwise moment of the upward pressure between the section and the left-hand end of the base. With less rigid bases, the relative flexibility may result in a redistribution of the ground pressures, the latter being high in areas immediately under the loads and less at intermediate areas, the variation being somewhat as shown in *Fig.* 21(*d*). Unless the base can be designed for a greater proportion of the load to be carried in the vicinity of the load (by increasing the width at these areas), the method just outlined for finding the bending moments should be adopted. If the alternative design can be adopted, considerable reduction in the longitudinal bending moment results (but higher transverse bending moment).

Wall Footings.—When the load on a strip footing is uniformly distributed throughout its length, as in the general case of a wall footing, the principal bending moments will be due to the transverse cantilever action of the projecting portion of the base. If the wall is of concrete, and is built monolithic with the base the moment is a maximum at the face of the wall, but if the wall is in brick or masonry the maximum moment occurs under the centre of the wall. Expressions for these moments are given on *Table* No. 18. When the overhang is less than the thickness of the base the moments can be neglected, but in all cases the thickness of the base should be such that the normal shear stresses are not excessive.

Whether wall footings are designed for transverse bending or not, if the safe ground pressure is less than 2 tons per square foot longitudinal reinforcement should be inserted to take possible longitudinal moments due to unequal settlement or non-uniformity of loading. A method of assessing the amount of longitudinal steel is to design the footing as though it might be called upon to span over a cavity (or soft area) of, say, 10 ft., according to the nature of the ground.

Balanced Foundations.—When it is not possible to place an adequate base centrally under a column or other load owing to site limitations, and when under such conditions the eccentricity would result in inadmissible ground pressures, a balanced foundation can be devised as illustrated diagrammatically in *Fig.* 22(*a*). This case is very common in designing the external columns on city sites. The column is supported on the overhanging end C of a beam BC, that is supported on a base at A and subjected to a counterbalancing effect at B. The reaction at A, which depends on the relative values of BC and BA, can be carried by an ordinary reinforced concrete or mass concrete base designed for a concentric load. The counterbalancing can usually be provided by the

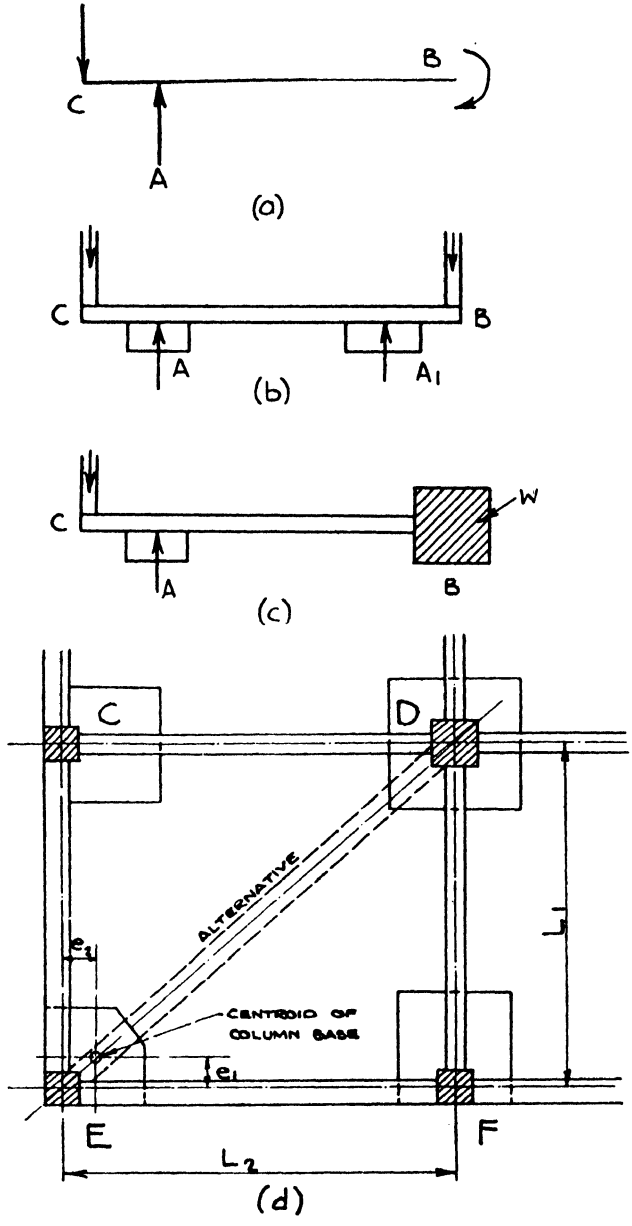


Fig. 22.—Balanced Foundations.

load from another column as in *Fig.* 22(*b*), in which case the dead load on this column at *B* should be sufficient to counterbalance the dead and live load on the column at *C*, and vice versa, with a sufficient margin of safety. Formulæ giving the resulting values of the reactions at A and A_1 are given on *Table* No. 18; from these reactions the shears and moments in the beam can be

readily calculated. In dealing with an external column of a building it is usual to be able to arrange that the internal column B is immediately over A_1.

If no column loads can be conveniently brought into service to counterbalance the column at C an anchorage must be provided at B by other means, such as the construction of a mass concrete counterweight or the provision of tension piles.

If the column considered is a corner column for which the foundation will be eccentric in two directions, one parallel to each main building line, it is sometimes possible to introduce a diagonal balancing beam that is anchored by the adjacent internal column D (*Fig.* 22(*d*)). In other cases, however, the two wall beams that meet at the column can be designed as balancing beams to overcome the double eccentricity. The overhung bending moment in the beam EC is equal to $W_E e_1$, where W_E is the column load and the upward reaction on column C is $\frac{W_E e_1}{L_1}$. Similarly the maximum moment in beam EF is $W_E e_2$ and the upward reaction on column F is $\frac{W_E e_2}{L_2}$.

Raft.—A numerical analysis of a raft foundation supporting a series of symmetrically arranged column loads, is given in the author's "Practical Examples of Reinforced Concrete Design." This example follows the general assumption of uniformly distributed ground pressure. If consideration is to be given to variable settlement of the ground with resulting non-uniform pressure, the reader is referred to Mr. A. L. L. Baker's study of the "Soil Line" method of analysis.

When the columns or other load points on a raft foundation are not equally loaded or symmetrically arranged, the raft should be designed so that the centroid coincides with the centre of gravity of the loads. As this will give uniformly distributed ground pressure, the area of the raft should be made equal to the total load (including the weight of the raft) divided by the safe bearing pressure. If this coincidence of centres of gravity is impracticable owing to the extent of the raft being limited on one or more sides, the plan of the raft should be made such that the eccentricity (e_w) of the total loading (W_T) is an absolute minimum, and this may produce a raft non-rectangular in plan, as in the example illustrated in *Fig.* 23. The maximum ground pressure, which should not exceed the permissible bearing pressure, occurs at A and is given by

$$p_{max.} = \frac{W_T}{A_R} + \frac{W_T e_w f}{I_R}$$

where A_R = total area of the raft

and I_R = moment of inertia about the axis $N - N$ which passes through the centroid of the base and is normal to the line joining the centroid and the centre of gravity of the loads.

The minimum ground pressure occurs at B and is given by

$$p_{min.} = \frac{W_T}{A_R} - \frac{W_T e_w g}{I_R}.$$

The ground pressure anywhere along the line NN will be $\frac{W_T}{A_R}$ and from these

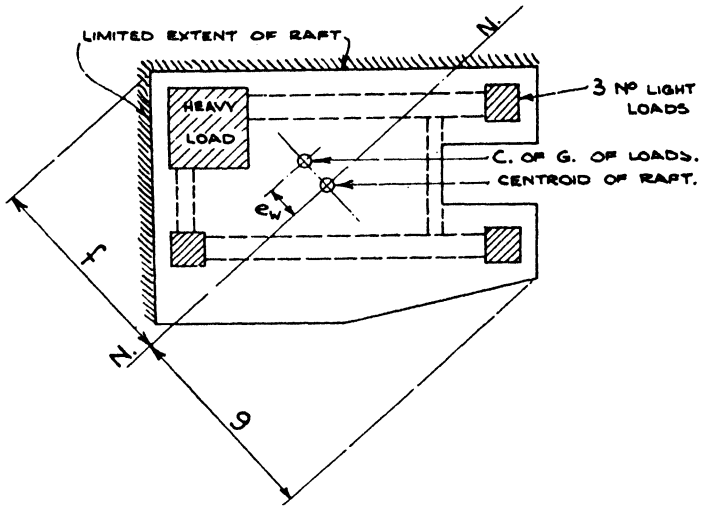


Fig. 23.—Eccentrically Loaded Raft.

three determined pressures that at any other point or the mean pressure over any area can be assessed. Having arranged a rational system of beams or ribs dividing the slab into suitable panels (as suggested by the dotted lines in *Fig.* 23), the slab panels and beams can be designed for the bending moments and shear forces due to the net upward ground pressures to which they will be subjected during the transference of the concentrated loads to the bearing pressures.

4.—Piled Foundations.

Safe Load on Pre-cast Piles.—Reinforced concrete piles are either pre-cast or cast *in situ*, and have been driven in general work in lengths up to 110 ft., although when exceeding 60 ft. it is necessary to give very special consideration to the design of the pile and of the handling and driving plant. Lengths less than 15 ft. are seldom economical. For ordinary work pre-cast piles have usually a square or octagonal section and are from 8 in. to 18 in. wide, although special piles have been designed in excess of these dimensions. For their support, piles either depend on direct bearing resistance on a firm stratum or on side frictional resistance in soft strata. The load that can be safely carried on a pile depends on the maximum load that the pile can carry when considered as a column and on the minimum load that would produce settlement or further penetration of the pile into the ground. So many are the factors that enter into a consideration of the settlement load for any particular pile that the results of calculations made to determine this load do not have quite the conclusive value of the results of test loads on driven piles. The value of calculations, however, is greatly enhanced when they can be used comparatively in conjunction with loading tests. Such tests are often inconvenient and expensive, and for most normal cases the engineer has to fall back on safe loads computed by one of the many pile formulæ that have been put forward from time to time. When a number of these formulæ

are applied to any one problem, the range of the results will be so wide that at first sight it would appear that the use of formulæ is most unreliable. This apparent inconsistency is largely due to the fact that each formula has its own limitations of application, and no particular formula should be used unless one is satisfied that the problem in hand falls within the conditions appertaining to the formula.

Pile driving formulæ fall into two main groups: (i) Impact formulæ which are applicable to bearing piles, and which, if carefully selected, can be quite reliable. (ii) Friction formulæ which are applicable to piles that are supported by the frictional resistance of the ground in which they are embedded; these formulæ are not very reliable and should be associated with test loadings.

For the sake of comparison of impact formulæ, those which follow have been set out in similar form using the notation

W_1 = safe load on pile (tons).
w = weight of moving parts of hammer (tons).
H = fall of hammer (inches).
n = number of blows per final inch of penetration.
k = a coefficient to allow for characteristics of pile, type of hammer and method of operation, nature of soil, condition of dolly, factor of safety, and other factors not directly represented in the formula. (This coefficient is often fallaciously referred to as simply the "factor of safety.")
P = weight of pile (tons), including helmet, dolly, cushioning, and any stationary parts of the hammer resting on the pilehead.

$$R = \frac{P}{w}$$

A = effective cross-sectional area of pile (square inches).
L = length of pile as cast (feet).
E = elastic modulus of material of which pile is made (tons per square inch).

$$K = \text{elastic constant} = \frac{AE}{12L}.$$

Impact formulæ can be conveniently divided into four classes:

CLASS I.—These formulæ involve only the weight of the hammer and the drop, and are sometimes accompanied by a statement regarding the set. McAlpine's formula and Haswell's formula are of this type and are useless for reinforced concrete piles.

CLASS II.—This class takes the simple form

$$W_1 = \frac{wHn}{k}.$$

The formulæ proposed by Sanders ($k = 8$), Merriman ($k = 6$), and Berg ($k = 10$) are in this class, but this type of formula is unreliable unless the value of k can be fixed with regard to all the variables encountered in pile driving. The Wellington series of formulæ for timber piles represents the factor k by an expression involving n, as does also the Nicholson formula for concrete piles. For well-driven concrete piles a value of k between 20 and 30 seems to be suitable.

CLASS III.—This class would include those formulæ that, in addition to the primary factors w, H and n, also involve the ratio of pile weight to hammer weight. The best known is possibly the Dutch formula in its simplified form

$$W_1 = \frac{wHn}{k(1 + R)}.$$

For concrete piles the value of k would lie between 3 and 9. For piles subjected to dead loads only and driven by a winch-operated drop hammer the value of k should be taken as 6 if a helmet and dolly are used, but if no dolly is provided $k = 4$, and these values can be decreased by 10 per cent. if a single-acting steam-hammer is used. The safe loads calculated by this formula with these values of k have been tabulated on *Table* No. 19 assuming the set to be $\frac{1}{10}$-inch per blow (i.e. $n = 10$) and the hammer drop to be 3 ft. For other values of n and H an expression for adjusting the tabulated loads is given, but this formula gives rather too high a value of W_1 for sets smaller than those represented by $n = 10$. High values of k should be used if the soil is resilient and up to 50 per cent. higher if the working load will be vibratory.

The Brix formula is also commonly used for reinforced concrete piles driven with drop hammers. This formula,

$$W_1 = \frac{wHnR}{k(1 + R)^2},$$

can be used for higher values of n than the Dutch formula, using the same range of values for k as advocated for that formula. An American range of formulæ of this class is of interest as indicating the effect of the type of the hammer:

Drop hammer, $$W_1 = \frac{wHn}{6(1 + R)}$$

Single-acting hammer, $$W_1 = \frac{wHn}{6(1 + 0{\cdot}1Rn)}$$

Double-acting hammer, $$W_1 = \frac{(ap + w)Hn}{6(1 - cRn)}$$

in which a = piston area (square inches).
p = steam pressure (tons per square inch).
c = constant = 0·1 to 0·3.

CLASS IV.—This class includes the more complex formulæ and embraces a group of formulæ that incorporates the elasticity of the pile. Representative of this group is Rankine's formula

$$W_1 = \frac{1}{k}\left[\sqrt{wHK + \left(\frac{K}{n}\right)^2} - \frac{K}{n}\right]$$

where $k = 3$ for dead loads, pile driven without dolly,
$= 4$ for general work (dead loads and dolly),
$= 5$ for vibrating working loads.

The formulæ proposed by Redtenbacher, Weisbach, and Bennett are somewhat similar, and such formulæ can be used for most concrete pile problems, or as an alternative the use of the more modern and comprehensive formula proposed by Mr. Hiley and extensively used is advocated.

This formula embraces all the major variants occurring in pile driving problems, such as weight and type of hammer, drop, penetration per blow, length of pile, shape of point, type of helmet, nature of ground, and material of which the pile is made. In the form given on *Table* No. 19 the constant c takes into account the energy absorbed in temporarily compressing the pile, the helmet, and the ground, and values of c for various degrees of severity of driving reinforced concrete piles of various lengths are given. These values assume that the pile is fitted with a helmet with packing and dolly of the type usually employed in connection with concrete piles. When no helmet is used the value of c should be reduced by 0·1 in. Since the quake of the earth below the pile shoe is included in this value it follows that the nature of the ground affects c, and the tabulated values are applicable to a firm gravel sub-stratum. If the pile bears on clay an additional 0·05 in. should be allowed, and if the material is of a peaty character with soft ground below it as much as 0·20 in. can be added to the tabulated value of c for normal driving conditions. The factor $2c$ is a quantity that is measurable on any pile that is being driven if proper arrangements are made, since it represents the difference between the permanent penetration for one blow and the maximum instantaneous depression of the pile-head as measured at the top of the helmet.

The factor e represents the efficiency of the blow and depends on the $\frac{P}{w}$ ratio; values of e are given on *Table* No. 19, together with values of $\frac{H_1}{H}$ which allow for the freedom or otherwise of the drop. Settlement loads as calculated by the Hiley formula are subject to a factor of safety allowance of $1\frac{1}{2}$ to 3 for normal structures.

CLASS V.—When piles are driven into soft material and depend solely upon friction for their support, the safe load can be calculated by either of the following formulæ:

(i) $W = L[k_1L(B + D) + k_2]$
(ii) $W = k_3(B + D)L + k_4BD$
where L = embedded length of pile (ft.)
B = breadth of pile (in.)
D = width of pile (in.).

The factors k_1, k_2, k_3, and k_4 depend on the character of the ground driven through and into, and appropriate values are given on *Table* No. 19. Formula (i) is more suitable when the material under the shoe of the pile is not of much better quality than that in which the greater length of the pile is embedded; formula (ii) is more applicable to cases where the lower few feet of the pile penetrate a firmer stratum than that surrounding the major portion of the pile.

When a pile is driven through a fairly soft material to refusal on hard strata (rock) the safe load is determined by the strength of the pile considered as a column.

Design of Pre-cast Piles.—Reinforced concrete piles should be designed to withstand the stresses due to handling, driving, and load carrying, with appropriate factors of safety. These operations are discussed more fully in the author's

book "Concrete Construction." Cracking due to overstressing young concrete during the handling and slinging operations can be guarded against by so arranging the position and number of the points of suspension that the stresses due to bending moments produced by the weight of the pile itself are within the safe tensile stresses. For square piles of $1 : 1\frac{2}{3} : 3\frac{1}{3}$ mix and having a normal percentage of longitudinal steel the maximum moment due to bending about an axis parallel to one side of the section should not exceed $60D^3$ in.-lb. approximately, where D = length of side in inches.

If toggle holes are provided some degree of assurance that the pile will not be lifted so as to bend about a diagonal is attained. The toggle holes should be so arranged that the minimum bending moments are experienced during lifting, and the appropriate positions of the holes for this condition for single and double-point lifting are given on *Table* No. 19. The size and length of the pile will determine whether single or two-point lifting should be adopted.

The maximum compression stresses experienced in a pile are usually those due to driving and occur at the head of the pile, generally as final set is approached. In the simplest case this stress would be approximately the driving force divided by the total effective cross-sectional area, but even with a well-centred pile it is probable that the maximum stress is at least double the mean stress. The driving force can be approximately calculated from the factors involved in the Hiley formula, and thus the probable maximum stress is given by

$$c_m = \frac{2wH_1en}{A(1 + cn)}$$

where A = total effective cross-sectional area of the pile.

If the value of c_m greatly exceeds the normal allowable compressive stress there is danger that the cover concrete may spall off at the head of the pile. Therefore if driving conditions are anticipated in which by calculation c_m exceeds the safe normal stress, either the richness of the mix or the effective cross-section of the pile should be increased, and helical binders should be provided in the topmost few feet of the pile. Octagonal piles usually have helical binders throughout their whole length.

In *Fig.* 24 is illustrated a typical detail of the reinforcement in a square reinforced concrete pile, in which helical binders at the head of the pile are provided. The arrangement of the toggle holes and spacers is also indicated. For driving into clays, gravels, or sands a pile shoe having a two to one overall taper, as shown in *Fig.* 24, is usually satisfactory, but for other types of soil other shoe shapes are necessary. If the pile has to be driven through soft material to take a bearing on a thin bed of gravel overlying softer ground it is necessary to have a blunter shoe to prevent punching through the thin strata. For friction piles driven into soft material throughout a shoe is not absolutely necessary, and a blunter end should be formed as in *Fig.* 24(*a*). When driving through soft material to a bearing on soft rock or stiff clay, the form of pile end shown in *Fig.* 24(*b*) is satisfactory so long as driving ceases as soon as the firm stratum is reached or is only just penetrated. When driving down to hard rock, or where heavy boulders are anticipated, a rock shoe as shown in *Fig.* 24(*c*) should be fitted.

Irrespective of the load the pile can carry before risk of settlement is incurred,

the stresses produced by the working load on the pile acting as a column should be considered. For calculation of the stress reduction due to slenderness (see Chapter XIII) the effective length of the pile can be considered as two-thirds of the length embedded in soft strata (or one-third of the length embedded in fairly firm strata) plus the length projecting above the ground, and usually the end conditions of a pile are equivalent to the general case of one end fixed

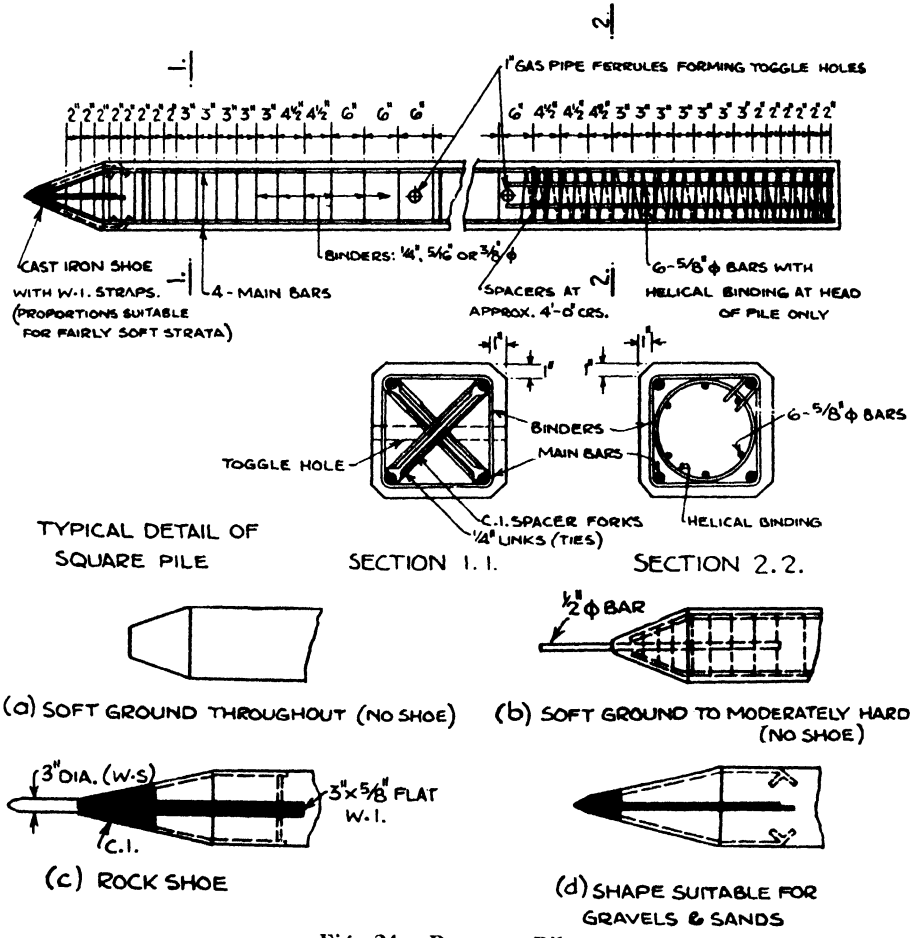


Fig. 24.—Pre-cast Piles.

and one end hinged. In no case should the effective column length be assumed at less than 5 ft. plus the projecting length.

Piling Plans.—In preparing pile lay-outs attention must be given to practicability of driving as well as to effectiveness for load carrying. In order that each pile in a group shall carry an equal share of the load the centre of gravity of the pile group should coincide with the centre of the superimposed load. The clear distance D between any two piles should generally be not less than 2 ft. 6 in., except in clays where experiments indicate that a clear distance

equal to the size of the pile is effective. So far as possible piles should be arranged to lie along a series of straight lines in both directions throughout any section of a particular job, as this form of lay-out minimises the amount of movement of the driving frame. The lay-out should also allow for driving to proceed in such a way that any displacement of earth due to the consolidation in the piled area shall be free to take place in a direction away from the piles already driven.

Pile Caps.—Pile caps should be designed primarily for punching shear around the heads of the piles and around the column base and for bending moment due to transferring the column or other load to the piles. The thickness of the cap should also be sufficient to provide adequate bond length for the pile bars

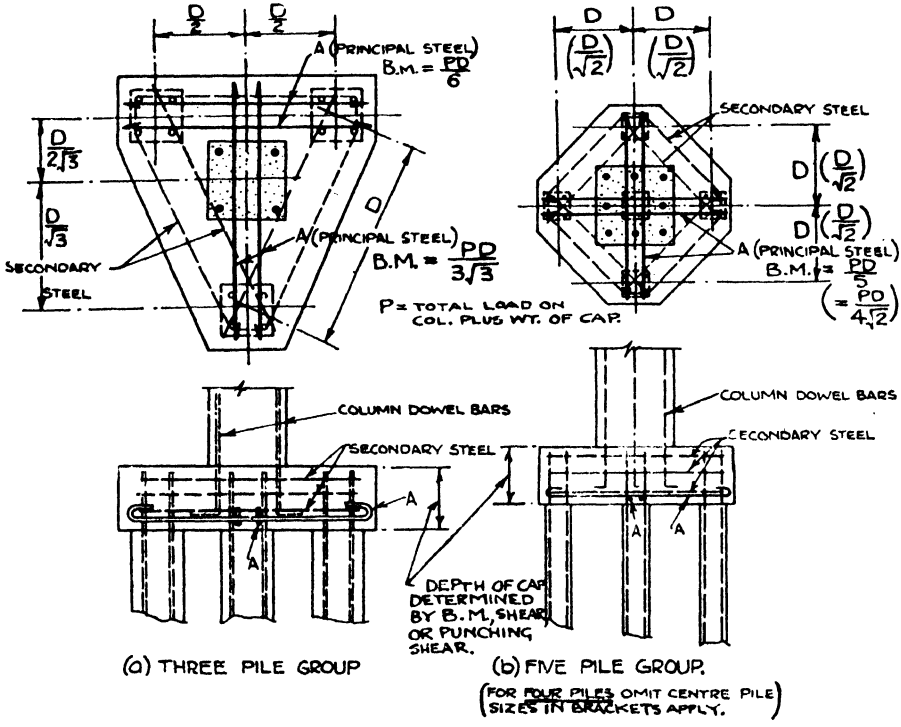


Fig. 25.—Typical Details of Pile Caps.

and column dowel bars. If the thickness is such that the column load can all be transmitted to the piles by dispersion no bending moments need be considered, but usually when two or more piles are placed under one column it is necessary to reinforce the pile cap for the bending moments produced. *Fig.* 25 illustrates typical details of pile caps suitable for three and five piles in a group, and in each case each of the bands of steel marked *A* is provided to give a resistance equal to a single pile load multiplied by the distance from the centre of the pile to the column. The shear and compressive stresses on the concrete must be investigated in these cases.

The secondary reinforcement should consist of $\frac{3}{8}$-in. or $\frac{1}{2}$-in. horizontal bars that will bind the concrete around the head of the piles.

Loads on Piles in a Group.—If the centre of gravity of the total load W_T on a group of N piles is coincident with the centre of gravity of the piles, each pile will be approximately equally loaded, and will be subjected to a load equal to $\frac{W_T}{N}$. If the centre of gravity of the load is displaced a distance e from the centre of gravity of the pile group, the load on any one pile is given by

$$W_T\left(\frac{1}{N} \pm \frac{ed_1}{\Sigma d^2}\right),$$

where Σd^2 = the sum of the squares of the distance of each pile measured from an axis passing through the centre of gravity of the pile group and at right angles to the line joining this centre of gravity to the centre of gravity of the applied load.

d_1 = the distance of the pile considered from the above axis ; this dimension will be positive if on the same side of the axis as the centre of gravity of the load, and negative if on the opposite side.

If the structure supported on the pile group is subject to a general bending moment, M, which is transmitted to the foundations, the expression given for the pile load can be used by substituting $e = \frac{M}{W_T}$.

Cast-in-situ Piles.—In certain conditions a cast-in-situ pile, of which type there are many different patented systems, is preferable to a precast pile. The following advantages are obtained with cast-in-situ piles, although all the points enumerated are not applicable to any one system.

(1) Variable length of piles is obtainable when depth of bearing stratum varies.

(2) Pile can be stopped-off at any level below ground.

(3) Formation of " club-foot " to give increased bearing area (bored types only).

(4) Punching through intermediate hard stratum (tube-driven types only).

(5) Increased friction in soils such as compact gravels.

(6) No effect on adjacent piles or on ground level due to consolidation of ground when many piles are driven in a constricted area (bored types only).

(7) Silent and vibrationless (bored types only).

(8) Small headroom required (bored types only).

(9) Cutting-off surplus lengths or lengthening in-situ is not required.

In general the advantages of a pre-cast pile over a cast-in-situ pile include :

(1) Hardening of concrete unaffected by deleterious ground waters.

(2) Pile can be inspected before driving into ground (but faults below ground are not evident).

(3) Pile size not affected by water below the ground. (This applies also to cast-in-situ types with a central core.)

(4) The piles can be driven into river beds, etc., for jetties, etc.

In any given instance the procedure to be adopted by the designing engineer is to set down the conditions and limitations applicable to the problem, and to select the pile (either pre-cast or one of the cast-in-situ type) that complies with the greatest number of these requirements.

CHAPTER VII

RETAINING WALLS AND CONTAINERS

METHODS of calculating the horizontal pressures due to retained earth, and due to liquids and dry materials contained in tanks, bunkers, and silos, are given in Chapter III, and the data ordinarily necessary for this calculation are given on *Tables* Nos. 5 and 6 (pp. 219 and 221). The present chapter deals with the general design of retaining walls and container structures, and with the calculation of the forces and moments produced by the pressure of the retained or contained materials. Where containers are required to be watertight, the recommendations given in the appropriate report of the Institution of the Civil Engineers have in general, been followed.

1.—Retaining Walls.

Types of Walls.—A retaining wall is essentially a vertical cantilever, and when constructed in reinforced concrete can be a simple cantilever wall, a counterforted wall, or a sheet-pile wall.

A simple cantilever type is suitable for walls of moderate heights and can be constructed either with a base projecting backwards under the filling, as in *Fig.* 26(*a*), or with a base projecting forward as in *Fig.* 26(*b*). The former type

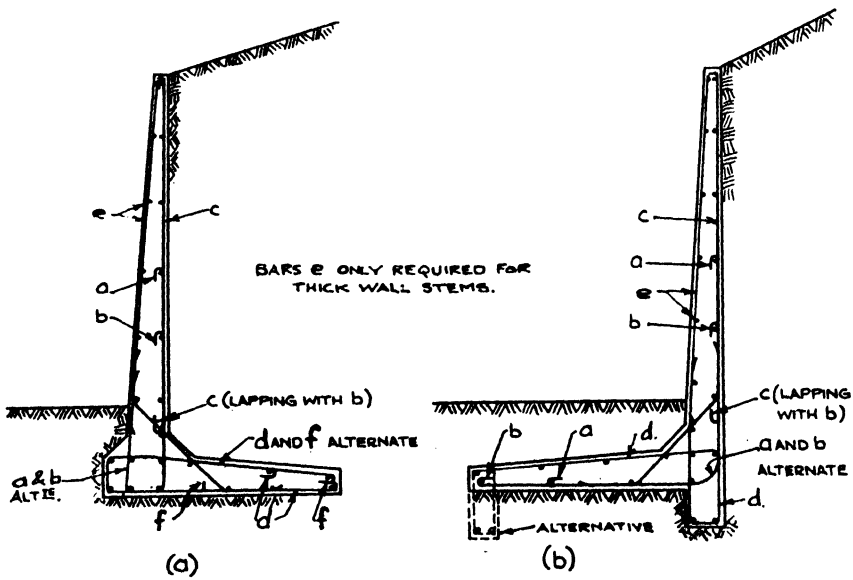


Fig. 26.—Typical Details of Cantilever Retaining Walls.

is generally more economical. Designing methods for both types are discussed later.

Counterforted design is suitable for high walls or great pressures. The slab panels span horizontally between vertical counterforts or buttresses as in *Fig.* 27(*a*), although in cases of high walls, the slab spans vertically on to horizontal beams (as in *Fig.* 27(*b*) which react on the counterforts. By graduating the spacing of the beams the maximum bending moments in each span can be kept equal and the slab maintained the same thickness throughout. When the horizontal shearing stresses will allow, the web of the counterfort can be cut out as indicated by the dotted lines in *Fig.* 27(*a*). Counterfort type walls are designed in accordance with the same principles with regard to stability, sliding, and ground pressures as are simple cantilever walls, except that these

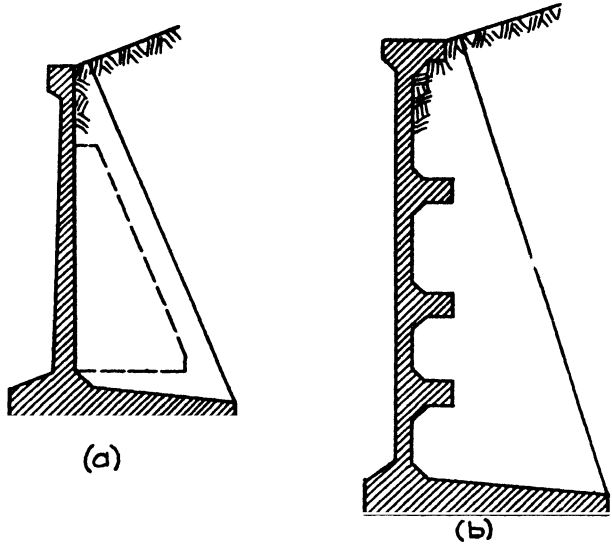


Fig. 27.—Counterfort Retaining Walls.

factors are investigated for a length of wall equal to the distance apart of the counterforts instead of a length of 1 ft.

Pressures behind Walls.—The value of the horizontal pressure due to retained earth is often taken as $27h$ lb. per square foot, where h is the depth (in feet) of earth above the point under consideration, as discussed in Chapter III. When the ground is compact a less conservative figure is often assumed, say $22h$, which corresponds to an angle of repose of 40 deg. and a density of 100 lb. per cubic foot. The values $27h$ and $22h$ assume a horizontal fill and would be increased or reduced for positive or negative surcharges.

In uncertain ground, or ground that may be accidentally waterlogged, it is often advantageous to design for a factor of safety of 4 against ground pressure, and of $2\frac{1}{2}$ against possible water pressure.

Cantilever Retaining Walls.—The factors affecting the design of simple cantilever walls are usually considered per foot length of wall when the wall is

of uniform height, but when the wall is of varying height a section of convenient length, say 10 ft., should be treated as a complete unit. Formulæ giving the limiting values of the moments, forces, and dimensions, etc., for simple cantilever retaining walls are given on *Table* No. 20 (page 257). The principal factors to be considered are as follows:

Stability.—A minimum factor of safety of 1½ against overturning should be allowed.

Ground Pressure.—The maximum pressure should not exceed the allowable pressure for the type of ground upon which the wall is founded.

Sliding.—The total vertical load multiplied by an adequate coefficient of friction should exceed the total horizontal pressure by at least 50 per cent. For sands, gravels, rock, and other fairly dry soils, a coefficient of 0·4 is suitable, but for less reliable clayey soils a rib should be provided on the underside of the slab to ensure a more definite resistance to sliding. In walls of the form shown in *Fig.* 26(*b*), where the vertical load is always small compared with the horizontal pressure, such a rib should always be provided either immediately below the wall stem as indicated or at the extreme edge of the toe as shown in the alternative (dotted) sketch.

Resistance Moments.—The safe resistance moment of the stem of the wall should be equal to the bending moment, which can be calculated from the pressure behind the wall, and may be critical at the top of the haunches at the base of the stem. The base slab should be made the same thickness as the bottom of the wall stem and the same steel should be provided. The base slab can be tapered as indicated in *Fig.* 26.

Expansion and Contraction.—Long walls should be provided with expansion joints, suitable details of which are given in the next chapter. To reduce the risk of cracking due to shrinkage, appropriate methods of construction should be specified (see the author's "Concrete Construction"), and as a further precaution against contraction and temperature cracks occurring on exposed faces of walls reinforced principally on the earth face, a mesh of reinforcement, say ⅜-in. diameter bars spaced at 12-in. centres horizontally and vertically, should be provided on the exposed face if the wall thickness exceeds 8 in.

Drainage behind Walls.—Tapering the base slab in front of and behind the wall serves not only to economise in concrete but also to assist drainage, the consideration of which is of some importance in retaining wall construction, especially for walls that have been designed for low pressures. Where the fill behind the wall is gravel or sand, a French drain of loosely-packed rubble should be constructed along the base of the back of the wall, and weep-holes, 3 in. to 6 in. diameter, provided at intervals of about 10 ft. A weep-hole should be provided in every "pocket" formed by counterforts, and the top surface of any intermediate horizontal beams should be given a slight slope away from the back of the wall. With backings of poor porosity, hand-packed rubble placed behind the wall for almost the whole height, in addition to weep-holes, materially assists effective drainage of the filling, and since the rubble is partly self-supporting the pressure on the stem is to a certain extent relieved. The filling behind the wall should not be tipped from a height into position, but should be carefully deposited in shallow horizontal layers.

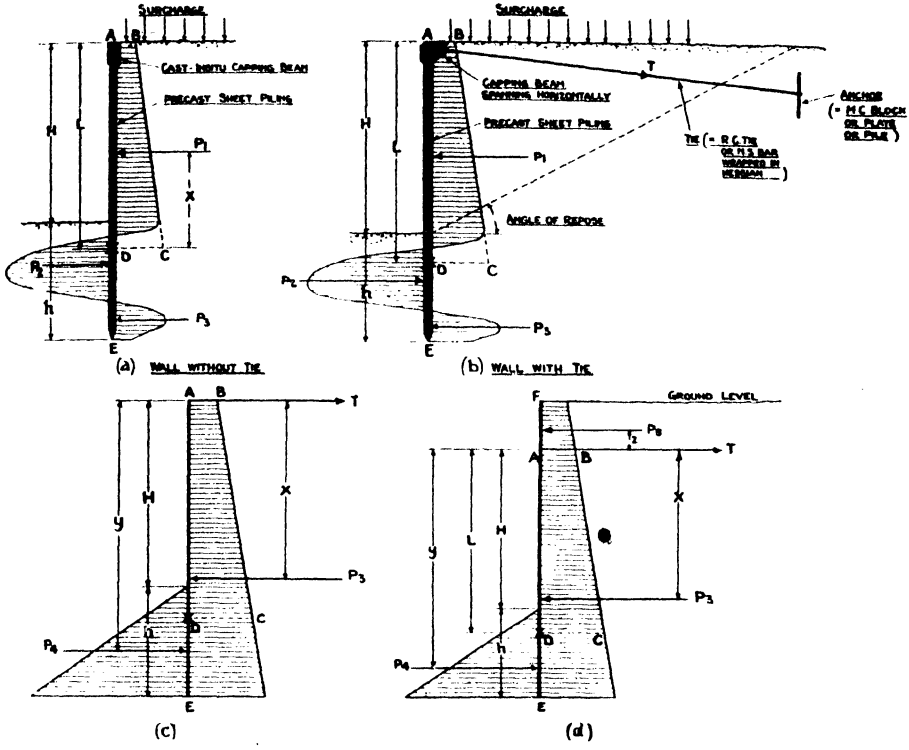


Fig. 28.—Sheet Pile Walls.

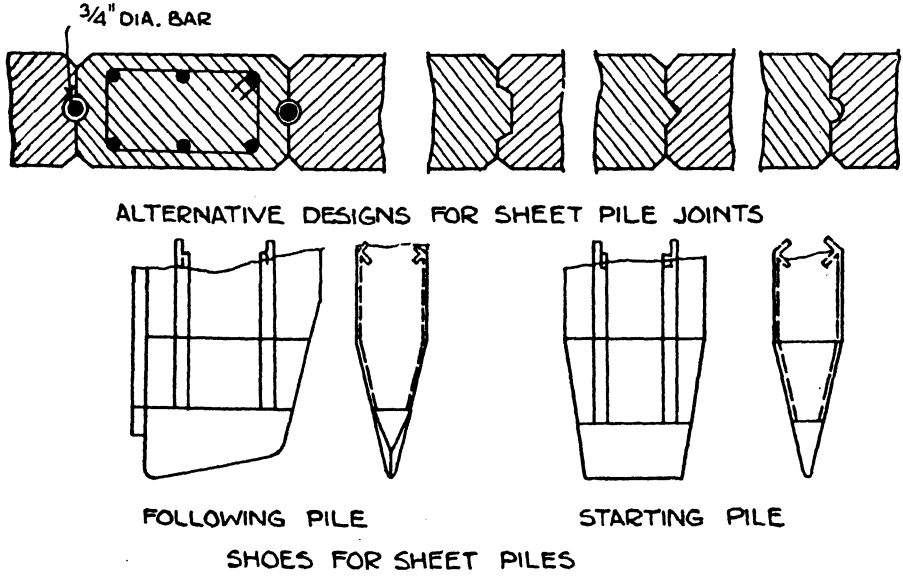


Fig. 29.—Sheet Piles.

2.—Sheet Pile Walls.

General Design.—When satisfactory bearing strata are not encountered at a reasonable depth below the surface of the ground in front of a retaining wall, sheet pile walls are usually adopted. Such walls are constructed by driving pre-cast reinforced concrete sheeting into the ground a sufficient distance to obtain an anchorage for the vertical cantilever effect and security against sliding forward and spewing. This type of wall is particularly suitable for waterside works, and in the simplest form the sheeting simply cantilevers out of the ground, the pile heads being generally stripped and bonded into a cast-in-situ capping beam.

Typical sections of sheet piles and common designs for the tongue-and-groove jointing that is necessary to maintain alignment during driving are given in *Fig.* 29, which also illustrates typical shoe shapes for "starting piles" and "following piles." (For suitable dimensions see "Concrete Construction.")

If the height of the wall and the pressure on the sheeting are such that an excessive pile section is required the introduction of a tie at capping beam level will reduce the maximum moment. This tie can be either constructed in reinforced concrete or can be a mild steel bar, properly anchored into the capping beam and wrapped with bitumenised hessian to protect it from corrosion. The capping beam must be designed to span between the ties and to take the horizontal reactions from the top of the sheeting. The remote end of the tie should be anchored behind the natural slope of the ground, and the anchorage should be provided by a block of mass concrete, a vertical concrete plate, or preferably by an anchor pile. Although the force in the tie is increased, moments can be reduced by placing the tie at a point below the top of the wall; a suitable beam must then be provided at this level.

Forces on Sheet Pile Walls.—The forces on a simple cantilever sheet pile wall are as indicated in *Fig.* 28(*a*), in which P_1 is the active pressure due to the filling and surcharge behind the wall and P_2 and P_3 are passive pressures producing the necessary restraint moment to resist the overturning effect of P_1. The maximum moment in the sheeting will occur at some point D, and the distance L is calculated by the factors k_1 given in the column headed "free" on *Table* No. 20 (page 257) for different angles of repose for the ground in which the pile is embedded; these factors, together with k_2 in the same column, are based on the more comprehensive range deduced by A. Freund. It is sufficient if the bending moment on the sheeting be calculated as P_1x, the value of P_1 being conveniently represented by the area of the trapezium $ABCD$ in *Fig.* 28(*a*), which value can be determined from *Table* No. 5 (page 219). The distance x locates the centroid of the area. The embedded length of the sheeting must be great enough to enable sufficient passive pressures to be produced, and the factors k_2 (*Table* No. 20) referred to enable this length to be calculated.

When a tie is introduced at the top of the sheeting the forces acting on the sheeting are as shown in *Fig.* 28(*b*), which are similar to those in *Fig.* 28(*a*) except for the introduction of the horizontal force T in the tie. It is not simple to determine the variations of the pressure by mathematics with any precision, and therefore it is recommended that design problems be tackled in the following

manner. The factors k_2 in the column headed "hinged" on *Table* No. 20 will give the minimum value for h for the production of sufficient restraint moment. The embedded length h must not, however, be less than either of the minimum values required to resist forward movement of the toe or to prevent spewing. The sheeting will be stable if

$$P_3x \not> \tfrac{2}{3}P_4y$$

where P_3 = total active pressure on the whole depth of the sheeting as shown in *Fig.* 28(*c*).

P_4 = total maximum passive pressure that can be brought into play in front of the sheeting.

The value of P_3 can be computed from the data on *Table* No. 5, and P_4 will be calculated from the formula given in Paragraph 6 of Chapter III. The factor $\tfrac{2}{3}$ is introduced in order to allow a margin between the theoretically calculated passive resistance and that actually required.

To prevent spewing in front of the sheeting the embedded length should not be less than $\dfrac{k_2^2W}{w}$

where W = weight per square foot at point E of earth and surcharge above this point.

w = weight per cubic foot of earth in front of sheeting.

k_2 = pressure factor taken from *Table* No. 5.

The bending moment on the sheeting can be calculated by first determining L from the factors given in the column headed "hinged" on *Table* No. 20. The sheeting can be considered as a propped cantilever of span L built in at D and propped at A and subject to a trapezoidal load represented by the area $ABCD$. This load can be divided into a uniformly distributed load and a triangularly distributed load, for both of which the moment coefficients can be read off *Table* No. 7 (page 223). This Table also gives the reaction on the prop, and in this case this reaction represents the force in the tie. Since the security of this design of wall depends on the efficiency of the anchorage, no risk of underrating the force in the tie should be incurred; it is better to increase the force to be provided for from the value given by the theoretical reactions to half the value of P_1. The value of this force should certainly not be less than $P_3 - \tfrac{2}{3}P_4$, which is the unbalanced part of the outward active pressure.

In the case of the anchored wall illustrated in *Fig.* 28(*b*) and (*c*) it is assumed that the connection between the tie and the head of the sheeting is equivalent to a hinge; that is, the moment at A is zero. If the wall is extended above A, either by continuing the sheeting or by constructing a cast-in-situ wall, as shown in *Fig.* 28(*d*), a moment is introduced at A equal to P_5z, where P_5 is the total active pressure on the extended portion of the wall AF. This moment will introduce a negative moment in the sheeting at A, but will reduce the positive moment in the sheeting between D and A and will also reduce the negative bending moment at D. If this moment at A is large enough to produce conditions amounting to complete fixity at A, then the span L can be calculated by the factors k_1 given in the column headed "fixed" on *Table* No. 20. In this case also the factor k_2 in the same column will give the minimum embedded

length h, but at the same time h must be sufficient to prevent spewing and forward movement as already described. The equation for stability will be

$$P_3x - \tfrac{2}{3}P_4y = P_5z.$$

The force allowed for in the tie will be given by

$$T = P_3 - \tfrac{2}{3}P_4 + P_5$$
$$\text{or } T = 0{\cdot}5P_3 + P_5$$

whichever is greater. The moment on the sheeting will be calculated from the pressure represented by the area of the trapezium $ABCD$, considering the beam fixed at both ends and using the appropriate coefficients given on *Table* No. 7 and the reduction factors tabulated below.

When the moment at A is insufficient for complete fixity, the moments, forces, and values of L and h will be intermediate between those for hinged and fixed conditions at A.

A horizontal slab supported on a system of king piles is sometimes provided at A, and this has a sheltering effect on the sheeting insomuch that if it is carried far enough back it can completely relieve the sheeting below A from any active pressure due to earth or surcharge above the level of A.

Reduced Pressures on Flexible Walls.—The pressures behind flexible walls adjust themselves in such a way that the moments on the walls are reduced. Mr. Stroyer has put forward a formula by which the amount of this reduction can be assessed for reinforced concrete sheet pile walls. Without restricting the thickness-height ratio, this formula gives the following reduction factors, which are more conservative than those put forward by the Danish Society of Engineers.

B.M. REDUCTION FACTORS FOR SHEET PILE WALLS.

Not applicable to simple cantilevered walls.

Factors result from redistribution of pressures, not from pressure reduction.

Angle of Repose		5 deg. or less	15 deg.	20 deg.	30 deg.	35 deg.	45 deg. or over
Ratio of wall thickness to span	0·02	1·00	0·80	0·70	0·56	0·48	0·36
	0·10	1·00	0·88	0·80	0·69	0·62	0·50
	0·20	1·00	0·92	0·86	0·78	0·72	0·60
	0·30	1·00	0·93	0·90	0·83	0·78	0·69
	0·40	1·00	0·96	0·91	0·87	0·83	0·76

3.—Tanks.

Direct Tension in Walls of Cylindrical Tanks.—The walls of a cylindrical tank are primarily designed to take the direct tension due to the horizontal pressures of the contained materials, and if p lb. per square foot is the unit pressure at any depth the value of the direct tension on a ring 1 ft. in depth is given by

$$T = 0{\cdot}5pD \text{ lb.}$$

where D = internal diameter of the tank (ft.). Sufficient horizontal steel must be provided to take this tension, and if t lb. per square inch is the safe tensile stress and A_T is the area of the cross section of the reinforcement (sq. in.) in a ring 1 ft. in depth

$$A_T = \frac{T}{t} = \frac{pD}{2t}.$$

For tanks containing liquids the value of t should be 12,000 lb. per square inch, but for dry materials a stress of 18,000 lb. per square inch can be allowed.

These stresses and the length of overlap allowed on the bars should be subject to the remarks in Chapter X. For cylindrical tanks containing liquids the thickness of the walls should be determined in relation to the total direct tension. In order that tension cracks should not become localised, the tensile stress in the concrete should be kept within reasonable limits; a suitable maximum value for the concrete tensile stress c_t for a $1 : 1\frac{2}{3} : 3\frac{1}{3}$ concrete having the properties required for impermeable construction (see *Table* No. 23) is 175 lb. per square inch. The minimum thickness of the wall d (in.), at any depth h, for any value of t adopted in the design of a tank containing a liquid weighing w lb. per cubic foot, is given by

$$d = 0{\cdot}042whD\left[\frac{1}{c_t} - \frac{(m-1)}{t}\right].$$

If the tank contains water ($w = 62{\cdot}4$ lb. per cubic foot) and is designed for $t = 12{,}000$, $c_t = 175$, and $m = 12$, this expression reduces to

$$d = \frac{hD}{80} \text{ in.}$$

when h and D are in feet.

It is not usually recommended that the thickness of walls should be less than 4 in. or 5 in. irrespective of the head of water.

Bending Moments in Walls of Cylindrical Tanks.—In addition to the horizontal tension in the walls of a cylindrical container the bending moments produced by fixity at the base of the walls must be considered. Unless a definite joint is made at the foot of the wall there will be a certain amount of continuity between the walls and the base slab that will cause vertical cantilever action for a certain height up the wall. In consideration of this problem there are three principal factors, namely, (*a*) the bending moment at the base of the wall, (*b*) the point at which the maximum ring tension occurs, and (*c*) the magnitude of the maximum ring tension.

The coefficients and formulæ for determining these factors given on *Table* No. 20 are derived from Mr. H. Carpenter's simplification of Dr. Reissner's exhaustive mathematical consideration of the subject. In the complete study the shape of the wall enters into consideration, but the difference between the moments at the bases of triangular or rectangular walls (other conditions being equal) is so small that the usual intermediate case of a trapezoidal section can be considered the same as a rectangular wall. The error involved would partly offset the error of assuming perfect fixity at the junction of the wall and base slab.

The design procedure is first to determine the maximum vertical moment and provide an equal resistance moment at the base of the wall. This is followed by the determination of the maximum ring tension and the point on the wall at which this occurs; sufficient steel (and concrete) must be provided at this point to take this maximum tension. Above this point the steel can be uniformly graded down to a nominal amount, and below this point the steel can be maintained equal to that required for the maximum ring tension.

Walls of Rectangular Tanks.—The bending moments and direct tension on the walls of rectangular tanks would be calculated in the same manner as described later (see p. 98) for bunkers. For impermeable construction, however,

using a concrete of $1 : 1\frac{2}{3} : 3\frac{1}{3}$ mix the maximum compressive stress in the concrete should not exceed 880 lb. per square inch, and the maximum tensile stress in reinforcement should not exceed 12,000 lb. per square inch. This stress, however, may be increased to 16,000 lb. per square inch for reinforcement on beam faces remote from the liquid. To minimise the risk of cracking, the tension in the concrete should not exceed 250 lb. per square inch. This requirement applies to all slabs with liquid on the tension face, but only to slabs less than 9 in. thick with tension on the face remote from the liquid. Neglecting the reinforcement, the resistance moment of a section at 250 lb. per square inch is $42bD^2$ where D is the total depth of the section. With a modular ratio $m = 12$ and minimum percentages of reinforcement the moment is $43bD^2$ and upwards. (See *Table* No. 31A.)

Bottoms of Elevated Tanks.—The design adopted for the bottoms of elevated tanks depends on the diameter of the tank and the head of water sustained. For very small tanks a flat beamless slab bottom is satisfactory, but beams are necessary for tanks from 10 ft. up to, say, 25 ft. in diameter. Alternative arrangements are indicated in *Fig.* 30(*a*) and (*b*). In *Fig.* 30(*a*) each beam is designed to span between opposite columns and takes one-quarter of the load on the tank bottom. The remainder of the load not taken by the two beams, together with the weight of the walls and the roof load, is taken directly on to the columns through the walls. In the arrangement shown in *Fig.* 30(*b*), each length of beam between columns takes the loading on the shaded area, and the remainder of the loading on the floor of the tank together with the weight of the walls and the roof load is equally divided between the eight cantilevered portions of the beams.

For large diameter tanks domical bottoms (and roofs) of either of the types shown in *Figs.* 30(*c*) and 30(*d*) are most economical, and although the shuttering is much more costly the saving in material over beam and slab construction is considerable. The ring beams marked R take the horizontal component of the thrust from the domes, and the thicknesses of the domes are determined by the magnitude of this thrust. The working compressive stress in the roof domes should be kept low, say 20 per cent. of the normal safe stress, in order to guard against localised increases due to incidental point loading or to non-uniform distribution of the superload. For the bottom domes, where the uniformity of the loading is more assured, a higher stress can be worked to, and about one-half to 1 per cent. of reinforcement should be provided in each direction. The shear around the periphery of the dome should also be investigated and sufficient concrete thickness provided to resist the shearing forces. Expressions for the maximum axial thrust and vertical shear around the edge of the dome, together with the resultant ring tension in the ring beams, are given on *Table* No. 22 (page 261).

The Intze form of tank bottom illustrated diagrammatically in *Fig.* 30(*d*) produces an economical design for large diameter tanks. The outward thrust from the top of the conical section is taken by the ring beam S, and the difference between the thrust from the bottom of the conical section and the thrust from the domical section is taken by the ring beam T. Referring again to *Table* No. 22, expressions are given for the forces, etc., in the various parts of such a tank bottom; the proportions of the rises and diameters of the conical and

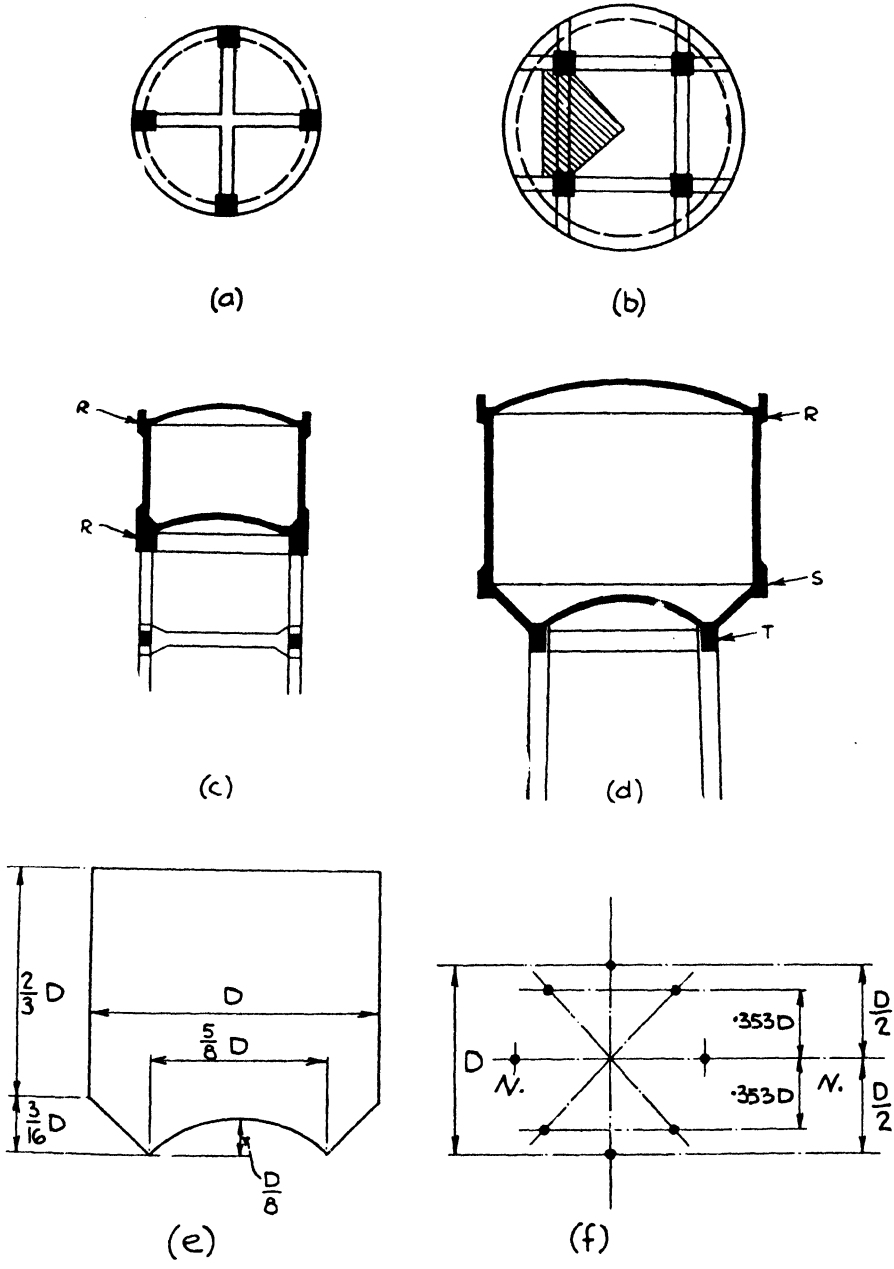


Fig. 30.—Water Towers.

domical sections can be so arranged that the resultant thrust on T is zero. Suitable proportions for bottoms of this type are given in *Fig.* 30(*e*), and the volume for a tank of diameter D adopting these proportions is $0{\cdot}585D^3$. The walls of the cylindrical portion of the tank should be designed in accordance with the principles described in the previous paragraph, due account being taken of the cantilever moment at the base of the wall and its effect as a transverse moment in the conical section.

The floors of tanks must be designed to take, in addition to any of the forces and moments already described, any direct tensions due to vertical spanning, or partial cantilever action, of the walls.

It should also be observed that the weight of liquid in any single compartment should be treated as a dead load when assessing bending moment coefficients, as all spans will be loaded simultaneously. When designing tee-beams in tank bottoms for negative bending moments, the section should be investigated to ensure that the tension in the concrete (on the water face) does not exceed 250 lb. per square inch. For this purpose the width of slab considered as part of the tee should be taken from *Table* No. 32 (page 281).

Columns Supporting Elevated Tanks.—It is important that there should not be unequal settlement of column foundations supporting elevated tanks and for this reason, if the ground is at all doubtful, a raft foundation is preferable to any other.

In addition to the bending moments and shears due to wind pressure on the tank, as described in Chapter V, (*Fig.* 17,), the wind moment induces direct forces in the columns; these are thrusts on the leeward side and tensions on the windward side. The value of this thrust or tension can be calculated simply for a group of four columns. If the wind moment is M_W ft.-lb. and the distance apart of adjacent columns is D ft. the following expressions apply:

Wind normal to D: $M_W = PH$ (*Fig.* 17,)

$$\text{Force in each column on leeward side} = +\frac{M_W}{2D}$$

$$\text{Force in each column on windward side} = -\frac{M_W}{2D}$$

Wind normal to diagonal:

$$\text{Force in leeward corner column} = +\frac{M_W}{D\sqrt{2}}$$

$$\text{Force in windward corner column} = -\frac{M_W}{D\sqrt{2}}.$$

For any other arrangement of columns, the force in any column can be calculated from the moment of inertia of the group. For example, consider a wind moment of M_W due to wind normal to the axis NN of the group of eight columns illustrated in *Fig.* 30(*f*). The moment of inertia of the group about NN is

$$\begin{aligned} 2 \times (0{\cdot}5D)^2 &= 0{\cdot}5D^2 \\ 4 \times (0{\cdot}353D)^2 &= 0{\cdot}5D^2 \\ \hline I &= 1{\cdot}0D^2. \end{aligned}$$

$$\text{Force on extreme leeward column} = + \frac{M_W \times 0{\cdot}5D}{1{\cdot}0D^2}.$$

The forces on the remaining columns would be determined similarly, substituting the appropriate "arm" for $0{\cdot}5D$.

Temperature Effects.—For tanks containing hot liquids, lower working stresses should be used for the bending moments and forces due to the pressure of the contained liquids. A method of determining the increased stress due to temperature changes is described in the next chapter.

In Great Britain the effects of temperature due to weather variations are seldom sufficiently serious to be considered in tank design, but elsewhere it may be necessary to protect the reinforced concrete structure from extreme exposure to the sun. External linings of timber, brickwork, or similar material may be necessary or, alternatively, it will be essential to design for bending moments due to differences of temperature on opposite faces.

Joints.—Details of joints suitable for reservoirs, swimming pools, tanks, etc. are given in the next chapter. The reader is also referred to the summary of experiences and recommendations given in the Institution of Civil Engineers Code of Practice for reinforced concrete structures for the storage of liquids.

Underground Tanks.—Underground or submerged tanks will be subjected to external pressures due to the surrounding earth or water, which will produce direct compression in the walls. The stress produced by this compression on the walls of cylindrical tanks will be a maximum when the tank is empty, and is given by

$$c = \frac{p_e D}{2[12d + (m - 1)A_T]}$$

where p_e = intensity of external pressure at the depth considered (see Chapter III).

This stress is usually well within safe limits. Unless conditions are such that the permanence of the external pressure is assured, the tensional relief provided by the compression should be disregarded altogether in the design of the tank when full. When empty, the structure should be investigated for flotation if it is submerged in a liquid or is in waterlogged ground.

Reservoirs with earth banked-up against the walls and over the roof should be designed for earth pressure from outside with the tank empty, the full pressure given by the Rankine or similar formula being assumed. When the container is full, in general no reduction should be made to the internal pressure by reason of the external pressure, but in cases where the designer considers such reduction justified, the amount of the reduction should be considerably less than the full theoretical Rankine pressure.

The earth cover over the roof of a reservoir should be considered as a live load, although it is ultimately a uniform load over all spans simultaneously. When the earth is being placed in position, conditions may occur whereby some spans are loaded and others are unloaded. Often, however, the designer can specify and ensure that the earth cover can be applied in such a way as to minimise the bending moments.

4.—Bunkers and Silos.

Properties of Contained Materials.—The weights of materials that are stored in bunkers, etc. are discussed in Chapter II and included on *Table* No. 4 (page 217), while the pressures due to these materials are dealt with in Chapter III and *Tables* Nos. 5 and 6 (pages 219 and 221). There are, however, usually two aspects of these properties to be considered, namely, (i) when proportioning a structure to give a specified capacity, the weight per cubic foot of the material should not be overrated nor too small a value given to the angle of repose; (ii) when calculating the weight to be carried on the bottom and the pressures to which the sides will be subjected, the weight should not be underestimated nor the angle of repose assumed too high. Usually two sets of properties are therefore considered in designing a container; examples of these are as follows.

Material.	Capacity calculations.		Strength calculations.	
	Unit weight (lb. per cubic foot)	Angle of repose (deg.)	Unit weight (lb. per cubic foot)	Angle of repose (deg.)
Unwashed coal (dry)	45	45	58	40
Washed coal (dry)	45	45	56	40
Coke and breeze	30	45	35	40
Shale (colliery dirt)	60	45	70	35
Coal slurry	50	40	62·4	20 to 0
Wet coal (15 per cent. of moisture)	45	45	56	25
Fine coal	45	40	56	20

Walls.—The walls of bunkers and silos have to be designed to take bending moments and direct tensions induced by the contained material. If the wall slabs span horizontally they have to be designed for the combined effect of the moments and direct forces in accordance with the methods described in Chapter XIV. If the walls span vertically horizontal steel should be provided to take the direct tensions and vertical steel to take the bending moments. In this case also the horizontal bending moments due to continuity at corners should be considered, and it is usually sufficient if as much horizontal steel is provided at any level at the corners as is normally required for vertical bending at this level; the maximum amount of steel provided for this purpose, however, need not exceed the amount of vertical steel required at one-third the height of the wall. The principal bending moments on walls spanning vertically will be due to the triangularly distributed pressure from the contained material. Bending moment coefficients for such loading are given on *Table* No. 7B (page 225).

For walls spanning horizontally the bending moments and forces will depend upon the number and arrangement of the compartments. For multi-compartment structures the intermediate walls act as ties between the outer walls, and on *Table* No. 21 (page 259) expressions are given for the negative moments in the outer walls of rectangular bunkers with various arrangements of intermediate walls or ties. The corresponding expressions for the reactions, which are a measure of the direct tensions in the walls, are also tabulated.

Any particular span of an exterior wall is subject to its maximum stresses when the adjacent compartment is loaded, since in this condition it is subjected to both the maximum moment and maximum direct force due to the filling. An intermediate wall is subject to maximum bending moment when the compartment on one side of it is filled, and to maximum direct force (but minimum moment) when both adjacent compartments are loaded.

For small bunkers the wall panels may be of such proportions that it is convenient to span them in two directions, in which case the bending moment coefficients and formulæ given on *Table* No. 14B are applicable as the pressure along the horizontal span will be uniformly distributed, while along the vertical span triangular distribution will occur.

In the case of an elevated bunker the whole load is usually transferred to the columns by the walls, and when the span exceeds twice the depth of the wall the latter can be designed as a normal beam. If the magnitude of the moment warrants, a compression head can conveniently be provided at the top of the wall, but there is usually ample space to accommodate the tension steel in the base of the wall. When the span between columns is less than twice the wall height the true beam action becomes less apparent; the wall is then more in the nature of an arch and requires reinforcement to take the tie force along the base of the wall. Sufficient reinforcement should be provided to take a direct force equal to one-quarter of the total load carried by the wall.

In addition to the stresses produced by vertical bending moment (if any) the stresses due to loads, other than the filling, carried by walls must be investigated. These loads may be due to the roof or superstructure or to cranes mounted above the bunker and to the weight of the wall itself. In the case of big bunkers the moments and forces due to wind pressure should be calculated, and in silos the direct compression force induced in the leeward walls by wind pressure is one of the principal forces to be investigated. The equally important stress due to the weight of filling sustained by friction on silo walls (see Chapter III) must be added to the stresses produced by wind pressure, and at the base and top of the walls there may be additional bending stresses due to continuity with the base slabs or covers.

Where walls differ in thickness between the top and base there are a number of factors to consider before deciding whether the wall should be tapered uniformly from top to bottom or whether the reduction in thickness should be made in a number of definite steps. The shuttering is somewhat more costly for tapered than for stepped walls, especially in the case of circular containers. Stepped walls, however, may induce high secondary stresses at the change in section, and since the daywork joints are usually arranged at the change in section liability to cracking is greater. Stepping on the outside is often objectionable as it provides ledges for the collection of coal, cement, or other dust that is usually in the air in an industrial works, and stepping on the inside may interfere with the free flow of the filling when emptying the bunker or silo.

In *Fig.* 31 typical arrangements of reinforcement in the walls and bottom of a bunker are illustrated, both for a structure where the walls span vertically and for one where they span horizontally. In the latter case the reinforcement will vary from a maximum at the bottom to a merely nominal amount at the top of the wall, and the vertical steel need only be sufficient to keep the horizontal bars in place; for general cases ⅜-in. bars at 12-in. to 15-in. centres are satisfactory. In the case of tall bunkers each lift of vertical steel should not exceed about 10 ft., to prevent distortion due to "whippiness."

The useful life of a reinforced concrete bunker is considerably in excess of that of a steel or timber structure, but this life can be further prolonged by careful detailing. By thus reducing the liability to cracking, the concrete does

not deteriorate so rapidly under service conditions due to abrasion from falling and moving coal or stone or other hard filling. If attention is given to the secondary stresses and care is exercised to obtain a good dense concrete, lining the wearing faces of the structure with tiles or plates is unnecessary except where coke is being stored or where coal or stone is dropped from a height. The

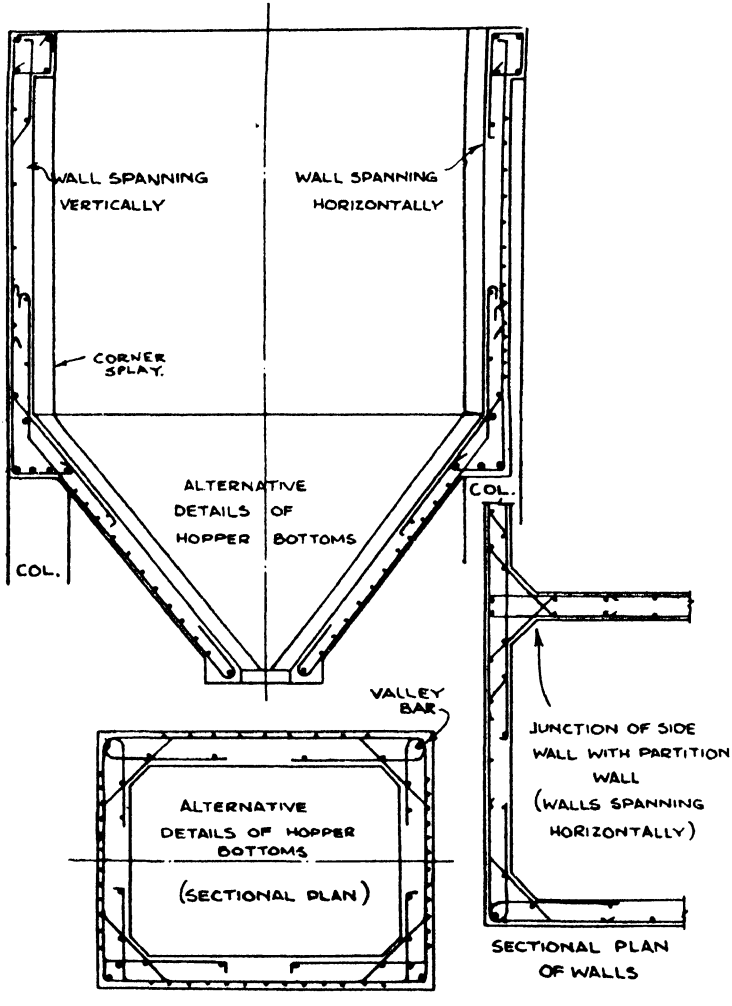


Fig. 31.—Typical Details of Bunkers.

top surfaces of ties and intermediate walls exposed to falling material should be made in the form of an inverted **V** and should be protected by a replaceable metal shield.

Hopper Bottoms.—The design of hopper bottoms in the form of inverted truncated pyramids consists of finding a " centre of pressure " and the normal pressure at this point for each sloping side. With a determined mean span the

bending moments at the centre and edge of the sloping side considered can be calculated. The horizontal direct tension is then computed and the horizontal steel determined. The direct tension acting along the slope at the mean centre and the moment at this point are combined to find the reinforcement necessary in the underside of the slab at this point. Likewise at the top of the slope the moment and the component of the hanging-up force are combined in order to calculate the steel required in the upper face of the sloping side at this point.

The "centre of pressure" and the mean span can be most economically found by inscribing on a normal plan of the sloping side considered a circle touching three of the sides. The diameter of this circle is "the mean span" and the centre is the "centre of pressure." The total load normal to the slab at this point is the sum of the normal components of the vertical and horizontal pressures at the "centre of pressure" and the dead weight of the slab; the values of the pressures, etc., and the resulting moments, together with the direct tensions both along the slope and horizontally, are given on *Table* No. 22 (page 261) for the critical parts of the structure. An example of a hopper bottom design is given in the "Additional Examples" that follow *Table* No. 40 (page 310).

In adopting this method of design it should be remembered that, although the horizontal span of any side is considerably reduced towards the outlet, the steel should not be reduced below that determined for the "centre of pressure," since in determining the moment based on the mean span adequate transverse support from the steel towards the base is assumed. In *Fig.* 31 alternative commonly adopted details of reinforcement for a hopper bottom are illustrated.

The "hanging-up" force along the slope will have both vertical and horizontal components, the former being resisted by the walls acting as beams. The horizontal component, acting inwards, will produce horizontal bending moments in the beam at the top of the slope, but in normal cases this bending moment is neglected as the inward horizontal force is usually resisted by a corresponding outward passive pressure from the contained material. When there is insufficient head of material above the top of the slope to create the necessary counter-pressure, it is necessary to calculate and design for the resulting horizontal bending moment.

5.—Impermeability and Protection of Concrete.

The design of impermeable structures is considered elsewhere in this volume, for example, in the earlier part of this chapter, in Chapter IX, etc. Allied to special design is first-class construction. The principles of the latter are dealt with in greater detail in the author's volume "Concrete Construction," where also the subjects of integral waterproofing and the provision of waterproof linings are discussed. Many structures have to be designed to resist corrosion by acids and other chemicals which have an injurious effect on concrete. The consideration of the effect on the concrete of these chemicals, fumes, etc., is necessary when designing storage tanks, chimneys, floors of certain factories, etc. The following list includes non-injurious as well as corrosive substances and liquids, and in the latter case indications of recommended treatments are given. This list is compiled from information already published in many countries, and in some cases the origin is obscure because the recommendations have been

copied so often in different publications. For this reason the list is given with some reserve.

Where corrosive liquids are stored the corrosion of the concrete is generally most marked near the normal level of the contained liquid, and certain liquids, for example, vegetable oils, which may not normally affect concrete may cause corrosion when the concrete surface is alternately submerged and exposed.

ACCUMULATOR ACID.—Sulphuric acid diluted in distilled water to a specific gravity of 1·22 approximately. Protection of concrete is necessary, bitumastic compositions, acid-resisting asphalt or paint being recommended. See also "Sulphuric Acid."

ACETIC ACID.—Causes slow disintegration from which concrete can be protected by the application of spar varnish, bituminous enamels, phenol-formaldehyde varnish, paraffin wax, hard rubber, or, for concentrations up to 60 per cent., acid-resisting paints.

ACIDS.—All acids attack concrete to some extent, strong acids being particularly virulent, although weak acids can be stored in untreated concrete tanks if the concrete is impermeable. The earlier remarks concerning impermeable construction should be observed. Greater immunity from attack by weak acids can be obtained by using aluminous cements or by impregnation with fluosilicate solution. The latter, and similar treatments, become ineffective if the skin is broken. The calcium salts contained in set cement are soluble in nearly all mineral or organic acids, and some types of acid-resisting preparations are admixtures that form insoluble substances upon combination with the acid-soluble portions of the cement; the admixture, without this chemical combination, of merely acid-proof insoluble materials is not sufficient to withstand the attacks of strong acids. For the most corrosive acids, linings of asphalt, bricks, tiles, paints, enamel, rubber, or proprietary acid-resistant coatings are usually recommended as given for specific acids in the present list. It should be noticed that while most common acids, for example, nitric and sulphuric acid, are more destructive in high concentrations, hydrochloric acid is most injurious in lower concentrations. Proprietary acid-resisting cements are obtainable, some of which are said to be resistant to all acids (except hydrochloric) in all strengths and up to high temperatures. For weak and medium strength acid solutions, treatment with paraffin wax or sodium silicate is adopted.

ACID VAPOURS.—Occur in dye-works, pickling plants, bleaching works, etc.; applications such as proprietary lacquers provide protection.

ALCOHOL.—Little or no effect on concrete, but some authorities recommend protection by acid-resisting paints.

ALKALIS.—See Ammonia, Caustic Soda, Calcium, etc.

ALMOND OIL.—Remarks as for coco-nut oil. Varnish is also recommended as a protective coating.

ALUMINIUM.—Aluminium (and to a less extent zinc and lead) can be seriously corroded when in contact with freshly placed Portland cement mortar or concrete. The corrosion is said to be due to the alkaline moisture that exudes from the concrete as a result of the setting process or changes in humidity. This moisture only attains an appreciable calcium hydroxide content after prolonged leaching out, but when the calcium hydroxide exceeds 50 per cent. of saturation value, crystalline calcium aluminate is precipitated, forming a protective coating which

retards further corrosion. Corrosion can be prevented by inserting an impermeable layer of bituminous material between the concrete and the aluminium lining. Glycerine-litharge mortars have been recommended in place of Portland cement and similar mortars when setting aluminium linings, and a fairly strong material can be made by mixing 9 parts of crushed limestone (run of crusher with fines) and one part of litharge with a solution of approximately 35 per cent. of crude commercial glycerine and 65 per cent. of water.

ALUMINIUM SULPHATE.—See "Sulphates."

AMMONIA.—Hard rubber or special paints recommended as protection.

AMMONIUM CHLORIDE.—Remarks as for magnesium chloride.

AMMONIUM LIQUOR.—Little effect on concrete; treatment sometimes recommended.

AMMONIUM NITRATE.—Disintegrates concrete, especially if the solution is in excess of ½ to 1 per cent., necessitating a protective coating as for sulphates.

AMMONIUM SULPHATE.—This is probably the most aggressive of the sulphate salts, and concrete floors of sulphate stores, etc. are protected by blue bricks laid on edge and set in pitch. In one reported case where sulphuretted hydrogen and ammonia, dissolved in moisture condensed on the surface of the concrete, had oxidised to ammonium sulphate causing disintegration of the concrete, a successful protection was obtained by treating the new concrete with a 30 per cent. solution of high-silica sodium silicate. (See also "Sulphates.")

AMMONIUM SULPHIDE.—Attacks concrete, necessitating protection as for sulphates.

ANHYDRITES.—Aluminous cement resists attacks.

ANTHROCENE.—No effect on concrete, but in some cases tanks are rendered with cement mortar to reduce seepage.

BEER.—Concrete fermentation tanks should be lined with enamel, glass, or "stainless" metal linings. Deleterious effects are most virulent when using Portland cement, aluminous cements being immune.

BENZINE.—Remarks as for petrol.

BENZOL.—No injurious effect on concrete, but slight seepage occurs. Recommended protection as for light fuel oils. A skimming coat of waterproof cement well worked in, followed by two coats of cement grout has proved successful.

BOILER FUMES.—See "Smoke."

BRINE.—Untreated concrete tanks are suitable for weak solutions, but strong solutions require protection by special brick linings, the jointing in which should be resistant to prevent seepage of brine behind the brickwork to attack concrete. Aluminous cements offer greater immunity than Portland cements.

CALCIUM CHLORIDE.—No effect on concrete.

CALCIUM NITRATE.—No effect on concrete.

CALCIUM SULPHATE (GYPSUM).—See "Sulphates." Often found in clays, the ground water from which may contain solutions of sufficient strength to attack Portland cement concretes, although aluminous cement concretes may be immune. See also observations on Ground Waters in "Concrete Construction."

CARBOLIC ACID.—Attacks poor concretes, although cases have been recorded where untreated 6-in. concrete, 1 : 1½ : 3 mix with a steel-float finish has been successful.

CARBOLINEUM.—Remarks as for creosote.

CARBONATES.—At normal cencentrations these have practically no effect on concrete.

CARBONIC ACID.—When dry has no effect on concrete, but causes slow disintegration when in solution. Recommended treatments include asphalt, bituminous paints, coal-tar paints, fluosilicate, spar varnish, or phenol-formaldehyde resin; see also "Ground Waters" in "Concrete Construction."

CARBONZOL.—No effect on concrete.

CAUSTIC SODA.—Caustic alkalis have little effect on Portland cement concretes, although they seriously attack aluminous cement concretes. Some protection of all concretes is recommended, but cases are reported where 4 per cent. solutions have been stored successfully without treatment. Alkali-resisting paints will provide protection for solutions of any concentration.

CHLORIDES.—Some chlorides have little or no effect on concrete except in high concentrations, while others cause slow disintegration and require protection as for sulphates. (See also separate chlorides.)

CHLORIDE OF LIME.—See "Calcium Chloride."

CHLORINE SOLUTION.—Little effect on concrete, but treatment as for acids generally is preferable.

CIDER.—Paraffin wax treatment reported successful.

CITRIC ACID.—See "Acids." Acid-resisting paints provide protection against all concentrations.

COBALT SULPHATE.—See "Sulphates."

COCO-NUT OIL.—Slowly disintegrates concrete, which can be protected by fluosilicate, sodium silicate, or linseed oil.

COPPER CHLORIDE.—Attacks concrete slightly; treatment as for sulphates.

COPPER NITRATE.—Protection afforded by acid-resisting paints.

COPPER SULPHATE.—See "Sulphates."

CORN SYRUP.—Remarks as for glucose.

COTTON-SEED OIL.—Authorities differ regarding the extent to which concrete is disintegrated by this oil; some suggest that no protective treatment is required as the attack is only slight. Others report virulent attack, particularly on Portland cement concrete but less so with aluminous cements.

CREOSOTE.—Has a tendency to disintegrate concrete slowly, requiring protective coatings as recommended for petrol.

CRESOL.—Remarks as for creosote.

CUMOL.—Remarks as for benzol.

EPSOM SALTS.—See "Magnesium sulphate."

FERRIC CHLORIDE.—Remarks as for magnesium chloride.

FERRIC NITRATE.—See "Copper Nitrate."

FERRIC SULPHATE.—See "Sulphates."

FISH OIL.—Only slowly attacks concrete, and treatment with fluosilicate, sodium silicate, or linseed oil is recommended.

FLUORIDES.—No effect on concrete.

FOOT OIL.—Remarks as for fish oil.

FORMALDEHYDE.—Protection provided by paints.

FORMIC ACID.—See "Acids." Acid-resisting paints provide protection against solutions up to 60 per cent.

FUEL OILS.—See "Oils."

GALLIC ACID.—See "Ink."

GASOLINE.—See "Petrol."

GLAUBER SALT.—See "Sodium Sulphate."

GLUCOSE.—Slowly attacks concrete requiring protection as for sulphates.

GLYCERINE.—No protection required.

GYPSUM.—See "Calcium Sulphate."

HYDROCHLORIC ACID.—Particularly destructive, especially when hot. Diluted hydrochloric acid can penetrate thick coatings of paraffin wax. Protective treatment as for sulphuric acid, including rubber linings.

HYDROFLUORIC ACID.—Concrete requires protection. Lead linings are recommended, and rubber is reported as effective up to 150 deg. F. and for 50 per cent. solutions.

INK.—Contains acid (usually gallic acid). Storage vats require protection by one of the coatings given for acids or by proprietary ink-resisting linings.

IRON.—See ferric sulphate, ferric chloride, etc. Iron or steel built into or attached to exposed concrete surfaces may lead to discolorations due to rust.

JAM.—Concrete requires protection from boiling jam, aluminium linings (see "Aluminium") usually being provided. Where floors are exposed to spillings from jam boiling, the concrete needs protection; see "Sugar Juices."

KEROSENE.—See "Paraffin."

LACTIC ACID.—Derived from milk and occurs i whey and sour milk. Portland cement concretes are attacked and can be treated as for carbonic acid in water, or coatings of linseed oil, paraffin wax, or paints. Aluminous cement concretes are attacked to a much smaller degree. Sodium silicate will provide temporary protection against deterioration of concrete floors subject to milk spillings, but tiles laid in asphalt are recommended for permanent protection.

LARD.—Remarks as for fish oil; this also applies to lard oil.

LEAD.—For the effect on lead in contact with concrete see "Aluminium." Lead pipes passing through concrete can be protected by a coating of pitch or tar or by embedding the pipes in clay.

LEAD NITRATE.—See "Copper Nitrate."

LEAD SULPHATE.—See "Sulphates."

LIME.—See various calcium compounds. Gasworks lime attacks Portland cement, Aluminous cement concretes being more resistant. Portland cement treated with proprietary paints is also effective. Piles driven through gasworks lime should be protected by a coating of tar.

LINSEED OIL.—No effect on concrete; is used as a protection.

LUBRICATING OIL.—See "Oils."

MAGNESIUM CHLORIDE.—Slowly attacks concrete; protection as for sulphates. No protection required if in weak solutions as in ground waters.

MAGNESIUM SULPHATE (Epsom salts).—See "Sulphates" and "Calcium Sulphate."

MANGANESE SULPHATE.—See "Sulphates."

MERCURIC CHLORIDE.—Remarks as for magnesium chloride.

MILK.—See "Lactic Acid."

MOLASSES.—Slight attack on concrete requiring treatment as for sulphates.

NAPTHALENE.—Remarks as for petrol.

NICKEL SULPHATE.—See "Sulphates."

NITRATES.—Some nitrates have a deleterious effect, while concrete may be immune from attack by others; see separate references. At the low concentrations met in practice these salts may have no effect.

NITRIC ACID.—Remarks as for sulphuric acid, but rubber lining is not recommended by some authorities except for low concentrations.

NITRO-HYDROCHLORIC ACID.—Particularly destructive, especially when hot. See "Hydrochloric Acid."

OILS.—Oils may be either mineral, vegetable, or animal products; vegetable oils only attack concrete to any degree. Oils in the present list should be referred to under separate headings. Light fuel oils, such as petrol, paraffin, benzol, etc., can be stored in concrete without injurious effect, but applications of spar varnish, linseed oil or sodium silicate are recommended to provide protection against seepage of oils of specific gravity less than 0·875 (54½ lb. per cubic foot) equivalent to not under 30 deg. Baumé. No protective treatment is required for heavier oils, as seepage may be negligible. Resinous coatings such as an alcoholic solution of a phenol condensation product, or an oil-resisting paint, are recommended as a protection against fatty oils. Concrete can be protected from the penetrating effects of lubricating oils or transformer oils by using oil-resisting lacquer.

OLIVE OIL.—Causes slow disintegration. Recommended treatments as for coco-nut oil, or with spar varnish or bakelite varnish.

ORGANIC MATERIALS.—See various oils, alcohol, formaldehyde, etc.

OXALIC ACID.—No deleterious effect on concrete, since it combines with free lime to form insoluble and non-corrosive calcium oxalate.

PARAFFIN.—Remarks as for petrol.

PEANUT OIL.—Remarks as for almond oil.

PETROL.—Although having no deleterious effect on concrete, the loss due to seepage may be considerable as petrol may find its way through pores that are impermeable to water. Application of fluosilicate, spar varnish, sodium silicate, or phenol-formaldehyde varnish is recommended.

PHENOL.—Remarks as for creosote.

PHOSPHATES.—These salts only attack concrete when in high concentrations.

PHOSPHORIC ACID.—Slowly disintegrates concrete due to the formation of soluble acid phosphates. Requires protection as for acids generally, acid-resisting paint providing protection from all concentrations. If rubber is used, it should be specially compounded.

PITCH.—No effect on concrete. Used as a protective coating in some cases. Tar tanks are sometimes rendered to prevent percolation through construction joints.

POPPYSEED OIL.—Remarks as for almond oil.

POTASSIUM CHLORIDE.—No effect on concrete.

POTASSIUM NITRATE.—No effect on concrete.

POTASSIUM SULPHATE.—See "Sulphates."

PULP (WOOD).—No effect on concrete.

PYRITES.—Slowly attacks concrete, requiring protection as for sulphates.

RAPE-SEED OIL.—Remarks as for olive oil.

RESIN.—No deleterious effect on concrete; used as a protective coating.

RUBBER.—Used as a protection for concrete containers holding corrosive

liquids. Most inorganic acids (except nitric acid) of any concentration and at temperatures up to 150 deg. F. can be stored in soft rubber or hard rubber (vulcanite) linings, although 50 per cent. concentration is the limit for hydrofluoric and sulphuric acids and 85 per cent. for phosphoric acid. If soft rubber (and in some cases, hard rubber) is used for certain acids, it is necessary to use a special grade of rubber. Similarly most solutions of inorganic salts and alkalis up to saturation point and 150 deg. F. temperature, and most organic materials up to the same limits, can be stored in rubber-lined tanks. Hard rubber is generally more resistant to chemical attack than soft rubber. Rubber is not generally recommended for protection against powerful oxidizing agents such as nitric and chromic acids, ozone, etc.

SALT (COMMON).—See "Brine" and "Sodium Chloride."

SALTS.—Soluble inorganic salts attack concrete; the descending order of virulence has been given as follows: sulphates, sulphides, nitrates, chlorides, and carbonates. See under each of these headings and separate substances for particulars of treatment.

SEWAGE.—Usually the acid content is low, otherwise sewage may have deleterious effect on concrete. Aluminous cements are more resistant than Portland cements. Protection and impermeability have been obtained by applying ½-in. rendering to walls and ¾-in. rendering to inverts, using a Portland cement mortar with a waterproof admixture, and applying the mortar to a concrete surface that had been well hacked immediately after the removal of the shuttering. Surface treatments with bitumen or fat are not permanent. Where disintegrating action has been very pronounced, concrete pipes have been lined with vitrified clay tiles and the joints pointed with bitumen emulsion. The attack arising from sewage, which results in the presence of hydrogen sulphide, is usually limited to the concrete at and above normal liquid level.

SILICATES.—These have no deleterious effect on concrete (see also "Sodium Silicate").

SMOKE.—Where concrete is exposed to smoke, the sulphur in which leads to the formation of sulphuric acid, protection is required. See "Sulphuric Acid."

SODIUM CHLORIDE.—No deleterious effect on concrete.

SODIUM NITRATE.—No effect on concrete.

SODIUM SILICATE.—No deleterious effect on concrete, and is widely used as a protective coating and pore filler.

SODIUM SULPHATE (Glauber salt).—See "Sulphates" and "Calcium Sulphate." Steam curing has been specially recommended for protection of concrete against sodium sulphate.

SOYA BEAN OIL.—Remarks as for almond oil.

STRONTIUM CHLORIDE.—No effect on concrete.

SUGAR JUICES.—These have a deleterious effect on Portland cement concrete but no effect on aluminous cement concretes.

SULPHATES.—Most sulphates vigorously attack concrete, the intensity of the attack increasing with the concentration of the solution up to 1 per cent., with little increased virulence beyond this limit. Recommended protective coatings include fluosilicate, sodium silicate, linseed oil, bitumens, glass linings, vitrified brick or tile laid in litharge, rubber and proprietary resisting paints. See also "Ammonium Sulphate," "Sodium Sulphate," etc.

SULPHIDES.—Most sulphides affect concrete, necessitating the protection used against sulphates. Sulphide ores only slowly attack concrete. Soluble sulphides, with the exception of ammonium sulphide, have no effect.

SULPHITE LIQUOR.—Slowly attacks concrete; protection as for sulphates recommended.

SULPHURIC ACID.—The disintegrating effect on concrete is guarded against by lining the container with glass, lead, or vitrified brick or tile. The effect of lead lining in contact with concretes should be considered. See "Lead" and "Aluminium." Rubber is also effective up to 150 deg. F. and for 50 per cent. solutions, while paints are effective for any concentration.

SULPHUROUS ACID.—Remarks as for sulphuric acid, but rubber is effective for all concentrations.

SWEET BOILING.—See "Jam."

TANNIC ACID.—Derived from tea and causes slow disintegration of Portland cement, but has little effect on aluminous cement. Treatment for Portland cement concrete as for carbonic acid in water.

TANNING LIQUOR.—If of the non-acid type, the concrete does not require protection, but if of the acid type it should be protected by acid-resisting bituminous paint.

TEA.—See "Tannic Acid."

TOLUOL.—Remarks as for benzol.

VINEGAR.—A weak solution of acetic acid which slowly disintegrates concrete. Protection by paraffin wax has been recommended.

WALNUT OIL.—Remarks as for almond oil.

WATER.—(See also "Brine" and "Concrete Construction" for impermeable construction.) Ordinary cold fresh water has no effect on concrete, but if hot water has to be stored it is necessary to ensure that temperature effects have been taken into account in the design.

WATERGLASS.—See "Sodium Silicate."

WHEY.—See "Lactic Acid."

WINE.—The treatment of the interior of vats with a solution of fluosilicate or with tartaric acid has been successful and had no ill-effect on the contents. Vats treated with potassium silicate solution are reported to contaminate the wine.

XYLOL.—Remarks as for benzol.

ZINC.—For remarks on the effect of zinc in contact with concrete, see "Aluminium."

ZINC SULPHATE.—See "Sulphates."

CHAPTER VIII

BRIDGES, BUILDINGS, AND OTHER STRUCTURES

ANY given structure is an assemblage of members each of which is subjected either to pure bending or to direct forces or to combined bending and direct force. The designer's art embraces not only the computation of and the provision for the moments and forces in a given assembly of members, but also the arrangement of these members in such a way that the moments and forces produced are reduced to minimum values consistent with the requirements of the structure and the limitations of the site. In previous chapters, in addition to outlining the methods of computing the moments, shears, and forces produced by the external loading, some indication has been given of the application of these methods to such structures as foundations, retaining walls, tanks, bunkers, and silos. In this present chapter it is proposed to deal briefly with other types of structures that are met with in designing practice. The necessary sections required to resist the calculated moments and forces on the various parts of the structure should be determined in accordance with the methods set out in later chapters.

1.—Bridges.

Bridge designs fall into two principal classes, namely, (i) arches and (ii) girder bridges. Bowstring girders can be considered as a special type of arch design, and rigid frame bridges as special types of girder design. The selection of a particular type depends on the span, nature of foundations, and headroom required.

Arch Design.—Within the scope of this volume it is impracticable to give to arch design the detailed consideration that the subject merits and necessitates. All that will be attempted is to give the data for the design of fixed-end arches for those cases when the designer has a free hand in choosing the profile of the arch, whether the ratio of span to rise is specified or not. The reader is referred to other volumes dealing solely with bridge construction that specially treat the simpler problem of hinged arches and the more difficult problem of designing fixed arches to specified profiles that do not approximate to parabolic.

Arches are of two principal types, namely, " hinged " or " fixed." A hinged arch can be either hinged at both supports, or at both supports and the crown. A fixed arch can either be rigidly fixed at the springings, or can be partly fixed if it is one of a series of continuous arches. A fixed arch is more economical in material than a hinged arch, but the calculations usually presuppose absolute

rigidity at the supports and freedom from settlement at these points. Unless these conditions can be absolutely assured it is safer to adopt a hinged design; further, the design procedure for a hinged arch is much simpler than that for a fixed arch, since at the hinges there is no bending moment.

Approximate Determination of Sections.—Any particular section of a fixed arch, whether it be an arch rib or an arched slab, is subjected to a bending moment and a thrust; the determination of the magnitude of these at the critical sections is the objective of the calculations. Arch design is a matter of trial and error since the section and shape of the arch rib or slab enter into the essential calculations, but it is possible to select a preliminary section that will give results that reduce the repetition of arithmetical work to a minimum. The following is a suggested method of determining the possible section at crown and springing, and is based on treating the fixed arch as a hinged arch. Referring to *Fig.* 32,

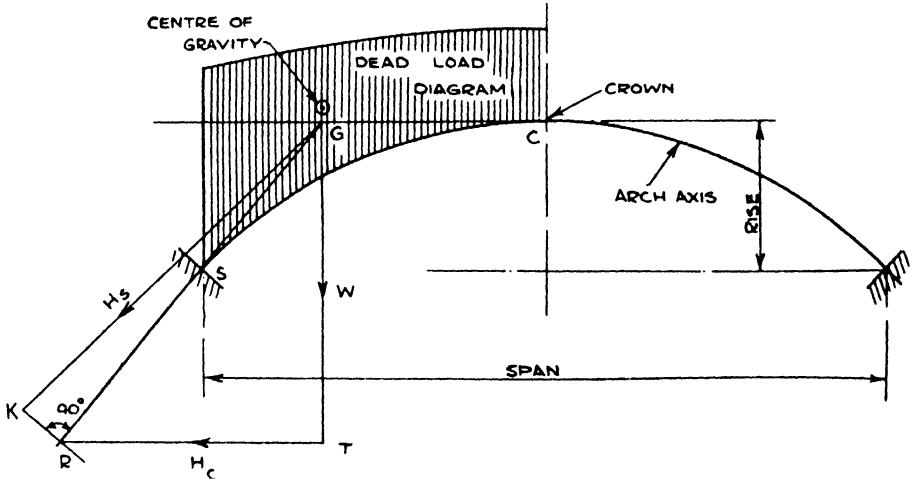


Fig. 32.—Approximate Method of Determining Thrusts.

draw a horizontal line through the crown C, and find G, the point of intersection with the vertical through the centre of gravity of the dead load on half the arch span. Set off GT equal to the dead load on the half span, drawn to a convenient scale; draw a horizontal through T to intersect GS produced in R. Draw RK perpendicular to GR, and GK parallel to the tangent to the arch axis at S. With the same units of weights used in drawing GT, scale off $TR = H_C$ and $GK = H_S$. If c is the maximum allowable compressive stress in the concrete, d is the arch thickness at the crown, d_s is the arch thickness at the springing, and b is the assumed breadth of arch rib (12 in. for an arch slab), then the following expressions approximately apply:

$$d = \frac{1{\cdot}7H_C}{cb}$$

$$d_s = \frac{2H_S}{cb}.$$

Having fixed approximate sizes thus, or otherwise, a calculation is made to determine the thrusts and moments, and the stresses produced by these are computed. These stresses will show whether the assumed sections are suitable.

Analysis of Fixed Arches.—Consideration in this paragraph will be limited to symmetrical, approximately parabolic, fixed-end arches that can be either open or closed spandrel arch rib or arch slab design as shown in *Fig.* 33, and the treatment is based on Mr. H. Carpenter's development of Dr. Strassner's methods. In all the cases considered it is assumed that the axis of the arch is made to coincide with the line of pressure due to the dead load, which gives the most economical design and simplest methods of calculation. If the increase in thickness of the arch ring from crown to springing is a parabolic variation, only the moments and thrusts at the crown and springing sections need be investigated.

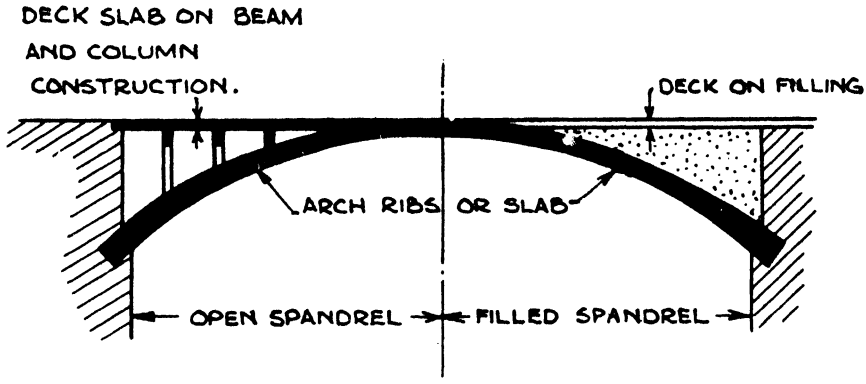


Fig. 33.—Arch Types.

Due to the dead loading alone the horizontal thrust is given by

$$H = \frac{k_1 w_d L^2}{R}$$

where w_d = dead load per unit length at the crown
L = span and R = rise of arch axis
and k_1 = a coefficient depending on the dead load at the springing; values are given on *Table* I (p. 113).

Due to the elastic deformation produced by the thrust along the arch axis the assumption of rigid abutments produces an anti-thrust = H_D which, while slightly reducing the thrust due to the dead load, renders this thrust eccentric, producing a positive bending moment at the crown and a negative bending moment at the springings. If d = thickness of arch at crown

$$H_D = -k_2\left(\frac{d}{R}\right)^2 H$$

where k_2 is a coefficient depending on the relative thicknesses at the crown and springing, and values of k_2 are obtained from *Table* II (p. 114).

Due to dead load and arch shortening, the resultant thrusts H_C and H_S at the crown and springing respectively and parallel to the arch axis at these sections, are given by

$$H_C = H - H_D$$

$$H_S = \frac{H}{\cos\phi} - H_D \cos\phi$$

where ϕ is the angle between the horizontal and the tangent to the arch axis at the springing. Values of $\cos\phi$ for various types of arches are given on *Table* III (p. 114).

The moments due to the eccentricity of H_C and H_S are given by

$$M_C = k_3 R H_D$$
$$M_S = (k_3 - 1) R H_D.$$

For values of k_3 see *Table* IV (p. 114).

For arches of small rise-span ratios, or with rings which are thick compared with the span, the stresses due to arch shortening may be excessive and the difficulty is overcome by introducing temporary hinges at the crown and springings. These hinges are filled with concrete after the arch shortening deformations have taken place; this method eliminates all bending stresses due to dead loading.

The additional horizontal thrust due to increase of temperature and the anti-thrust due to fall in temperature are given by the expression

$$H_T = \pm k_4 \left(\frac{d}{R}\right)^2 dt$$

where t = rise or fall in temperature in deg. F. and k_4 has the values given on *Table* V (p. 115). If d and R are in feet, H_T will be in lb. per foot width of arch. The values of k_4 are based on an elastic modulus for concrete $E_C = 2{,}000{,}000$ and a linear expansion coefficient $e = 0{\cdot}0000066$ per deg. F. If other values, say E_1 and e_1 are adopted, then k_4 should be multiplied by $0{\cdot}076\ E_1 e_1$. A value of t equal to 15 deg. F. is ample for structures in the British Isles, but careful consideration should be given to those factors that may necessitate an increase or may justify a decrease in the temperature range.

At the crown the increment or decrease in normal thrust due to temperature change is equal to H_T and the moment produced is given by $-k_3 R H_T$; due account must be taken of the sign of H_T when substituting. The normal thrust at the springing due to temperature change is given by $H_T \cos\phi$, and whether the thrusts due to dead load, etc., are augmented or decreased thereby depends upon the sign of H_T. At the springing the moment is given by $(1 - k_3) R H_T$ and the sign will be the same as that of H_T.

The shrinkage that takes place when concrete hardens produces anti-thrusts equivalent to a drop in temperature, and with the usual sectional method of constructing arch rings shrinkage may be allowed for by assuming it equal to

a 15-deg. F. drop in temperature and using the formula for H_T previously given.

The disposition of live loading on an arch to produce the maximum stresses on the critical sections follows from a study of influence lines, and the following approximate conclusions have been deduced.

(*a*) The maximum positive bending moment at the crown occurs when the mid-third section of the arch is loaded.

(*b*) The maximum negative moment at the springing occurs when four-tenths of the span adjacent to the springing considered is loaded.

(*c*) The maximum positive moment at the springing occurs when the whole span is loaded except the length of four-tenths of the span adjacent to the springing considered.

The maximum bending moments and thrusts are given by the following expressions, in which w is the intensity of uniformly distributed loading equivalent to the specified live load:

Maximum positive B.M. at crown,	$k_5 wL^2$
Corresponding horizontal thrust,	$k_6 w\frac{L^2}{R}$
Maximum negative B.M. at springing,	$k_7 wL^2$
Corresponding horizontal thrust,	$k_8 w\frac{L^2}{R}$
,, vertical reaction,	$k_9 wL$
Maximum positive B.M. at springing,	$k_{10} wL^2$
Corresponding horizontal thrust,	$k_{11} w\frac{L^2}{R}$
,, vertical reaction,	$k_{12} wL$.

The value of the coefficients k_5 to k_{12} are given on *Tables* VI, VII, and VIII (pp. 115-6). In the foregoing expressions, the live load is expressed in terms of an equivalent uniformly distributed load. The table in *Fig.* 1 (p. 14) gives the Ministry of Transport values for the distributed load, but it should be remembered that these figures have to be combined with the prescribed knife-edge load. If H and V are the appropriate horizontal thrust and vertical reaction, the corresponding normal thrust at the springing is given by

$$T = H \cos \phi + V\sqrt{1 - \cos^2 \phi}.$$

TABLE I.—HORIZONTAL THRUST DUE TO DEAD LOAD

Values of k_1

Rise ÷ span	0·10	0·15	0·20	0·25
Uniform dead load	0·125	0·125	0·125	0·125
Open spandrel	0·135	0·140	0·144	0·148
Filled spandrel	0·160	0·176	0·190	0·204

TABLE II.—HORIZONTAL THRUST DUE TO ARCH SHORTENING

Values of k_2

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	$\frac{d_s}{d}$ = 1·25	1·10	1·07	1·03	0·99
	1·50	1·42	1·37	1·32	1·25
	1·75	1·68	1·63	1·58	1·53
Open spandrel	$\frac{d_s}{d}$ = 1·25	1·13	1·08	1·03	1·00
	1·50	1·44	1·39	1·33	1·27
	1·75	1·73	1·68	1·63	1·58
Closed spandrel	$\frac{d_s}{d}$ = 1·25	1·19	1·13	1·08	1·00
	1·50	1·53	1·48	1·42	1·33
	1·75	1·86	1·82	1·76	1·69

TABLE III.—INCLINATION OF ARCH AXIS AT SPRINGING

Values of cos ϕ

Rise ÷ span	0·10	0·15	0·20	0·25
Uniform dead load	0·930	0·848	0·781	0·709
Open spandrel	0·918	0·820	0·740	0·650
Closed spandrel	0·893	0·764	0·665	0·565

TABLE IV.—MOMENTS DUE TO ARCH SHORTENING AND TEMPERATURE CHANGE AND ECCENTRICITY OF THRUST

Values of k_3

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	$\frac{d_s}{d}$ = 1·25	0·284	0·293	0·300	0·307
	1·50	0·248	0·253	0·258	0·263
	1·75	0·223	0·227	0·231	0·235
Open spandrel	$\frac{d_s}{d}$ = 1·25	0·279	0·280	0·281	0·282
	1·50	0·240	0·243	0·247	0·251
	1·75	0·218	0·220	0·222	0·224
Closed spandrel	$\frac{d_s}{d}$ = 1·25	0·255	0·261	0·265	0·270
	1·50	0·224	0·226	0·228	0·230
	1·75	0·200	0·200	0·200	0·200

TABLE V.—HORIZONTAL FORCE DUE TO TEMPERATURE CHANGE

Values of $k_4 \div 10^3$

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	$\frac{d_s}{d} = 1·25$	2·54	2·42	2·32	2·22
	1·50	3·42	3·27	3·14	3·01
	1·75	4·26	4·10	3·94	3·78
Open spandrel	$\frac{d_s}{d} = 1·25$	2·58	2·46	2·33	2·19
	1·50	3·49	3·34	3·18	3·02
	1·75	4·37	4·21	4·04	3·83
Closed spandrel	$\frac{d_s}{d} = 1·25$	2·72	2·53	2·38	2·17
	1·50	3·74	3·55	3·34	3·12
	1·75	4·69	4·48	4·29	4·12

TABLE VI.—HORIZONTAL THRUSTS DUE TO LIVE LOAD

Values of k_6, k_8, *and* k_{11}

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	k_6	0·059	0·059	0·059	0·059
	k_8	0·039	0·039	0·039	0·039
	k_{11}	0·086	0·086	0·086	0·086
Open spandrel	k_6	0·062	0·064	0·065	0·066
	k_8	0·038	0·038	0·037	0·037
	k_{11}	0·088	0·089	0·090	0·092
Closed spandrel	k_6	0·070	0·074	0·077	0·080
	k_8	0·037	0·035	0·033	0·032
	k_{11}				
	$\frac{d_s}{d}$ 1·25	0·093	0·097	0·098	0·100
	$\frac{d_s}{d}$ 1·50	0·095	0·098	0·101	0·103
	$\frac{d_s}{d}$ 1·75	0·097	0·100	0·104	0·106

TABLE VII.—VERTICAL REACTIONS DUE TO LIVE LOAD

Values of k_9 and k_{12}

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	k_9	0·358	0·358	0·358	0·358
	k_{12}	0·149	0·149	0·149	0·149
Open spandrel	k_9	0·354	0·352	0·350	0·349
	k_{12}	0·150	0·151	0·153	0·155
Closed spandrel	k_9	0·342	0·337	0·330	0·321
	k_{12}	0·160	0·164	0·170	0·177

TABLE VIII.—BENDING MOMENTS DUE TO LIVE LOAD

Values of k_6

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	$\frac{d_s}{d}$ = 1·25	0·0048	0·0049	0·0051	0·0052
	1·50	0·0045	0·0046	0·0046	0·0047
	1·75	0·0042	0·0043	0·0043	0·0044
Open spandrel	$\frac{d_s}{d}$ = 1·25	0·0052	0·0054	0·0057	0·0060
	1·50	0·0048	0·0050	0·0052	0·0054
	1·75	0·0044	0·0046	0·0048	0·0050
Closed spandrel	$\frac{d_s}{d}$ = 1·25	0·0060	0·0069	0·0077	0·0084
	1·50	0·0056	0·0062	0·0068	0·0075
	1·75	0·0052	0·0058	0·0063	0·0068

Values of k_7

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	$\frac{d_s}{d}$ = 1·25	0·019	0·019	0·018	0·018
	1·50	0·021	0·021	0·020	0·020
	1·75	0·022	0·022	0·022	0·022
Open spandrel	$\frac{d_s}{d}$ = 1·25	0·018	0·018	0·017	0·017
	1·50	0·020	0·020	0·019	0·018
	1·75	0·022	0·021	0·020	0·020
Closed spandrel	$\frac{d_s}{d}$ = 1·25	0·017	0·015	0·014	0·014
	1·50	0·018	0·017	0·016	0·015
	1·75	0·020	0·018	0·017	0·016

Values of k_{10}

Rise ÷ span		0·10	0·15	0·20	0·25
Uniform dead load	$\frac{d_s}{d}$ = 1·25	0·019	0·019	0·018	0·018
	1·50	0·021	0·020	0·020	0·020
	1·75	0·022	0·022	0·022	0·022
Open spandrel	$\frac{d_s}{d}$ = 1·25	0·020	0·021	0·021	0·021
	1·50	0·022	0·023	0·023	0·023
	1·75	0·024	0·025	0·025	0·025
Closed spandrel	$\frac{d_s}{d}$ = 1·25	0·024	0·025	0·025	0·026
	1·50	0·026	0·027	0·028	0·028
	1·75	0·029	0·030	0·031	0·031

An example of the application of the foregoing analytical method of determining the moments and forces in an arch ring indicates the procedure.

DATA.—A fixed-end arched slab ; open spandrels.

Span: 150 ft. (measured horizontally between the intersection of the arch axis with the springing section)

Rise: 22 ft. 6 in. (being the rise of the arch axis within the 150-ft. span).

Slab thickness: 3 ft. 4½ in. at springing, 2 ft. 3 in. at crown.

Dead load: 250 lb. per square foot, excluding the weight of the arch slab.

Live load: 300 lb. per square foot. (This is approximately equal to the Ministry of Transport loading ; since the live load should extend over one-third or four-tenths of the span to give maximum bending moments, the corresponding loaded length will be between 50 and 60 ft. ; from *Fig.* 1 (p. 14) the equivalent distributed load is 220 lb. per square foot. The difference between this figure and 300 lb. per square foot will allow a margin for the single knife-edge load of 2,700 lb. per foot width of slab.)

Temperature range: ± 15 deg. F. ; $E_C = 2{,}000{,}000$; $e = 0{\cdot}0000066$.

Shrinkage: equivalent to temperature drop of 15 deg. F.

FACTORS.—Rise : span ratio $= \dfrac{22{\cdot}5}{150} = 0{\cdot}15$.

$$\frac{\text{Thickness at springing}}{\text{Thickness at crown}} = \frac{d_s}{d} = \frac{3{\cdot}375}{2{\cdot}25} = 1{\cdot}5.$$

Dead load at crown : as above = 250 lb. per square foot.
27-in. slab = 338 lb. ,, ,,

Total : $w_d = 588$ lb. ,, ,,

Angle of inclination of arch axis at springing (from *Table* III, p. 114) $\cos\phi = 0{\cdot}820$.

A strip of slab 12 in. wide will be considered.

Horizontal thrusts due to dead load, etc.:

Dead load (*Table* I, p. 113) $H = 0{\cdot}140 \times 588 \times \dfrac{150^2}{22{\cdot}5} = 82{,}200$ lb.

Arch shortening (*Table* II, p. 114) $H_D = -1{\cdot}39\left(\dfrac{2{\cdot}25}{22{\cdot}5}\right)^2 82{,}200 = -1{,}145$ lb.

Temperature change (*Table* V, p. 115) $H_T = \pm 3{\cdot}34 \times 10^3\left(\dfrac{2{\cdot}25}{22{\cdot}5}\right)^2 \times 2{\cdot}25 \times 15$
$= \pm 1{,}130$ lb.

Shrinkage (as for temperature) $= -1{,}130$ lb.

Crown:

Maximum positive B.M.	Moment in.-lb.	Thrust lb.
Dead load and arch shortening		
$H_C = 82{,}200 - 1{,}145 =$		80,855
(*Table* IV, p. 114)		
$M_C = 0{\cdot}243 \times 22{\cdot}5 \times 1{,}145 \times 12$	= 75,100	

Temperature drop: thrust H_T as before (*Table* IV)		− 1,130
B.M. = 0·243 × 22·5 × 1,130 × 12	= 74,300	
Shrinkage (as for temperature)	74,300	− 1,130
Live load (*Tables* VI and VIII, pp. 115 and 116)		
Thrust: $0{\cdot}064 \times 300 \times \frac{150^2}{22{\cdot}5}$	=	18,600
B.M.: 0·0050 × 300 × 150² × 12	= 405,000	
Total moment and thrust:	628,700	97,395

Springing: Maximum negative moment:	Moment in.-lb.	Thrust lb.
Dead load and arch shortening:		
$H_S = \frac{82{,}200}{0{\cdot}820} - 1{,}145 \times 0{\cdot}820 =$		99,260
M_S (*Table* IV, p. 114) = 0·757 × 22·5 × 1,145 × 12 =	− 234,200	
Temperature drop:		
Thrust = − 1,130 × 0·820		− 925
B.M. (*Table* IV) = − 0·757 × 22·5 × 1,130 × 12 =	− 231,000	
Shrinkage (as for temperature)	− 231,000	− 925
Live load (*Tables* VI, VII and VIII, pp. 115 and 116)		
$H = 0{\cdot}038 \times 300 \times \frac{150^2}{22{\cdot}5} = 11{,}400$ lb.		
$V = 0{\cdot}352 \times 300 \times 150 = 15{,}800$ lb.		
Normal thrust = $(11{,}400 \times 0{\cdot}820) + (15{,}800\sqrt{1 - 0{\cdot}820^2}) =$		14,500
Moment = 0·020 × 300 × 150² × 12 =	− 1,580,000	
Total moment and thrust:	− 2,276,200	111,910

Springing: Maximum positive moment:	Moment in.-lb.	Thrust lb.
Dead load and arch shortening as before	− 234,200	99,260
Temperature rise and shrinkage neutralise.		
Live load (*Tables* VI, VII, VIII, pp. 115 and 116).		
$H = 0{\cdot}089 \times 300 \times \frac{150^2}{22{\cdot}5} = 26{,}700$ lb.		
$V = 0{\cdot}151 \times 300 \times 150 = 6{,}800$ lb.		
Normal thrust: $(26{,}700 \times 0{\cdot}820) + 6{,}800\sqrt{1 - 0{\cdot}820^2} =$		24,200
Moment = 0·023 × 300 × 150² × 12 =	1,830,000	
Total moments and thrusts:	1,595,800	123,460

With the corresponding thrusts and moments thus determined the area of reinforcement and the stresses at crown and springing are found in accordance with the appropriate methods described in Chapter XIV. There only then remains to determine the intermediate sections and the profile of the arch axis. If the dead loading is uniform throughout (or practically so) the axis will be a parabola, but if it is non-uniform the axis must be shaped to coincide with the line of pressure for dead load. The latter can be plotted by force and link polygons (after the manner of ordinary graphic statics), the necessary data being the magnitude of the dead loading and the value of the horizontal thrust due to dead load and vertical reaction (which equals the dead load on half the span) at the springing. The line of pressure, and therefore the arch axis, being established, and given the requisite thicknesses of the arch at the crown and springing, the lines of the extrados and intrados can be plotted to give a parabolic variation between the two extremes. Thus, the thickness (normal to the arch axis) at any point is given by $[(d_s - d)r + d]$ where r has the following values:

Ratio of distance of point from springing measured along arch axis to half length of arch axis	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
Value of r	0·563	0·250	0·063

Girder Bridges.—Certain aspects of the design of girder bridges have already been dealt with, viz. loading for road bridges, wind loading, pavement loads, dispersal of wheel loads and width of slab considered as carrying a point load (all in Chapter II), influence lines for point loads passing over a system of continuous spans, etc. (Chapter IV). Bridges of less than 15 ft. width are most economical if the deck slab is spanned transversely between two outer longitudinal girders. These girders may be the parapets of the bridge, but for major structures the use of the parapets as principal structural members is not considered good practice. If the width of the bridge exceeds 15 ft. the most efficient design is produced by providing several longitudinal girders, usually spaced at about 7-ft. centres for bridges designed for the maximum loadings. The roadway is generally a multiple of 10 ft. in width. Unless the more simple Ministry of Transport loading (*Fig.* 1) is adopted, the deck slab is designed for one or more greatest wheel loads placed in positions to give maximum moments, and the longitudinal girders are designed for the maximum bending moments produced by a train of wheels directly over the beam with a parallel train as near as possible.

Footpaths are sometimes cantilevered off the principal part of the structure, and water, gas, and electric services are generally installed in a duct provided under the footpaths. The clearances required for the loading gauges of British railways are given on *Table* No. 39 (p. 297). See *Table* No. 41 (p. 300) also.

Girder bridges do not usually exceed 50 or 60 ft. in span, as beyond this limit an arch design is generally more economical. Even when a bridge has an arch-shape soffit, if the span-rise ratio exceeds ten, or if it is impracticable to provide adequate abutments to take the thrusts from an arch design, it is usual to design the bridge as a girder; in assessing the bending moments in such a case it is essential to take into account the variation in moment of inertia

(Chapter IV), since the inertia is usually considerably higher at the supports than at the crown. The moment of resistance at midspan should be about 25 per cent. in excess of the theoretical bending moment at this point to allow for increases due to slight settlement of supports and for other variations from the design assumptions.

Bridge Piers and Abutments.—The piers for girder bridges are usually subjected to vertical loading only due to the reactions from the girders; the abutments of girder bridges have to be designed for the vertical reaction from the ends of the girders and the horizontal thrusts due to earth back-filling. It is usual to provide the girders with a sliding seating on the abutments unless continuity with the abutments has been assumed as in rigid-frame bridges.

The piers and abutments of arch bridges have to be designed to take both vertical reactions and the horizontal thrusts from the arches. Stability is obtained by constructing massive piers in plain concrete or masonry, or by providing tension and compression piles, or by a cellular reinforced concrete box construction filled with earth. Part of the horizontal thrust on the abutments can be resisted by the active (or passive) earth pressure behind the abutment, but in the case of fixed arches passive pressure should only be assumed when the structure reacts upon very compact ground and care should be exercised before considering the counter-thrust from active earth pressure to ensure that this pressure will always be effective. Adequate resistance to sliding should be assured, and the possibilities of uplift from water-pressure should be investigated.

Mid-river piers, if not protected by independent fenders, should be designed to withstand blows from passing vessels or floating débris, and should be provided with cutwaters.

2.—Buildings.

Floors.—The floors of buildings are either beam-and-slab construction or "mushroom" (flat slab) construction (Chapter IV). If the spans of the slabs between the beams exceed 10 ft. it is more economical to provide a hollow-tile floor, which is light in weight and uses less concrete. Such a floor consists of a thin top slab (1½ in. to 3½ in. thick) overlying a series of concrete ribs extending through the full thickness of the floor construction. These ribs may be provided at 6-in. to 24-in. centres and may be from 2 in. to 5 in. wide. The spaces between the ribs can be kept open, but in order to simplify the shuttering they are usually filled with hollow clay tiles (comparable with permanent shuttering). The combined depth of the rib and slab is determined in the same way as the depth of a solid slab, and the thickness of the top slab is made sufficient to provide adequate compression area. The tensile steel is located in the bottom of the rib, as indicated in the design example on p. 315. The width of the rib is primarily determined by the shear force. The space or tile between the ribs replaces the useless concrete below the neutral axis and saves dead weight. Typical weights of solid and hollow tile floors are given on *Table* No. 1 (p. 211). There are several proprietary types of hollow-tile floor construction, but the simplest construction is not covered by patents.

Roofs.—Concrete roofs can also be constructed economically with hollow-tile slabs. Although more expensive in shuttering, domed roofs also lead to

light-weight construction; the forces produced and the method of design of such a type have already been dealt with when considering tanks (see Chapter VII). Roof slabs should not be less than 4 in. thick whether treated for water-tightness or not, and should be laid to a fall of 1 in. in 10 ft. to facilitate drainage.

Holes in Slabs.—Holes formed in roof or floor slabs should be trimmed on all sides with reinforcing bars, unless the hole is large compared with the span of the slab (for example, stair wells or lift openings), in which case trimmer beams should be provided. When the holes are small, as in manholes in tank roofs, ventilation ducts, etc., the cross-sectional area of the trimmer bars placed parallel to the principal reinforcement should be at least equal to the area of principal reinforcement cut out by the hole. Other trimmer bars need only be nominal in size.

Stairs.—Stairs can be designed to span transversely or longitudinally. When spanning transversely (parallel to the nosings) supports must be provided at both sides of the flight, either by walls or stringer beams. In this case the "waist" or thinnest part of the stair construction need only be, say, 3 in. thick, the effective lever-arm for resisting the bending moment being about one-half the maximum thickness from the nosing to the soffit measured normally to the soffit. When spanning longitudinally the thickness of slab required to resist bending determines the thickness of the "waist." The loadings for which stairs should be designed have been discussed in Chapter II, and the bending moments should be calculated from the total weight of the stairs and the total superimposed load combined with the span as measured on plan. The stresses produced by the longitudinal thrust are only small and are usually neglected. Unless circumstances dictate otherwise a reasonable profile for a step is 7 in. rise with 10-in. "going," with 1-in. nosing or undercut giving an 11-in. tread.

As an example of the design of a simple flight of stairs by the alternative methods, consider the problem illustrated in *Fig.* 34.

(*a*) The flight supported by longitudinal stringers along both edges; assume a minimum "waist" of 3 in., as in *Fig.* 34(*a*).

Clear span = 4 ft. Effective span = 4 ft. 6 in.

Dead load:	Step	$= 11 \times 7 \times \frac{1}{2}$	$= 38\frac{1}{2}$	lb. per ft. run of step.
	Waist	$= 13\frac{1}{2} \times 3$	$= 40\frac{1}{2}$	,, ,, ,,
	Granolithic	$= \frac{1}{2} \times 11$	$= 5\frac{1}{2}$	,, ,, ,,
			$84\frac{1}{2}$	,, ,, ,,

Total dead load $= 84\frac{1}{2} \times 4{\cdot}5 = 380$ lb. per step.

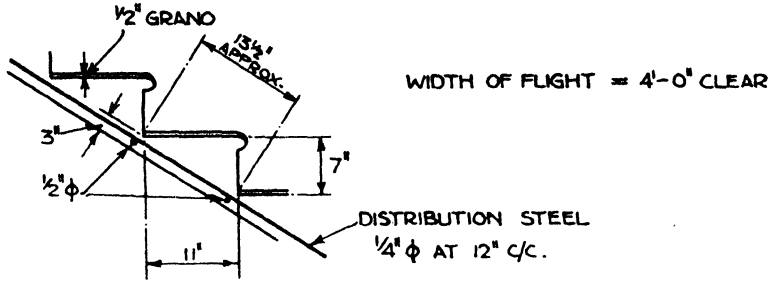
Live load: (See *Table* No. 2) $= \frac{11}{12} \times 100 \times 4{\cdot}0 = 367$ lb. per step.

Minimum total live load (*Table* No. 2) $= \frac{3}{8}$ ton $= 840$ lb. per ft. width.

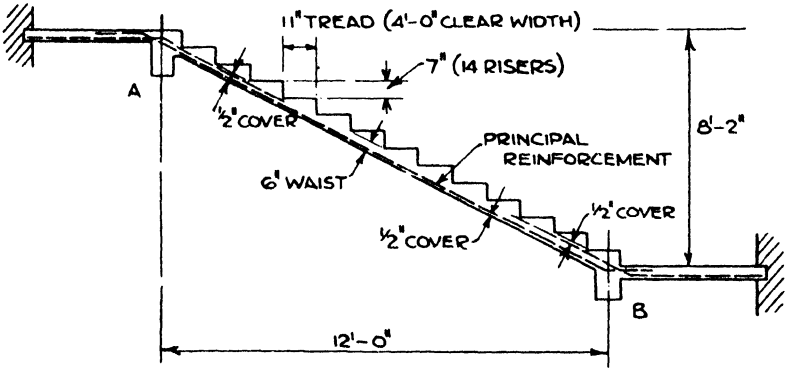
Total load $= 380 + (\frac{11}{12} \times 840) = 1{,}150$ lb. per step.

B.M. $= \frac{1}{8} \times 1{,}150 \times 4{\cdot}5 \times 12 = 7{,}763$ in.-lb. per step.

Assuming maximum stresses of 18,000 lb. and 750 lb. per square inch, from *Table* No. 31 (page 279), a slab a little over 3 in. thick is required; this is less than provided by the shape of the step, the mean effective depth of which is about 4 in.



(a) SUPPORTED ON STRINGER BEAMS.



(b) SPANNING BETWEEN BEAMS A AND B.

Fig. 34.—Details of Stairs.

Area of reinforcement

$$= \frac{7{,}763}{0{\cdot}87 \times 4{\cdot}0 \times 18{,}000} = 0{\cdot}123 \text{ sq. in. say, one } \tfrac{1}{2}\text{-in. bar per step.}$$

(*b*) The flight supported at the top and bottom on trimmer beams (or walls), beyond which the flight is continuous with landing slabs as shown in *Fig.* 34(*b*). Assume 6-in. waist; hence minimum effective depth = 5·25 in. Effective span = 12 ft.

Dead load:	Step = 11 × 7 × ½	=	38½	lb. per ft. run of step.	
	Waist = 13½ × 6	=	81	,,	,, ,,
	Granolithic = 11 × ½	=	5½	,,	,, ,,
Live load = 100 × $\frac{11}{12}$		=	92	,,	,, ,,
Total load		=	217	,,	,, ,,

Total load on 1 ft. width of flight (12 steps net) = 12 × 217 = 2,604 lb.

B.M. = $\frac{1}{10}$ × 2,604 × 12·0 × 12 = 37,598 in.-lb.

From *Table* No. 31, a 6-in. slab (waist) is required.

Principal longitudinal reinforcement

$$= \frac{37{,}598}{0{\cdot}87 \times 5{\cdot}25 \times 18{,}000} = 0{\cdot}466 \text{ sq. in. say, } \tfrac{1}{2}\text{-in. bars at 5-in. centres.}$$

Walls.—Curtain, panel, or filling walls in building structures, that is, walls not designed to take vertical loading or horizontal pressures, should be made not less than 4 in. thick for constructional reasons, and should be reinforced so as to span, if necessary, across their shortest dimension. Usually nominal reinforcement, say $\frac{3}{8}$-in. bars at 12-in. centres placed horizontally and vertically on both faces, is sufficient. Heavier bars should be provided above and at the sides of all door and window openings, and $\frac{1}{2}$-in. bars 5 ft. long should be placed diagonally across the corners of all such openings.

3.—Industrial Structures.

Utility rather than appearance is the predominant factor in the design of structures in connection with colliery works, steel works, by-product and gas works, quarries, and other essentially industrial plants, although some attention is given to the architecture of factories and such prominent structures as water towers, cooling towers, and tall bunkers. In addition to the capability of the various members to sustain the principal moments and forces to which they are subjected, there are other considerations peculiar to each type of industrial structure. Vibration must be allowed for in the design of the substructures for coal screening and stone crushing and screening plants. Provision against impact and extra security are obtained in pit-head gear design by adopting a factor of safety of ten on the breaking strength of the ropes for all forces due to suspended loads. Watertightness is an essential quality required in slurry basins, coal draining bunkers, settling tanks, and similar hydraulic structures, while airtightness is essential in gas purifiers and in airlock structures in connection with colliery work. The suction head on airlocks is usually 5 in. to 8 in. of water (that is, 26 lb. to 43 lb. per square inch). The freedom of concrete from corrosion by the fumes that are inseparable from certain industrial processes is one of the qualities that recommend the material for industrial construction, but protection is needed to prevent contact with certain liquids (see Chapter VII). Provision should be made for expansion in connection with steelworks, coke ovens, gas retorts, etc. Boiler foundations, especially on clay, should be made sufficiently thick to prevent undue heating and drying out of the subsoil. Firing floors, coke benches, and rolling-mill floors should be protected from extreme temperatures and abrasion by being covered with steel plates or brick linings.

Structures in mining districts should be designed to resist the moments and forces produced by a not unreasonable settlement of part of the ground upon which they are founded. Thus raft foundations that have at any section equal resistance to negative and positive moments are commonly adopted. If isolated foundations are provided, as for gantries, the beams should be designed for freely-supported moments at midspan, although some negative moment should be allowed for over the supports.

4.—Chimneys.

Determination of Section.—The two principal loadings on a chimney are the horizontal wind pressure and the dead weight of the stack. At any section the cantilever bending moment due to the former is combined with the direct

force due to the weight of the chimney above the section considered to find the maximum stresses. Suitable values for wind pressures on circular and other stacks are given in Chapter II, and an example showing the method of determining the stresses is described in Chapter XIV. Generally the preliminary determination of the section required is a matter of trial and error, but Messrs. Taylor, Glenday, and Faber have established the following expressions for circular stacks :

$$\text{Total area of vertical steel required} = \frac{M}{ar} - \frac{W}{b} \text{ (sq. in.)}$$

$$\text{Total thickness of concrete} = \frac{M}{cr^2} + \frac{W}{dr} \text{ (in.)}$$

where M and W are the moment (in.-lb.) and direct force (lb.) at any section and r is the mean radius at the section (in inches) ; the factors, a, b, c and d have the following values :

Stress in concrete, lb. per square inch	600	600	600	400
Stress in steel, lb. per square inch .	12,000	14,000	16,000	16,000
a	6,408	7,551	8,685	8,600
b	7,776	8,990	10,230	9,698
c	4,582	3,409	2,867	1,376
d	1,372	1,410	1,419	911

The preliminary section should be selected by these expressions, adopting stresses low enough to allow ample margin for temperature effects. Having decided upon a tentative section, this can then be more accurately analysed, as described later, to determine the maximum stresses under the worst conditions.

Temperature Stresses.—The following consideration of stresses due to restrained temperature effects, can be applied to the design of chimneys, tanks containing hot liquids, retort foundations, etc.

The first stage in the investigation of temperature stresses is to determine the change of temperature (T) through the concrete. The coefficient of heat transmission through a wall built up of different layers of materials, the successive thicknesses of which are d_1, d_2, d_3, etc., is given by

$$\frac{1}{k} = \frac{1}{a_i} + \frac{d_1}{k_1} + \frac{d_2}{k_2} + \quad . \quad . \quad . \quad . \quad + \frac{1}{a_0}$$

$$= \frac{1}{a_i} + \frac{1}{a_0} + \sum \frac{d}{k}$$

where k_1, k_2, k_3, etc., are the coefficients of transmission for the successive materials, and a_i and a_0 represent the drop at the internal and external faces respectively. Suitable values of the coefficients, if d is in feet, are

$k = 1{\cdot}0$ for concrete
$= 0{\cdot}5$ for firebrick
$= 0{\cdot}1$ for slag wool,
$a_0 = 2{\cdot}0$ (max.).

In the case of chimneys and tanks containing hot liquids, there may be

little difference between the temperature of the gas or liquids and the internal face of the concrete or lining in contact with the gas or liquid. Hence a_1 should generally be considered as zero. If the temperature of the flue gases (or hot liquid) is T_G and the external air temperature is T_A, the value of the change of temperature through a concrete wall d_c feet thick is given by

$$T = \frac{(T_G - T_A)kd_c}{k_c}$$

where k_c = coefficient of transmission through concrete. All the temperatures should be in degrees Fahrenheit.

On an uncracked reinforced concrete section, or on a cracked section that is in compression all over, the change in the compressive stress on the concrete due to a temperature difference of T deg. F. is

$$c_T = \pm \frac{T\varepsilon_c E_c}{2}$$

where ε_c = coefficient of linear expansion for concrete (say 0·000055 per degree F.)
E_c = modulus of elasticity of concrete. (Use the value given on *Table* No. 23 for "special calculations.")

On a cracked section subject to tension, the concrete being neglected except as covering and connecting the reinforcement, the change in stress in the reinforcement is

$$t_T = \pm \frac{fT\varepsilon_s E_s}{2}$$

where fd = the distance between the centres of the reinforcement on opposite faces of the wall.
ε_s = coefficient of linear expansion for steel (say 0·000006 per degree F.)
E_s = modulus of elasticity for steel = 30,000,000 lb. per square inch.

If the section to which the temperature strains have to be added is already stressed in tension on one face and compression on the other (as may occur in the case of the wall of a tank containing hot liquid), then to the bending moment on the section should be algebraically added a temperature bending moment of

$$M_T = \frac{T\varepsilon_c E_c I_c}{d_c}$$

where I_c = moment of inertia of the section expressed in concrete units and ignoring any concrete area that may be cracked in tension. In impermeable construction designed to prevent the concrete cracking, the value of I_c would be based on the total concrete section together with an allowance for the reinforcement with a modular ratio corresponding to the value assumed for E_c.

It should be observed that the bending moment M_T due to temperature change tends to produce compression on the face subject to the higher temperature.

Maximum Stresses.—The maximum stresses on any horizontal plane of a chimney shaft should be investigated for the following conditions:

(*a*) When subject to direct load only, that is under the weight of the concrete shaft and the lining, the maximum compressive stress should not exceed the values given for direct stress on *Table* No. 23 (page 263). For this purpose, the

modular ratio used in calculating the effect of the reinforcement can be taken as 15.

(*b*) The stresses due to combining the wind moment with the maximum direct load should be ascertained either by comparison with the factors tabulated above or by graphical analysis as given in Chapter XI. To the maximum compressive stress on the inner face of the wall should be added the stress, c_T, due to temperature change, and to the maximum tensile stress in the reinforcement on the outer face should be added the stress t_T due to temperature change, thereby giving the approximate maximum stresses realised. The maximum compressive stress in the concrete should not exceed the values given on *Table* No. 23 for bending.

The various stresses are inter-related, and the imposition of, say, the temperature stresses on the combined bending and direct stresses may alter the basis of calculating these stresses by modifying the position of the neutral axis by subjecting more of the concrete section to tensile stresses beyond the ultimate, thereby resulting in cracking.

Transverse Stresses.—The preceding analysis deals solely with stresses normal to a horizontal section. Stresses normal to a vertical section are also produced both by wind pressure and temperature differences. On chimney shafts of normal dimensions the transverse bending moment resulting from wind pressure is usually negligible, but is not necessarily so for the case of large diameter cooling towers. A wind pressure of p lb. per square foot produces a maximum bending moment of $\pm \frac{pD^2}{12}$ ft.-lb. on 1-ft. height of a cylinder D ft. in external diameter. This bending moment causes a compressive stress on the outer face of a wall normal to p and tension on the inner face. An equal moment of opposite sign acts on surfaces parallel to the line of action of p.

The difference in temperature between the two faces of a concrete wall results in a transverse bending moment equal to $\frac{T\varepsilon_c E_c I_c}{d_c}$ (the symbols being as explained before). Since the concrete in a chimney shaft is unlikely to be cracked in the direction in which this moment operates, the concrete stress due to this moment can be written as $0{\cdot}5\ T\varepsilon_c E_c$, being a compression on the high temperature face and a tension on the opposite face.

5.—Culverts.

Bending Moments.—The bending moments produced in rectangular culverts of normal design are determined by considering the four sides as a continuous beam of four spans with the moments at the end supports equal; thus by applying the Theorem of Three Moments three equations containing three unknowns are obtained for unsymmetrical loading. The loading can be conveniently divided into the following components:

(*a*) Uniformly distributed loading on top slab and equal reaction from earth below bottom slab.

(*b*) Superimposed point loads on top slab and equal reaction from earth below bottom slab.

(*c*) Upward pressure on bottom slab due to the weight of the walls.

(*d*) Triangularly distributed horizontal pressure on walls due to earth.
(*e*) Uniformly distributed horizontal pressure on walls due to earth and surcharge.
(*f*) Internal horizontal pressure from contents of culvert.

These loads are indicated in *Fig.* 35 and the bending moments at the corners due to the various types of loading can be found from the following formulæ which are applicable when the thickness of the top slab, walls, and bottom slab are practically equal.

For all formulæ: $k = \frac{H}{L}$

For all loadings: $M_A = M_B$; $M_C = M_D$.

All moments are expressed in ft.-lb. per foot run of culvert.

Loading as in *Fig.* 35(*a*), $M_A = -\frac{w_1 L^2}{12(k+1)} = M_C$.

Loading as in *Fig.* 35(*b*), d = width of slab assumed to be supporting point load.

$$M_A = -\frac{WL}{12d}\left[\frac{2k+4{\cdot}5}{(k+3)(k+1)}\right]$$

$$M_D = -\frac{WL}{24d}\left[\frac{k+6}{(k+3)(k+1)}\right].$$

Loading as in *Fig.* 35(*c*), $M_A = +\frac{w_2 L^2}{12}\left[\frac{k}{(k+3)(k+1)}\right]$

$$M_D = -\frac{w_2 L^2}{12}\left[\frac{3+2k}{(k+3)(k+1)}\right].$$

Loading as in *Fig.* 35(*d*), $M_A = -\frac{p_1 H^2}{60}\left[\frac{(2k+7)k}{(k+3)(k+1)}\right]$

$$M_D = -\frac{p_1 H^2}{60}\left[\frac{(3k+8)k}{(k+3)(k+1)}\right].$$

Loading as in *Fig.* 35(*e*), $M_A = -\frac{p_2 H^2 k}{12(k+1)} = M_D$.

Loading as in *Fig.* 35(*f*), Moments as for loading in *Fig.* 35(*d*), but reverse sign.

Loading as in *Fig.* 35(*g*), Moments as for loading in *Fig.* 35(*e*), but reverse sign.

Loading.—The loading w_1 on the top slab would include the weight of the filling, any uniformly distributed superload, and the weight of the top slab. When a trench has been excavated in consolidated ground for the construction of the culvert and the depth from the surface of the ground to the roof of the culvert exceeds $3L$, it is permissible to take the maximum load on the culvert roof as due to a head of earth equal to $3L$. Although the roof of a culvert passing through even a newly-filled embankment is probably not subjected to the weight of the full head of earth above the roof, there is little reliable information concerning the actual loading carried, and therefore any load reductions due to

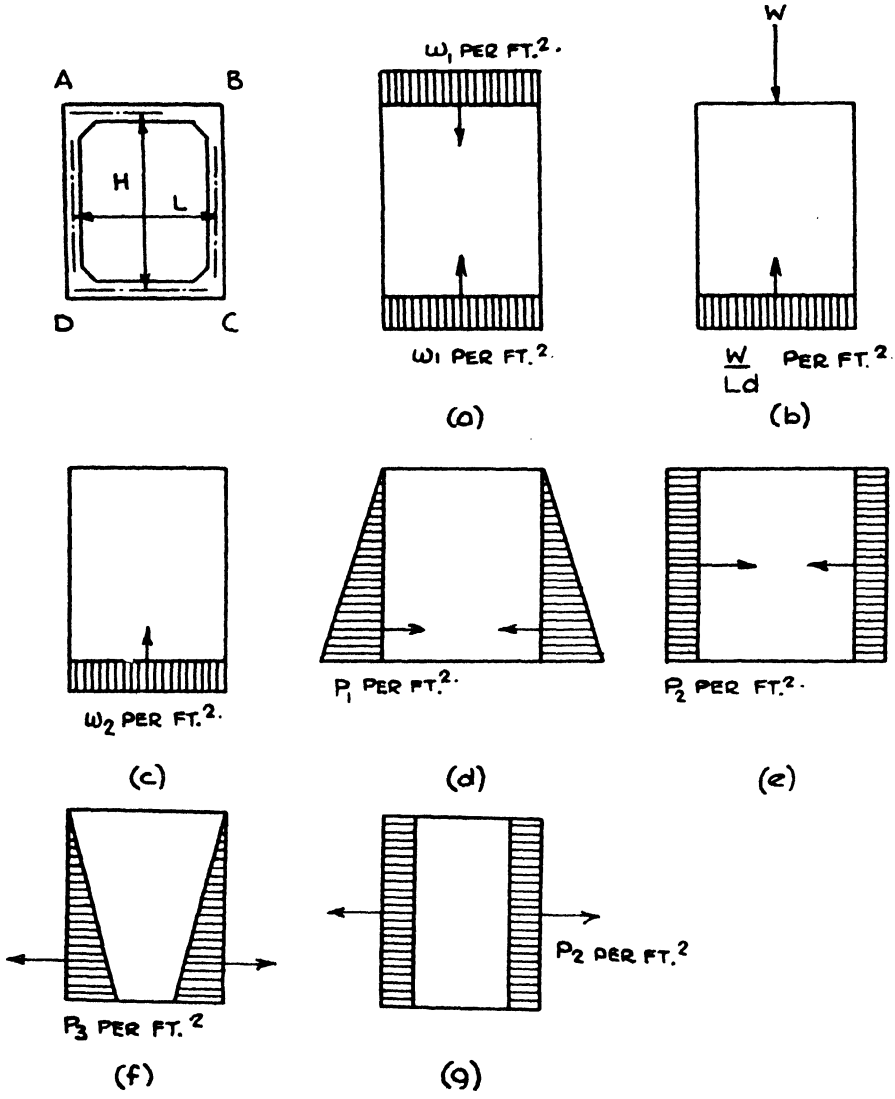


Fig. 35.—Loading on Culverts.

"earth arching" should be made with discretion. If there is no filling and the wheel loads or other point loads can bear directly on the roof of the culvert, the total point load should be considered as carried on a certain width of slab ($= d$) as determined by the methods described in Chapter II. The point load effect is somewhat modified if there is any filling above the culvert roof, and if the depth of filling is D the point load W can be considered as spread over an area of $4D^2$. When D equals or exceeds $\frac{L}{2}$ the point loading is equivalent to a load of $\frac{W}{4D^2}$ lb.

per square foot uniformly distributed over a length of culvert equal to $2D$. For values of D intermediate between $\frac{L}{2}$ and zero the moments will be between those due to a uniformly distributed load and a central point load.

The weights of the walls of the culvert can be assumed to be resisted by a uniform upward ground pressure as in *Fig.* 35(*c*). The weights of the bottom slab and the contents of the culvert are resisted directly by equal reactions from the ground below the slab and thus do not produce bending moments, although these factors must be taken into account when assessing the maximum ground pressures. The horizontal pressure due to the fluid contents of a culvert would produce an internal triangular loading as shown in *Fig.* 35(*f*), or even a trapezoidal load distribution if the surface of the water outside the culvert is higher than the roof. In the latter case, there will be an upward pressure on the underside of the roof slab, the moments being as for (*c*) reversed.

The magnitude and distribution of the horizontal pressure due to the earth against the sides of the culvert would be calculated in accordance with the formulæ given in Chapter III, consideration being given to possible water-logging of the ground which will result in increased pressures and potential flotation.

The maximum bending moments at the various critical sections of the culvert can be calculated by considering the possible incidence of loading, and generally there are only two conditions to consider:

(*a*) Culvert empty; full loading and surcharge on top slab (*Fig.* 35(*a*) and/or (*b*)), dead weight of walls (*Fig.* 35(*c*)) and maximum earth pressure on walls (*Fig.* 35(*d*) and (*e*)).

(*b*) Culvert full; minimum loading on top slab (*Fig.* 35(*a*)) and minimum (or zero) pressure from earth on walls (*Fig.* 35(*d*) and (*e*)), dead weight of walls (*Fig.* 35(*c*)) and maximum horizontal pressure from contents of culvert (*Fig.* 35(*f*) and (*g*)), and possible upward pressure on roof.

In special circumstances these conditions may not produce the maximum positive or negative moments at any particular section, and the effect of every probable combination should be considered. With the moments should be combined the direct thrusts and tensions due to various loadings.

6.—Roads.

Reinforced concrete roads can be divided into two classes:

(*a*) Reinforced concrete slabs underlying macadam, granite sets, asphalt, wood blocks, or other surfacing.

(*b*) Reinforced concrete slabs that form the complete road.

Preparation of Foundation.—With either type the ground upon which the slab is laid should be carefully prepared. All vegetation or patches of existing roads should be stripped off the site and hollow places filled with earth laid in superimposed layers 8 in. to 12 in. thick. Each layer should be well watered and rolled, a suitable weight for the roller being about 5 tons. Where the whole site has to be filled to the formation level of the road the filling should be left for about two months to settle before the concrete is laid. In awkward places where a roller cannot operate, a rammer weighing up to ¼ ton and operated by several men should be used. Immediately before the concrete is placed the

ground should be tamped with a ¼-cwt. hand rammer and well watered. When the surface lacks homogeneity it is advisable to roll in a layer of hardcore, and for very dry, dusty, or sandy surfaces the application of a sheet of waterproofed paper or a thin layer of lean concrete will prevent the water in the slab concrete from soaking into the ground. If the top of the ground is shaped in conformity with the camber of the finished road surface, the advantages of improved formation, drainage and economy of concrete accrue.

Reinforcement.—For all-concrete roads the slab can be from 4 in. to 8 in. in thickness, depending on the weight of traffic and the type of subsoil; the reinforcement should be between 6 lb. and 10 lb. per square yard, provided in a single layer in the bottom of the slab. With exceptionally heavy traffic on poor subsoils a layer of reinforcement in the top and bottom of the slab giving a total weight of 10 lb. to 15 lb. per square yard may be required. The provision of fabricated mesh reinforcement merits consideration in preparing road designs, and if cold-drawn hard steel is used a weight equal to only two-thirds of the required weight of rolled mild steel is necessary.

The arrangement of the reinforcement depends on the width of the road and spacing of the transverse joints. If these joints are at distances apart equal to the width of the road the steel should be arranged to give equal strength in both directions (square mesh), but if transverse joints are not provided, or are provided only at long intervals, at least nine-tenths of the total reinforcement should be parallel to the length of the road. For panels of intermediate sizes an intermediate ratio of between 0·5 and 0·9 of the total steel should be placed longitudinally.

The transverse bars should be bent up into and be attached to the longitudinal bars in the kerb if this is cast monolithic with the main slab. Often a granite kerb set on an independent mass concrete base is provided. If the ground under the edges of the road slab is likely to be affected by ground water, frost, etc., the outer 2 ft. or 3 ft. of the slab should be tapered from the normal thickness to 2 in. or 3 in. more at the extreme edge.

Concrete.—The concrete for road bottom slabs should not be leaner than a 1 : 2 : 4 nominal mix (see *Table* No. 23, page 263), and for all-concrete road construction not leaner than 1 : 1½ : 3. For the wearing surface, gravel or other rounded aggregates are definitely not advocated and a hard crushed stone should be used. In districts where such crushed aggregates are costly an economical and durable road slab can be obtained by making the bulk of the slab of 1 : 2 : 4 concrete with a selected gravel aggregate, and the top 1½ in. of 1 : 1½ : 3 concrete with a crushed granite aggregate graded from ½ in. to ¼ in. and clean coarse sand.

Exposure to weather and abrasion from traffic impose a severe test on concrete used for roads, and thus every reasonable precaution to attain a high standard of excellence should be taken. The water content should be strictly controlled and slump tests should be made periodically, the maximum slump being never more than 1½ in. and the nearer it is to zero the better so long as concreting is not hindered (see Chapter IX). Some specifications require the water content to be from 8 per cent. to 10 per cent. of the weight of the dry materials. For small water contents it is essential adequately to mix the materials, and it is sometimes recommended to mix dry for half a minute, and, after admitting

the water, mix for 1 to 1½ minutes with a drum speed of 15 to 20 revolutions per minute.

When the concrete has been placed it should be well and continuously tamped by hand or mechanical tampers or vibrators in order to bring to the surface the small amount of excess water before the immediate commencement of the finishing operations, the first of which is further tamping of the concrete surface with a timber beam shaped to the required camber of the finished profile. This beam is usually about 2 in. wide, extending the full width of the road, and worked off the side forms. This operation is sometimes followed by further consolidation by a wide roller weighing about 1 cwt. dragged transversely across the slab by workmen standing at the edges. There should then follow the minor operations involving surfacing by hand at joints, accidental hollow places, etc., previous to the final operation of drawing a canvas belt to and fro across the surface of the concrete. After the initial set has taken place the slab should be flooded with water, and later completely covered with a few inches of sand or earth from the excavation. This covering should be maintained saturated for about three weeks, and the road should not be opened to traffic until one month after pouring; in frosty weather this period should be lengthened. If rapid-hardening Portland cement or high-alumina cement has been used the interval can be reduced.

Joints.—Although many concrete roads are constructed without transverse joints the provision of such joints, and in wide roads a central longitudinal joint, assists in eliminating temperature cracks, shrinkage cracks, and the opening of construction joints. A convenient interval between joints is 30 ft., and the end of each day's concreting operations should coincide with a joint. A clear gap of about ½ in. should be left between the faces of adjacent panels and the space filled with a bitumastic material. The filling material is often in the form of sheets or strips shaped to the profile of the road and kerbs and projecting from the joint so that it can be flattened down on the top surface of the slab, as shown in *Fig.* 36, to protect the top edges of the panel faces. In the centre of the slab, and at intervals of about 2 ft. 6 in. across the width of the road, ½-in. bars 2 ft. or 3 ft. long should project horizontally from one panel to connect with the succeeding panel; these bars are usually greased in order to allow free expansion while preventing any one panel rising relatively to its neighbour.

7.—Permanent Joints.

Joints in monolithic concrete construction are required to allow free expansion and contraction due to temperature changes and shrinkage in such structures as retaining walls, reservoirs, roads, and long buildings, and to provide for unrestrained deformation in the case of vertical walls of cylindrical containers when it is undesirable to transfer any bending moment from the walls to the bottom slab. In *Fig.* 36 a number of typical designs for joints are illustrated.

At (*a*) the profile of a vertical joint in a cantilevered retaining wall is indicated. Such joints are frequently provided in the stems of walls, the spacing depending on the height of the wall, but the maximum interval should not exceed 60 ft. For low walls with thin stems a simple dry joint is usually sufficient, but the objection to the latter type is that any unequal deflection or tilting of one section relative to the next would show at the joint. A keyed joint, as indicated at

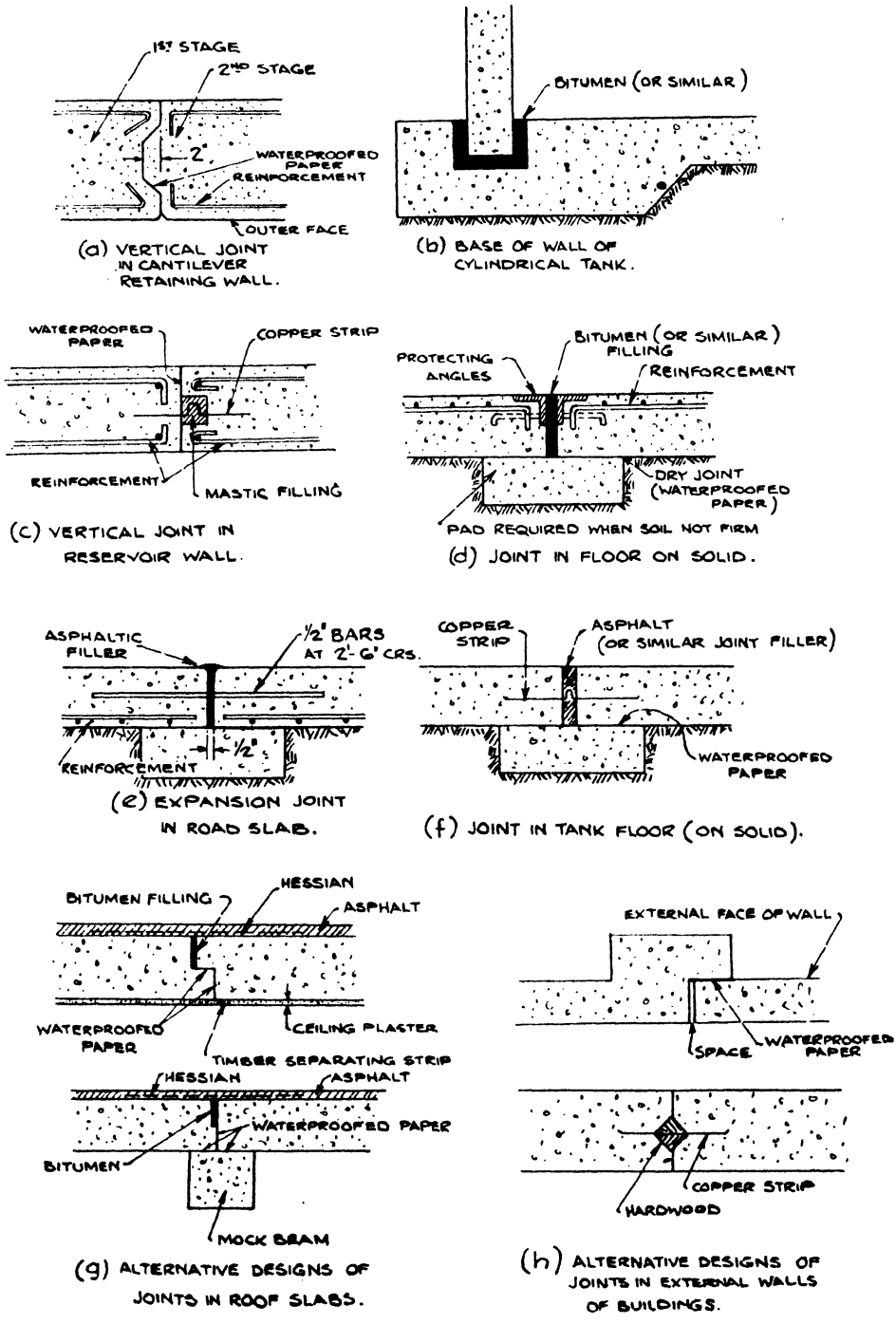


Fig. 36.—Typical Joints.

(*a*), is therefore preferable for walls more than 4 ft. high, and this construction also reduces the amount of percolation of moisture through the joint. A double chamfer on the exposed face of the joint improves the appearance. The type of joint illustrated allows for shrinkage and expansion, not greater than the shrinkage, to take place and usually such an allowance is sufficient. Where it is necessary to provide for a larger amount of expansion, a space ¼ in. wide should be left between the faces of the concrete and filled with resilient material.

In *Fig.* 36(*b*) a method of housing the base of cylindrical tank walls in the bottom slab of the tank is illustrated. This method, which could also be applied to the base of walls of cylindrical water towers, ensures that moments are not produced in the wall due to the restraining action of the base and that the base slab is not prevented from shrinking by the walls.

In the vertical walls of reservoirs a joint must be provided that allows movement due to shrinkage and temperature contraction to occur and also prevents water escaping through it. A joint of this kind, applicable to cantilevered walls of reservoirs, is shown in *Fig.* 36(*c*) and includes a copper strip which provides an effective water stop.

In slabs laid directly on the ground (and not forming part of a foundation raft), construction joints should be made permanent joints in predetermined positions, the latter usually conforming to the ends of day's work, restricted sections, changes of sections, or other positions where cracks are likely to occur. *Fig.* 36(*d*) shows the design for such a joint making a definite break in the continuity of the slab. This type of joint should be provided at intervals of about 15 ft. in the bases of reservoirs or in floors laid directly on the ground. The pads prevent one panel settling relatively to the other when laid on soft ground, but on very firm subsoils the pad may not be necessary When the floor is subject to abrasion, and especially in factories where wheeled containers may travel across the joint, the edges of the joint should be protected by building in strips of rolled steel angle section as indicated. If the subsoil is waterlogged, the detail at (*f*) can be adopted, the copper strip being inserted to render the joint watertight. This design is also suitable for the floors of reservoirs.

In the case of cantilevered retaining walls and reservoir walls it is not usually necessary to extend the joint from the walls into the base slab, but a longitudinal joint parallel to the wall should be provided to separate the base of the wall from the general area of the floor when the latter is considerable in size.

In *Fig.* 36(*e*) a common type of joint adopted in road construction is illustrated.

Where permanent joints are provided in buildings it is necessary to carry the joint through the floors, walls, and roof slab. Alternative proposals for the design of joints in roof slabs and external walls are given in *Fig.* 36(*g*) and (*h*).

The reader is referred to the author's "Concrete Construction" for particulars of the construction of the foregoing and other types of joints, and for descriptions of methods of construction which aim at eliminating or reducing shrinkage stresses. Reference should also be made to the Institution of Civil Engineers code relating to the design and construction of structures to contain liquids, in which the provision of joints in the floors, walls, and roofs of reservoirs is dealt with in detail.

CHAPTER IX

CONCRETE MATERIALS, MIXES, AND STRESSES

1.—Materials.

THE essential qualities of sand, stone, and cement are given in all regulations and specifications and are fully described in the author's "Concrete Construction." Only those factors that directly affect the designer are discussed in the present chapter.

Most of the materials employed in reinforced concrete are the subject of British Standards and these should be referred to for a complete specification of the requirements.

Cement.—The types of cement suitable for reinforced concrete are normal-setting ordinary, rapid-hardening, and blastfurnace Portland cements and high-alumina cement. Different types should not be combined. Quick-setting Portland cement is not used in normal construction work, although the admixture of calcium chloride to either ordinary or rapid-hardening Portland cement is frequently practised to accelerate initial set, either for cold-weather concreting or to enable earlier removal of the moulds from pre-cast units. Generally, cements with any such admixture do not conform to the requirements of current British Standards.

Normal-Hardening Portland Cement.—The setting times of ordinary Portland cement as specified in B.S. No. 12 (1940) are not less than 30 minutes for initial set and not more than 10 hours for final set. The setting times for quick-setting Portland cement are 5 and 30 minutes respectively. The fineness requirements are such that at least 90 per cent. should pass a No. 170 B.S. sieve (nominal size of aperture = 0·0035 in.). The minimum tensile stress on a 1 to 3 mortar is 300 and 375 lb. per square inch at 3 and 7 days respectively, or, as an alternative, a compression test on mortar cubes should exhibit strengths not less than 1,600 and 2,500 lb. per square inch at 3 and 7 days respectively.

Rapid-Hardening Portland Cement.—The principal physical difference between ordinary and rapid-hardening Portland cement is the finer grinding of the latter, the residue on a No. 170 B.S. sieve being not more than 5 per cent. to conform to B.S. No. 12 (1940). The residue when sieving an ordinary Portland cement as commercially supplied may be as low as 6½ per cent., while in the case of a rapid-hardening Portland cement it may be less than 2½ per cent. The setting times required conform to those specified for ordinary Portland cement, but the minimum tensile strength of mortar at 3 days is 450 lb. per square inch and the minimum compressive strength on mortar cubes at 3 days is 3,500 lb. per square inch. The more rapid rate of hardening of concretes made with such cement usually results in offsetting the extra cost by the savings accruing from the ability to strike the shuttering at an earlier date.

High-Alumina Cement.—This cement is characterised primarily by its rapid-hardening qualities due mainly to the increased proportion of alumina (as Al_2O_3). In Portland cement the amount of this constituent is usually between 5 and 6½ per cent., but it may be up to 40 per cent. in high-alumina cement; B.S. No. 915 (1940), specifies a minimum of 32 per cent. of alumina. The fineness is intermediate between ordinary and rapid-hardening cement, with a specified maximum residue of 8 per cent. on a No. 170 B.S. sieve. Initial set is between two and six hours, and the final set within two hours after the initial set. The compressive strength of mortar cubes is required to be double that of rapid-hardening cement at three days. The time saved as a result of using high-alumina cement for certain types of construction, and the immunity of high-alumina cement concrete from attack by sea-water and most corrosive liquids, are the principal factors which offset the higher cost.

The London Building By-laws specify certain requirements for high-alumina cement which do not in all respects conform to British Standard No. 915 (1940).

Portland Blastfurnace Cement.—The requirements for this cement as specified in B.S. No. 146 (1941) include a slag content of between 35 and 65 per cent. Otherwise the specified fineness, setting-times, and mortar strengths are the same as those for ordinary Portland cement. The use of this cement is accepted by the Code of Practice and the London Building By-laws.

Quality of Aggregates.—Cleanness and hardness are the essential characteristics of the fine aggregate (sand) and coarse aggregate (stone), but lack of porosity and freedom from crusher dust, laminated and wedge-shaped particles, and splinters are also important. Gravels and crushed stone are the best materials for coarse aggregate. Broken brick is a cheap form of aggregate for mass concrete work and produces a good fireproofing concrete, whereas flint, and to a certain extent granite, are not the most suitable materials for this purpose. Coke-breeze and other coal residues should not be used for load-supporting members, but they produce an economical light-weight concrete for partition walls and similar construction.

Gauge and Grading of Aggregate.—The grading and gauge of aggregate materials vary considerably with the nature and source of the material, and the requirements in this respect depend upon the class of work. For ordinary building work the fine aggregate should be graded from $\frac{3}{16}$-in. down to dust with not more than 3 per cent. passing a No. 100 B.S. sieve. The coarse aggregate should be graded from $\frac{3}{16}$ in. up to ¾ in., and between these limits the grading should be such as to produce a concrete that is dense and workable without requiring excess water. The maximum gauge of the coarse aggregate should be related to the cover of concrete over the reinforcement and the space between the bars. When the cover or lateral distance between bars is less than 1 in. it is usual to specify a maximum gauge of ¼ in. less than the smaller of these two dimensions. In no case is it advisable for the maximum gauge to exceed 20 per cent. of the minimum concrete dimension. Thus, if a ¾-in. aggregate is specified, neither the cover nor the lateral distance between any two bars should be less than 1 in., and the thickness of any slab or beam rib should not be less than 3¾ in.

For certain work special consideration of the maximum gauge is required; in the construction of a chimney shaft a maximum of ½ in. is often specified,

while for hollow clay-tile floors or similar narrow ribbed construction $\frac{3}{8}$ in. is the usual maximum.

For unreinforced concrete larger sizes, up to, say, 2 in., are permissible and for road slabs, particularly in the base slab of two-course construction, $1\frac{1}{2}$-in. or 2-in. maximum gauge is usual. Where the dimensions of plain concrete work warrant it, as in massive bridge piers, heavy foundations, concrete filling, etc., the use of hard stone "plums" can be permitted, subject to reasonable precautions in spacing and laying them.

In reinforced concrete work, the maximum gauge of the coarse aggregate may also exceed $\frac{3}{4}$ in., as long as it does not exceed 75 per cent. of the cover or lateral spacing between the bars, whichever is the smaller.

For work subject to attrition, such as road slabs, floors of garages, factories, workshops, etc., where special finishes are not applied, a coarse sand, rich in intermediate sizes, is preferable to the finer sands permissible in normal building construction. For impermeable construction the aggregates should be given special consideration so as to obtain as dense a concrete as possible. The limiting sizes of $\frac{3}{16}$ in. and $\frac{3}{4}$ in. already mentioned are generally suitable, but not less than 5 per cent. nor more than 30 per cent. of the fine aggregate should pass a No. 52 B.S. sieve. Not less than 95 per cent. of the coarse aggregate should exceed $\frac{3}{16}$ in., but between 45 per cent. and 75 per cent. should be $\frac{3}{8}$ in. or less. If the maximum gauge is other than $\frac{3}{4}$ in., between 45 per cent. and 75 per cent. should be equal to or less than the size corresponding to half the maximum gauge.

2.—Concrete Mixes.

Expressions for Mixes.—The proportions in which the three constituents—cement, fine aggregate, and coarse aggregate—are mixed are usually expressed for convenience in volumetric proportions based on unit volume of cement, for example, 1 : 2 : 4 meaning two parts by volume of fine aggregate to four parts by volume of coarse aggregate to each part by volume of cement. Since it is important to ensure that the quantity of cement is not less than that anticipated, it is imperative that the cement should be measured by weight. If Portland cement is taken at a nominal weight of 90 lb. per cubic foot, by a 1 : 2 : 4 mix is actually meant 90 lb. of Portland cement to 2 cu. ft. of fine aggregate to 4 cu. ft. of coarse aggregate. Since the basis of a batch of concrete is usually a hundredweight bag of cement, this mix would be equivalent to 112 lb. of cement to $2\frac{1}{2}$ cu. ft. of fine aggregate to 5 cu. ft. of coarse aggregate, and in specifications and on working drawings the concrete mix should be expressed in these terms.

So long as it is realised that the method of expressing concrete mixes in nominal volumetric proportions is merely a convenience, its use can greatly facilitate reference in conversation or writing to any particular mix and for this purpose is preferable to the alternative methods, namely, (1) expressing the mix in terms of volumes of fine and coarse aggregates to a specified weight of cement, which leads to cumbersome expressions, or (2) giving different mixes distinctive labels, such as "Mix A" or "Mix I." Since the Code of Practice and the London By-laws contain different references of this type for identical mixes, confusion may result. There is also the difficulty of selecting a suitable reference for mixes, in pro-

portions intermediate between, or variations upon, the principal referenced mixes.

For these reasons nominal volumetric proportions are adopted in this volume, and on *Table* No. 23 (page 263) are given the corresponding references in accordance with the Code and the London By-laws, together with the volumes of aggregate required with 1 cwt. of cement when using the more common mixes.

Proportions.—The proportion of cement to aggregate depends on the requirements of the concrete as regards strength, impermeability, etc. Experience shows that a nominal 1 : 2 : 4 mix produces a concrete suitable for general construction both as regards cost and strength. Leaner mixes are not generally recommended for reinforced concrete work unless the mixture has been properly designed and tested with respect to the actual materials and water content. A nominal 1 : 3 : 6 mix is suitable for unreinforced concrete or for temporarily placed concrete that has to be cut away later. Workable mixes richer in cement than 1 : 2 : 4, for example 1 : 1½ : 3 and 1 : 1 : 2, are stronger and more expensive owing to the higher cement content, but are not usually economically justified in members subject to simple bending, since so much of the concrete is in the tension area and is therefore ignored according to the generally accepted methods of calculation. The richer mixes, however, often produce economical designs for heavily loaded columns or for members subject to combined bending and direct thrust, when the direct force predominates. Richer mixes than 1 : 1 : 2 contain so large a proportion of cement that, apart from the question of cost, the shrinkage upon hardening is serious. Instead of using a very rich mix it is almost invariably more economical to obtain the necessary compressive strength by careful selection of the aggregates and control of the water content. On *Table* No. 23 recommendations are given for the most suitable mix for various parts and types of structures under normal conditions.

For impermeable construction, as is necessary in liquid-containing structures, the nominal proportions should not be less than approximately 1 : 1⅔ : 3⅓, that is 2 cu. ft. of fine aggregate and 4 cu. ft. of coarse aggregate to 1 cwt. of cement. If, however, the thickness of the concrete walls or other slabs exceeds 24 in., a nominal 1 : 2 : 4 mix is permissible.

It is also usual to specify a mix of 1 : 1⅔ : 3⅓ for pre-cast piles, concrete deposited under water, unprotected roof slabs, and in other places where a concrete of a little better quality than 1 : 2 : 4 is required. In hollow tile floors and similar narrow ribbed construction, a 1 : 1½ : 3 mix is often specified.

It is usual to " blind " the bottom of an excavation with concrete to obtain a clean working surface. The blinding layer should be composed of concrete mixed in the proportions of 1 part of Portland cement to 10 parts of combined aggregate.

Proportions of Fine and Coarse Aggregates.—The ratio between the amounts of fine and coarse aggregate necessarily depends on the grading and other characteristics of these materials in order that the volume of " sand " shall be sufficient to fill the voids in the coarse aggregate, thus producing a dense concrete. Until the material for a particular job has been delivered it is not possible to say what will be the exact grading of either the sand or the stone. Therefore this information will not usually be available when the specifications are being

prepared. Alternative courses are open to the engineer when specifying the proportions for a concrete mix.

The proportions of a particular sand and a particular stone, with whose properties the engineer is acquainted, can be specified. The gradings of these materials should be fairly constant throughout all consignments. It is preferable if one or two independent sources of supply are available in order to obtain competitive prices, and the material should be obtainable within reasonable distance of the site so as to reduce transport costs. Even when the material is defined in this way it is wise to specify the limits of variation of the essential properties.

Alternatively the proportions of a coarse and fine aggregate having definite gradings (within limits) can be specified by the engineer, leaving it to the contractor to supply what is required.

A third, and probably the best method, is to specify the provisional ratio of fine to coarse material, the maximum and minimum sizes (with such percentages of intermediate sizes as are thought necessary), and insert a clause that allows adjustment of the proportions after examination of the actual materials. The engineer should be cautious when putting this clause into effect, since alteration of the proportions may require the provision of more of the expensive material and less of the cheaper, thereby leading to discussion of unit prices.

Generally the proportion of fine to coarse aggregate should be such that the volume of fine aggregate should be about 5 per cent. in excess of the voids in the coarse material. Since the volume of voids may be up to 45 per cent., the common ratio is one part of fine to two parts of coarse aggregate, as for example in $1:2:4$, $1:1\frac{1}{2}:3$, etc. Such proportions, however, relate to dry materials. Whereas the water content of a damp coarse aggregate does not appreciably affect the volume, the dampness of fine aggregate can increase the volume by as much as 30 per cent. over the dry (or fully saturated) volume. It should therefore be realised, and included in the specification, that the proportions given apply to a dry " sand " and must be adjusted to allow for bulking due to dampness.

The $1:2$ ratio of fine (dry) and coarse aggregate will, of course, be altered if testing shows that with given materials a more dense concrete can be obtained by using other proportions. The Code and London By-laws recognise this variation, but set as limits to the amount of variation the ratios of $1:1\frac{1}{2}$ and $1:2\frac{1}{2}$. Thus for a nominal $1:2:4$ mix, variation of the proportions may result in the equivalent extreme mixes of approximately $1:2\frac{1}{3}:3\frac{2}{3}$ and $1:1\frac{2}{3}:4\frac{1}{3}$.

Water Content.—The quality of concrete depends to a great extent on the amount of water used in mixing. There is a certain amount of water for any proportions of given materials that will produce a concrete of maximum strength. A smaller amount of water than this leads to a decrease in strength, and about 10 per cent. less is insufficient water to ensure complete setting of the cement besides producing an unworkable concrete. More than this critical amount increases workability but decreases strength, a 10 per cent. increase reducing the strength by approximately 15 per cent., while a 50 per cent. excess reduces the strength by one half. With more than 50 per cent. excess the concrete becomes too wet and liable to segregation. The use of excess water not only leads to low strengths but increases shrinkage and decreases density.

A rule recommended for determining the amount of water to give a strong concrete of reasonable workability is

$$W = 0{\cdot}28W_C + 0{\cdot}04W_A$$

where W = weight of water
W_C = weight of cement
W_A = total weight of aggregate.

If dry aggregate has an average weight of 90 lb. per cubic foot, the amount of water required in a batch of 1 : 2 : 4 concrete containing 1 cwt. of cement is (using the quantities given on *Table* No. 23)

$$W = (0{\cdot}28 \times 112) + [0{\cdot}04 \times 90(2{\cdot}5 + 5{\cdot}0)] = 58 \text{ lb.}$$
$$= \frac{58}{10} = 5\tfrac{3}{4} \text{ gallons.}$$

This rule assumes that the materials are non-absorbent and dry, and allowances should be made for other conditions.

Concrete placed by efficient mechanical vibrators can often be mixed drier than normal tamped concrete, thereby obtaining higher strengths.

A practical method of assessing the amount of water is by making trial mixes with the materials and only sufficient water to make a plastic concrete that is capable of being worked among and around the bars. When the best mix has been determined, a slump test should be made and the value of the slump should be checked throughout the subsequent mixings by frequent testing. The slump test has the practical advantage of allowing for the porosity and dampness of the aggregates, and is a comparative measure of the wetness of the concrete placed.

A suitable maximum slump for reinforced concrete work is 6 in. but a smaller value is often desirable. For impermeable construction a slump of 2½ in. is generally recommended, but if the reinforcement is particularly intricate the slump may be increased to 6 in. In the latter case, however, efficient vibration would allow a drier and therefore denser and stronger concrete to be used. For plain concrete in heavy foundations, roads, and dams, the slump may be almost zero.

3.—Properties of Concrete.

Weight.—The weight of plain and reinforced concrete is discussed in Chapter II. In design 144 lb. per cubic foot is a convenient and acceptable weight to assume in calculating the dead load, since each square inch of cross section then corresponds to 1 lb. weight per lineal foot of the member considered. In the design of shuttering a weight of not less than 150 lb. per cubic foot should be allowed for wet concrete. The horizontal pressure exerted by wet concrete can be taken as 140 lb. per square foot of vertical surface per foot depth, but for depths exceeding a few feet and for narrow widths, drier concretes, and where the reinforcement is plentiful and intricate, this unit pressure can be decreased.

Ultimate Compressive Strength.—With given materials the ultimate compressive strength of concrete depends primarily upon age, cement content, and the ratio of cement to water, increase in any of these factors giving an

increase in strength. Maximum strengths vary from 1,500 lb. per square inch for lean concretes to more than 8,000 lb. per square inch for specially controlled mixes. The rate of increase of strength with age is almost independent of the cement content, and in 28 days about 60 per cent. of the strength attained in a year is reached; 70 per cent. of the 12-month strength is reached in two months and about 95 per cent. in six months. For design purposes the working stresses are usually based on the strength at 28 days. At this age concretes of various mixes should not have an ultimate strength less than those specified on *Table* No. 23, which agree with the London By-laws and the Code for ordinary quality of concrete.

If a superior concrete is used, for example Quality A (London By-laws) or High-Grade (Code), working stresses based on higher ultimate strengths can be adopted and the limiting values are tabulated on *Table* No. 23 for corresponding mixes. In order to adopt in design working stresses based on these higher ultimate values it is necessary to carry out preliminary tests at 28 days on laboratory-made cubes to ensure that a minimum factor of safety of $4\frac{1}{2}$ on the tabulated working stress is obtained. When corresponding tests are made on site-made cubes, the least factor of safety should be three.

For the minimum ultimate strengths for concretes mixed in proportions other than those given on *Table* No. 23, and for concrete to be used in impermeable construction and for plain concrete, the reader is referred to the remarks given later on "Working Stresses."

For the construction of bridges the Ministry of Transport specifies that the ultimate strength at 28 days with ordinary Portland cement, and at 7 days with rapid-hardening Portland cement, should be $(15A + 900)$ lb. per square inch, where A = weight of cement (lb.) used with 2 cu. ft. of fine aggregate and 4 cu. ft. of coarse aggregate. The corresponding ultimate strengths are given on *Table* No. 23.

Compression tests in this country are usually made on 6-in. cubes; these should be made, stored, and tested in accordance with the standards laid down in the London By-laws, for example. For cubes made on the site for routine tests it is usual, where not controlled by regulation, to cast four from a single batch of concrete and make distinguishing marks on these. Two of these should be tested at 7 days and two at 28 days. For the results to be acceptable, the strengths of each cube in any pair should not vary by more than 30 per cent. unless the minimum strength exceeds the ultimate strengths required. The 7-day tests form a guide to the rate of hardening, and generally the strength at this age for normal hardening Portland cement should be not less than 60 per cent. of the strength required at 28 days.

Tensile Strength.—Although neglected in orthodox calculations for members in bending, the tensile strength of concrete enters into the consideration of shear and the design of tanks; these aspects are discussed later under "Working Stresses."

Modulus of Elasticity.—Young's modulus for concrete (E_c) increases with the richness of the mix, age, repetition of stress, and similar factors. Actual values lie between the wide limits of 750 and 1,500 times the ultimate compressive strength. Thus E_c for a 1 : 2 : 4 concrete might average 3,000,000 lb. per square inch and, when combined with a steel modulus of E_s = 30,000,000 lb.

per square inch, a modular ratio $m = \frac{E_s}{E_c} = 10$ is obtained. In the generally accepted formulæ for calculations an arbitrary modular ratio of 15 (corresponding to $E_c = 2{,}000{,}000$) is usually adopted for all mixes and is accepted under the London By-laws. This ratio seems to compensate in part for the errors involved in the consideration of reinforced concrete as a theoretically elastic substance and for neglect of the tensional value of concrete in bending.

In design calculations the Code specifies a value of for the modular ratio equivalent to $\frac{40{,}000}{c_u}$, where c_u is the minimum ultimate compressive strength at 28 days. These ratios are tabulated on *Table* No. 23; for a 1 : 2 : 4 ordinary concrete a modular ratio of $\frac{40{,}000}{2{,}250} = 17{\cdot}8$ is obtained, the corresponding ratio for High-Grade concrete of the same mix being 14.

For building construction, except when using Special-Grade concrete, it is suggested that a modular ratio of 15 should be adopted for all mixes, but when calculations involving actual deformations are concerned the ratios in the Code should be used.

When designing impermeable structures it is recommended to take $m = 12$ for 1 : 1⅔ : 3⅓ Quality A concrete and $m = 14$ for a 1 : 2 : 4 Quality A mix.

Temperature Coefficient.—The coefficient of linear expansion and contraction due to temperature changes increases with an increase in cement content and for practical mixes may vary from 0·000005 to 0·0000065, an intermediate value of 0·0000055 per degree F., being a practical value which is about the same as that for mild steel. This factor enters into the design of chimneys, tanks containing hot liquids, exposed or long lengths of construction such as roads, arches, warehouses, etc., and adequate provision must be made in the design to take up the stresses due to temperature changes or to limit the strains by providing joints.

Shrinkage.—Unrestrained concrete members exhibit progressive shrinkage over a long period while they are hardening. For concrete that can dry completely, and where shrinkage is unrestrained, the linear coefficient is approximately 0·00025 at 28 days and 0·00035 at three months, after which period the increase is less rapid until at the end of twelve months it may approach a maximum value of 0·0005. In reservoir construction and similar cases where concrete cannot become completely dry a maximum value of 0·0002 may be reached. Concretes rich in cement, or made with finely-ground cements or with a high water content, exhibit greater shrinkage than lean mixes.

If the length of a concrete member is restrained so that shrinkage cannot take place, tensile stresses are set up. When it is realised that a shrinkage coefficient of 0·0002 may correspond to a stress of 500 lb. per square inch when restrained, it will be appreciated that it is important to reduce or neutralise these stresses by using strong concretes containing the minimum of cement and water, by proper curing, and by the provision of joints.

Bulk Shrinkage upon Mixing.—When the constituents of concrete are mixed with water and tamped into position, a reduction in volume to about two-thirds of the total volume of the dry unmixed materials takes place. The

actual amount of reduction depends on the nature, dampness, grading, and proportions of the aggregates, and on the cement content, the amount of water, the thoroughness of mixing, and the amount of tamping or vibration. With so many variables it is difficult to assess closely the amount of each material required to produce a cubic yard of wet concrete in place. The quantities given on *Table* No. 23 are normal values for gravel concretes but do not allow for waste. Alternative quantities are given for dry and damp fine aggregates, the latter allowing for 30 per cent. of bulking. The quantities of fine and coarse aggregates for other mixes can be obtained by proportion.

4.—Working Stresses.

Compression due to Bending.—The working compressive stress due to bending (referred to as c in the following pages) is usually taken as one-third of the specified minimum ultimate crushing strength at 28 days. The normally accepted values of c for the more common mixes not leaner than 1 : 2 : 4 are given on *Table* No. 23, and these stresses correspond to the requirements of the London By-laws and the Code, values being given for both ordinary quality and Quality A (or High-Grade) concretes. Thus for 1 : 2 : 4 concrete, the value of $c = \frac{2{,}250}{3} = 750$ lb. per square inch for ordinary quality and $c = \frac{2{,}850}{3} = 950$ lb. per square inch for Quality A. The Code recognises a grade of concrete superior to High-Grade which is termed "Special Grade," and for which under conditions of strict site control, preliminary and progress testing, and monolithic frame design, the value of c can be increased up to 25 per cent. in excess of High-Grade (or Quality A) stresses, subject to there being a factor of safety of 3 on site-made cubes at 28 days.

For mixes where the ratio of fine to coarse aggregate differs from 1 : 2, the stresses can be taken as equivalent to those for a mix having the same sum of fine and coarse aggregate. Thus a nominal mix of $1 : 2\frac{1}{2} : 3\frac{1}{2}$ or a mix of $1 : 1\frac{3}{4} : 4\frac{1}{4}$ can both be taken as equivalent to 1 : 2 : 4 as regards working stresses since the sum of the proportions of fine and coarse material is six in each case.

For proportions intermediate between those tabulated, the value of c can be computed by proportion between the values given for the two nearest mixes based on the sum of the separate volumes of the fine and coarse aggregate. In all such adjusted cases, it is essential that the minimum 28-day ultimate strength should be $3c$. Thus with a mix of $1 : 2 : 3\frac{1}{2}$ (ordinary) for example, the sum of the volumes of fine and coarse aggregate $V = 2 + 3\frac{1}{2} = 5\frac{1}{2}$, which is intermediate between 1 : 2 : 4 ($V_2 = 6$) and $1 : 1\frac{2}{3} : 3\frac{1}{3}$ ($V_1 = 5$). Thus

$$c = c_2 + \frac{(V_2 - V_1)(c_1 - c_2)}{V_2 - V_1}$$

where c_1 and c_2 are compressive working stresses for the two adjacent mixes. Thus for a nominal mix of $1 : 2 : 3\frac{1}{2}$

$$c = 750 + \frac{(6 - 5\frac{1}{2})(880 - 750)}{6 - 5} = 815 \text{ lb. per square inch,}$$

since 880 and 750 are the values of c for 1 : 2 : 4 and $1 : 1\frac{2}{3} : 3\frac{1}{3}$ ordinary mixes respectively.

For bridge construction the Ministry of Transport memorandum specifies that c should be taken as one-third of the ultimate strength calculated by the formulæ given previously. Thus $c = 5A + 300$, in which A is the weight (lb.) of cement used with 2 cu. ft. of fine aggregate and 4 cu. ft. of coarse aggregate. For 112 lb. of cement the volume of coarse aggregate for $1 : 1\frac{1}{2} : 3$ mix is $3\frac{3}{4}$ cu. ft.; thus $A = \frac{112}{3\frac{3}{4}} \times 4 = 120$ lb. Hence $c = (5 \times 120) + 300 = 900$ lb. per square inch, compared with the stress of 850 lb. per square inch used in buildings for ordinary concrete of this mix.

For impermeable construction, adopting a nominal $1 : 1\frac{2}{3} : 3\frac{1}{3}$ Quality A mix, a maximum compressive stress of 880 lb. per square inch should be adopted for slabs and beams subject to bending, although for the ribs of beams remote from the face in contact with the water a stress of 1,100 lb. per square inch can be used, since the 28-day ultimate strength of a concrete used for impermeable construction should not be less than 4,950 lb. per square inch for laboratory-made cubes and 3,330 lb. per square inch for cubes made on the works. In thick construction a $1 : 2 : 4$ mix is used having a 28-day ultimate strength of 4,275 lb. per square inch for laboratory-made cubes and 2,850 lb. per square inch for works cubes. The value of c can be taken as 760 lb. per square inch for slabs and beams and 950 lb. per square inch for beam ribs remote from the water face. These strengths and stresses are indicated on *Table* No. 23.

Direct Compression.—For members such as axially-loaded columns in direct compression, the maximum compressive stress is normally taken as 80 per cent. of the permissible compressive stress in simple bending. These stresses are given on *Table* No. 23. In orthodox design this stress is taken over the whole area of an ordinary column section and on the core section of columns with helically bound cores. In impermeable construction, however, as c is already taken at a low value (except for beam ribs remote from the water face) the same stress can be adopted in direct compression as in bending.

Combined Bending and Direct Force.—When a member is subject to bending moment combined with a direct thrust (as in arches, columns forming part of a frame, etc.) or bending moment combined with a direct pull (as in the walls of rectangular bunkers, tanks, etc.) it is usual to work to the same permissible compressive stress (c) in the concrete as if the member were acted on by bending alone. Thus for a column constructed in $1 : 2 : 4$ ordinary quality concrete, the compressive stress should not exceed 600 lb. per square inch for direct load alone, nor exceed 750 lb. per square inch when the direct load is combined with a bending moment.

Direct Tension.—In the design of members subjected to bending it is usual to neglect the value of the concrete in tension, but in certain cases, such as structures containing liquids and more commonly in the consideration of shear resistance, the tensile strength of the concrete becomes an important factor in the design. Tests seem to indicate that the tensile strength is not directly proportional to the compressive strength but varies more nearly as the compressive strength to the two-thirds power.

For members such as hangers which are in direct tension and where cracking is not necessarily detrimental, the tensile strength of the concrete is neglected and reinforcement is provided to take the whole load. In construction that

must be impermeable, and therefore free from cracks, the tensile stresses in the concrete must be limited. For members in direct tension, such as the walls of cylindrical containers, the tensile stress for a 1 : $1\frac{2}{3}$: $3\frac{1}{3}$ Quality A concrete should be limited to 175 lb. per square inch, but for members subject to bending the section should be designed as described in Chapter VII so that the maximum tensile stress in the concrete does not exceed 250 lb. per square inch. The corresponding stresses for 1 : 2 : 4 concrete should be reduced approximately in proportion to the ultimate strength.

Shear Stresses.—The shear stress calculated on the lever-arm section of a beam should not exceed 10 per cent. of the maximum permissible compressive stress in bending (that is $0{\cdot}1c$), and when the whole of the diagonal tension due to the shearing forces is taken on the reinforcement the shear stress should not exceed $0{\cdot}4c$.

Punching shear, as in column footings and pile caps, should be limited to $0{\cdot}2c$.

The foregoing stresses, which are tabulated on *Table* No. 23, are applicable to normal building work, but discretion should be used before adopting the extreme higher values. The stresses in brackets on *Table* No. 23 are the reduced shear stresses that should not be exceeded in impermeable construction.

Adhesion.—The basic adhesion stress between concrete and plain round bars is usually assumed to be $(0{\cdot}1c + 25)$ lb. per square inch when investigating the bond or anchorage of reinforcement, but the factors discussed in Chapter X should also be taken into consideration. The bond stress for 1 : 2 : 4 ordinary concrete in accordance with this expression is 100 lb. per square inch, and stresses corresponding to other mixes and qualities are given on *Table* No. 23. The values in brackets again apply to impermeable construction.

Rapid-Hardening Cement Concretes.—It is not generally recommended to work to higher stresses when rapid-hardening Portland cement is used, although from a comparison of ultimate concrete strengths at 28-days higher results are often obtained than with normal-hardening Portland cement concretes. Since the atmospheric temperature during the initial hardening stages and the fineness of the sand may have appreciable effects on the strengths obtained by rapid-hardening Portland cement concretes, it is not advisable to adopt higher working stresses unless frequent periodical tests are made on site-made cubes. Subject to such tests and given suitably coarse sands and moderate temperatures, there is some justification for higher stresses since there is less risk of damage due to overstressing by incidental loading during the first few weeks after placing the concrete. Since 1 cu. ft. of rapid-hardening Portland cement usually weighs less than an equal volume of normal cement, it is imperative that the former cement should always be proportioned by weight, otherwise the quantity of cement in the mix may not only be uncertain but insufficient.

Similarly with high-alumina cement concretes, the tests results of which exhibit higher strengths than normal-hardening Portland cement concretes, higher working stresses may be permitted if a suitably hard aggregate is used, but any excess beyond normal values should only be considered if tests are made on the actual materials used.

Bearing on Plain Concrete.—Plain concrete mixed in leaner proportions than 1 : 2 : 4 are used for filling under foundations and for piers, walls, etc.

Suitable working stresses for direct compression are given on *Table* No. 23 and conform generally to the London By-laws.

Modifications to Specified Stresses.—The working stresses tabulated on *Table* No 23 and the values specified in regulations must always be related to the method of computing the bending moments and forces to which the member is subjected. These stresses must necessarily be reduced if certain departures are made from the prescribed method of stress calculation, load assessment, or bending moment computation.

The working stresses should be reduced when any of the following conditions prevail:

(*a*) If tests on laboratory-made cubes made with the best available aggregates give results lower than 4·5 times the compressive stress in bending, the working stresses should be reduced to two-ninths of the ultimate strength and a further reduction should be made if tests on site-made cubes do not show a corresponding factor of safety of three.

(*b*) When the work is to be constructed by inexperienced men and organisations and where expert whole-time supervision is unavailable.

(*c*) When the calculations are only approximate or when the maximum loading is uncertain. In preliminary schemes secondary effects, such as temperature stresses in chimney shafts or torsional stresses on beams, may be omitted and a reduced working stress used. It is preferable, however, when preparing working drawing calculations to take account of these factors and adopt normal stresses.

(*d*) Slender column design requires stresses less than normal values (see Chapter XIII).

(*e*) When the structure is subject to vibration or impact the normal stresses should be reduced unless an allowance for these effects has been made in the assumed magnitude of the live loading (see Chapter II).

(*f*) If the maximum loading can occur before the concrete has obtained sufficient ultimate strength to ensure a factor of safety of four on the stresses due to the combined dead and live loads.

(*g*) If all the load is dead load the stresses should be reduced, unless means can be taken (for example, by introducing temporary supports) to prevent the full load acting on the structure until three months after completion.

(*h*) When the work is difficult to construct and concrete is to be deposited under water, in underpinning, or in similar difficult positions.

Under more favourable conditions reasons may exist for increasing the working stresses, although the 28-day ultimate strength with a factor of safety of three should invariably be adopted as a guide. Among such conditions are the following:

(*a*) When specially designed concrete mixes are used with the object of obtaining a high-strength concrete for special structures such as long-span arches.

(*b*) If the moments and forces are precisely determined in accordance with the elastic theory and the loading is accurately known and other requirements of Special-Grade concretes are complied with, an increase up to 25 per cent. of the Quality A stresses can be adopted subject to satisfactory test results.

(*c*) Certain methods of design for columns with helically bound cores allow increased stresses on the core.

(*d*) If extreme or infrequent live loading is designed for, stresses for these loads can be increased at the designer's discretion, but the stresses due to the dead load alone should not exceed normal values. No adjustment under this heading should be made if an allowance has already been made under heading (*b*).

(*e*) When the calculated stresses include those due to the bending moments and forces caused by wind pressures on buildings, the stresses either for ordinary or Quality A concrete can be increased by $33\frac{1}{3}$ per cent. If this increase, which is permitted by the London By-laws, is made, the designer should assure himself that without wind the normal working stresses are not exceeded. This increase should not be made when designing chimney shafts, water towers, or similar structures in which the effects of wind pressure are a predominating factor in the design.

In adopting the foregoing reductions or increases much has to be left to the judgment of the designer, the consideration of the conditions peculiar to the given structure, and the requirements of the controlling authorities.

CHAPTER X

REINFORCEMENT

1.—Material.

REINFORCEMENT is most commonly plain round mild steel bars conforming to B.S. No. 785 (1938). Included in the same specification, but less frequently used, are medium and high-tensile steels. The latter exhibits a higher yield point than mild steel by virtue of the increased carbon content. High-yield stress steels in common use include cold-worked bars embraced by B.S. No. 1144 (single twisted square bars and twin-twisted round bars) and cold-drawn mild steel wire (B.S. No. 785). The latter material, expanded metal (B.S. No. 405), and small-gauge twisted square bars are normally supplied in the form of diamond, oblong or square mesh fabrics for road and slab reinforcement. Where available, and convenient to use, old rails, disused wire ropes, and light structural steel sections are occasionally embedded in concrete as reinforcement.

2.—Working Stresses.

Tension.—For rolled mild steel having an ultimate strength of 28 tons to 33 tons per square inch the safe working stress in tension can be taken as 18,000 lb. per square inch for ordinary building work, bunkers, etc. For bridge construction under the Ministry of Transport the maximum permissible stress is 16,000 lb. per square inch. For impermeable construction the maximum tensile stress should be limited to 12,000 lb. per square inch when the concrete is ignored (see Chapter VII). The limiting stresses for diagonal tension due to shear are discussed in Chapter XII.

Since the yield point of rolled mild steel is about 32,000 to 36,000 lb. per square inch, a working stress of 18,000 lb. per square inch represents a factor of safety of two on the yield-point stress, and this factor is the true measure of the margin of security. For high-yield-point steel the maximum working tensile stress permitted is usually 50 per cent. of the yield-point stress, with a limit of 27,000 lb. per square inch.

Some steels of foreign origin do not always comply with the British Standard Specification, and when these are used the designer should decide whether the nature of the structure justifies a stress equal to a quarter of the ultimate strength or equal to half the yield-point stress.

The requirements of the London By-laws and the Code of Practice are that a maximum tensile stress of 18,000 lb. per square inch should be adopted for ordinary mild steel in buildings. For high yield-point steel the London By-laws accept a working stress of one-half the yield-point stress with a maximum of 27,000 lb. per square inch, but the Code of Practice limits the tensile stress to

0·45 times the yield-point stress (up to a limit of 25,000 lb. per square inch) as long as the total area of the tension steel does not exceed 1 per cent. of the effective area of the slab.

When deciding on a suitable tensile working stress for reinforcement in any particular part of the structure the modifying factors given in Chapter IX should be considered. Factors representing a variation in the ultimate strength of the concrete only cannot be regarded as operative when considering the steel stresses.

In impermeable construction, where cracking is to be avoided, there is no advantage, as far as tensile stress is concerned, in using high yield-point steel, since the stress of 12,000 lb. per square inch already mentioned should not be exceeded. Reinforcement on the faces of ribs remote from the liquid face can, however, be stressed to 16,000 lb. per square inch, and a similar stress can be adopted for tanks if the container is lined with an impermeable membrane which is sufficiently elastic to give with the strained concrete.

The working tensile stress on ordinary mild steel in building works can be increased by one-third when the increase is due solely to increased moments, etc., caused by wind pressure.

Compression.—The compressive stress to which reinforcement is subjected is controlled by the compressive stress in the surrounding concrete. Since the strain in the two materials is equal so long as the bond is not destroyed, the stresses will be proportional to the elastic moduli. Mild steel has an elastic modulus of 30,000,000 lb. per square inch, and with a modulus for concrete of 2,000,000 lb. per square inch, the compressive stress in the steel would be fifteen times that in the concrete, or generally

$$c_t = mc$$

where c_t = the compressive stress in steel

m = the modular ratio $= \frac{E_S}{E_C}$ (See *Table* No. 23 (page 263) and Chapter IX).

and c = compressive stress in the concrete.

In practical calculations it is more convenient to calculate the additional compressive resistance due to the steel; this is $c(m - 1)A_C$ where A_C is the area of compressive reinforcement. When this expression is used the resistance of the concrete must be calculated on the gross sectional area, no deductions being made for the reduction in concrete area to allow for the area occupied by the steel.

When the moment of resistance of a beam section is calculated on the steel-beam theory (see Chapter XI) the stress in the compression reinforcement is taken as equal to that in the tensile reinforcement, that is, generally 18,000 lb. per square inch.

For high yield stress steel the maximum compressive stress is usually specified as four-tenths of the yield stress.

3.—Adhesion.

Bond Length.—For a reinforcing bar efficiently to resist the tensile forces it must be adequately anchored in the concrete. There must be sufficient length of bar beyond any particular section to develop by adhesion between the steel and the concrete a bond force equal to the total tensile force in the bar at that section. The normal safe adhesion stress between 1 : 2 : 4 concrete and plain

round steel bars is usually taken as 100 lb. per square inch. Thus, a bar stressed to 18,000 lb. per square inch requires a length of 45 times its diameter, and for other stresses the bond length is in proportion. More generally, each 2,000 lb. per square inch of tensile stress requires five diameters of straight bar to provide sufficient adhesion exclusive of any end anchorage.

Anchorage of Tension Bars.—At the end of the net length of bar required for adhesion it is usual to provide an anchorage. This may be an additional length of straight bar, a properly formed semi-circular hook, a 60-deg. or 90-deg. bend, or some form of mechanical anchorage. In order to take advantage of the anchorage value of hooks and bends it is important that they shall be properly formed; suitable dimensions are given on *Table* No. 24 (page 265).

With a semi-circular hook no additional straight length of bar is required, but with a 60-deg. bend an additional straight length of five diameters should be provided and, with a 90-deg. bend, an additional length of ten diameters. In the absence of a hook or bend, an additional straight length of fourteen diameters should be provided.

The minimum lengths for these different types of anchorage are given on *Table* No. 24 in terms of bar diameters for various tensile stresses; it should be remembered that the adhesion length must be measured from the point where the bar commences to deviate from the straight. The minimum stress tabulated is 10,000 lb. per square inch and, although the maximum working stress in a bar may be less than this, it is recommended that in no case should a less anchorage length be provided than is required by this stress. For straight bars forty diameters should be the minimum.

A mechanical anchorage can be either in the form of a hook embracing a lateral bar, the internal diameter of the hook being equal to the diameter of the anchor bar, or the end of the reinforcing bar can be threaded and provided with a plate and nut. The minimum size of the plate should be such that the compression on the concrete at, say 600 lb. per square inch of net area of contact (that is, gross area of plate less the area of bar) should be equivalent to the bond resistance on a bar 14 diameters in length. This type of anchorage can also be provided on a bar where it is not possible to obtain the full bond length required by the working stress, in which case the plate should be of such a size that it makes up the difference in the net bond length and end anchorage.

The foregoing discussion and the anchorage lengths given on *Table* No. 24 apply to a permissible bond stress of 100 lb. per square inch which may be used with an ordinary quality 1 : 2 : 4 concrete. Values are also given for Quality A concrete of 1 : 2 : 4 mix, for which the bond stress is 120 lb. per square inch. If a stronger concrete is used the bond stress can be increased accordingly (see *Table* No. 23), and the net bond length thereby reduced. This length can be determined from the expression

$$N = \frac{t}{4s_b}$$

where N = the number of bar diameters in the length

t = the tensile stress in the bar at the section considered (lb. per square inch)

s_b = the maximum permissible bond stress (lb. per square inch).

The bond length obtained from this expression is sufficient if a semi-circular hook is provided, but the additional lengths of 5 diameters (with a 60-deg. bend), 10 diameters (with a 90-deg. bend), and 14 diameters (with no hook or bend) previously mentioned still apply.

For impermeable construction with a $1 : 1\frac{2}{3} : 3\frac{1}{3}$ Quality A concrete the bond length should be based on a stress of 100 lb. per square inch. The adhesion length with various anchorages given for 12,000 lb. per square inch and 1 : 2 : 4 ordinary concrete on *Table* No. 24 would therefore apply to impermeable construction using the richer mix. A semi-circular hook is the best anchorage for all bars in direct tension, both in impermeable construction and in such cases as hanging columns, etc. In slab reinforcement hooks can generally be omitted and straight anchorage lengths provided, but for any bar stopping in the tension area of a beam or similar member some form of hook or bend is preferable.

It is generally allowed that the anchorage length of the top portion of a bent-up shear bar can be measured from the point where the bar intersects the neutral plane.

Bond Length for Compression Reinforcement.—Where a bar is in compression, as in a column or in the compression side of a beam, a straight anchorage length equal to 24 diameters is usually considered sufficient, although where the bar can be subjected to either tension or compression, anchorage should be provided for the maximum stress. In impermeable construction a straight length of 30 diameters is recommended for compression bond lengths.

A compression bond length of bar need not be hooked, but some form of turned end is desirable if the end of the bar is near an outer end face of the member.

Bond in Beam Reinforcement.—The bond stress resulting from variation in the tensile stress in beams should be investigated. This bond stress is given by the formulæ

$$s_b = \frac{S \pm \frac{M}{d} \tan \alpha}{ao}$$

where S = maximum shear force at the section under consideration (lb.)
a = lever arm (in.)
o = total perimeter of the tension bars at the section (in.)
M = bending moment (in.-lb.)
d = effective depth (in.)
α = angle between top and bottom edges of the beam at the section considered.

The positive sign applies when the bending moment decreases with the increase in d (as at the haunches at the ends of freely supported beams). The negative sign applies when the bending moment increases with the increase in d (as at haunches adjacent to the interior supports of continuous beams).

When the beam is of equal depth throughout, or the top and bottom edges are parallel, the expression reduces to

$$s_b = \frac{S}{ao}.$$

The permissible value of s_b is usually twice the permissible bond stress for adhesion, that is, for a 1 : 2 : 4 ordinary quality concrete the allowable beam bond stress would be 200 lb. per square inch.

Values of bar perimeters are given on *Table* No. 25 (page 267).

4.—Detailing Reinforcement.

Lengths and Sizes of Bars.—Careful attention paid to a number of points regarding lengths, sizes, etc., of reinforcing bars is repaid by saving due to ease of fabrication of the bars on the site and by a general rise in the efficiency of construction and design. To this end as few different sizes of bars as possible should be used in any structure or part of a structure, and the largest diameter bars consistent with non-excessive spacing should be used, thus reducing the number of bars to be bent and placed. Also it should be borne in mind that large-diameter bars are cheaper than small. The basic rate is usually quoted for $\frac{5}{8}$-in. diameter bars, all larger bars being supplied at this rate, while smaller bars cost more for each $\frac{1}{8}$ in. of diameter below $\frac{5}{8}$ in. diameter.

Generally the longest bar economically obtainable should be used, but due regard should be paid to the facility with which a long bar can be transported to the site and placed in any particular member. Consideration should also be given to the maximum length that can be handled without being too "whippy." Such maximum lengths are of the order of 20 ft. for bars $\frac{5}{16}$ in. diameter and less, 25 ft. from $\frac{5}{16}$ in. to $\frac{1}{2}$ in., 40 ft. for $\frac{5}{8}$ in. diameter, 60 ft. for 1 in. diameter, and 75 ft. for bars over $1\frac{1}{4}$ in. diameter.

When proposing to use long bars it must be remembered that the basic rate only applies to bars up to 40 ft. long and an extra that may amount to 1*s*. per ton for every foot over 40 ft. is charged, although bars up to $\frac{3}{8}$ in. diameter can be obtained in long length coils at normal rates and sometimes lower. Above certain lengths it is more economical to lap two bars than to buy long bars, the extra cost of the increase in total length of bars due to overlapping being more than offset by the alternative increased tonnage charge for long lengths. Long bars cannot always be avoided when reinforcing long piles, but bars over 24 ft. and more especially over 40 ft. require special railway wagons, involving transport delays and often additional freight charges.

Bending Schedules.—The careful preparation of bending schedules can result in economies in construction and in material supply. The total length of each bar should for preference be given to a multiple of 3 in. and as many bars as possible should be of one length, thus keeping the number of different lengths of bars as low as possible.

The method of giving bending dimensions and marking up bars should be uniform throughout a given structure. A well-tried system of standard bending dimensions is illustrated on *Table* No. 24.

Cover of Concrete.—For proper protection of the reinforcement and in order to ensure that the thickness of concrete around a bar is adequate to develop adhesion, it is necessary to provide sufficient cover of concrete over the bars and the minimum space between adjacent bars. The minimum cover of concrete around a reinforcing bar should be in accordance with the following schedule :

Slabs:	$\frac{1}{2}$ in. minimum and not less than the diameter of the bar.
Beams:	1 in. minimum and not less than the diameter of the bar for top or bottom cover to main bars.
	$\frac{1}{2}$ in. minimum for binders.
	1 in. side cover irrespective of diameter of the bar.
	2 in. end cover over main longitudinal bars.
Columns:	1 in. minimum for main bars in columns less than 12 in. square.
	$1\frac{1}{2}$ in. minimum for main bars in columns 12 in. square or over.
	$\frac{1}{2}$ in. minimum for binders.
Piles:	$1\frac{1}{2}$ in. minimum to main bars.
Sea Work:	$1\frac{1}{2}$ in. to 2 in. over main bars.
Impermeable Construction:	1 in. and not less than the diameter of the bar with $1 : 1\frac{2}{3} : 3\frac{1}{3}$ concrete.
	$1\frac{1}{4}$ in. and not less than the diameter of the bar with 1 : 2 : 4 concrete.
Foundations:	$1\frac{1}{2}$ in. where concrete is in contact with earth.
Retaining Walls:	$\frac{3}{4}$ in. and not less than the diameter of bar on earth face.

The minimum clear distance between the bars in any one layer in beams should be not less than the diameter of the bar or 1 in. (or maximum size of the aggregate plus $\frac{1}{4}$-in.), whichever is the greater. The minimum clear distance between successive layers of beam bars should be $\frac{1}{2}$ in. and this distance should be maintained by the provision of $\frac{1}{2}$-in. diameter spacer bars placed every three or four feet throughout the length of the beam wherever two or more layers of reinforcement occur. Where the bars from transverse beams thread between the layers, special spacer bars are of course neither necessary nor effective. If the beam bars exceed 1 in. in diameter, it is preferable to increase the clear space between layers to $\frac{3}{4}$ in.

5.—Steel Areas and Weights.

The following data with reference to the areas and weights of round mild steel bars from $\frac{3}{16}$ in. to $1\frac{1}{2}$ in. diameter are given on *Tables* Nos. 25 (page 267) and 25A (page 266) and 26 (page 269):

(*a*) Cross-sectional area of any number of bars up to 20 (sq. in.).
(*b*) Cross-sectional area per foot width for bars spaced at 3-in. to 24-in. centres (sq. in.).
(*c*) Weight per foot of each size of bar (lb.).
(*d*) Length of each bar in one ton weight (ft.).
(*e*) Weights of any number of bars one foot in length (lb.).
(*f*) Weight per square yard for bars at different spacings (lb.).

In Chapter XV are given a number of methods of estimating and calculating the weights of reinforcement.

CHAPTER XI

BEAMS AND SLABS

1.—Rectangular Sections.

Reinforced in Tension only.—The basis of design of structural members subject to flexure is that the internal resisting couple shall be equal to the bending moment produced by the externally applied load. For members of reinforced concrete it is assumed that the strain at any point in a section is proportional to the distance of the point from the neutral axis, and that the tensile resistance of the concrete below the neutral axis is neglected. From these fundamental relations the general formulæ given below can be deduced. Referring to *Fig.* 37(*a*) the notation adopted is as follows, all units being in pounds and inches:

c = maximum compressive stress on the concrete,
t = maximum tensile stress on the steel,
b = breadth of beam,
d = effective depth of beam or slab,
D = total depth of beam or slab,
n = depth of neutral axis = $n_1 d$,
a = lever arm of section = $a_1 d$,
A_T = area of tensile reinforcement,
r = ratio of maximum stresses = $\frac{t}{c}$,
m = modular ratio,
T = total tension = $A_T t$,
C = total compression = $0{\cdot}5bnc$,
M = external bending moment,
$R.M.$ = safe resistance moment of a section, and
p = percentage of tensile reinforcement = $\frac{100A_T}{bd}$.

The neutral axis factor expressed in terms of the stress ratio is given by

$$n_1 = \frac{1}{1 + \frac{r}{m}}$$

and in terms of the percentage of steel by

$$n_1 = \sqrt{(0{\cdot}01mp)^2 + 0{\cdot}02mp} - 0{\cdot}01mp.$$

The lever arm factor in relation to the depth of the neutral axis is

$$a_1 = 1 - \frac{n_1}{3}.$$

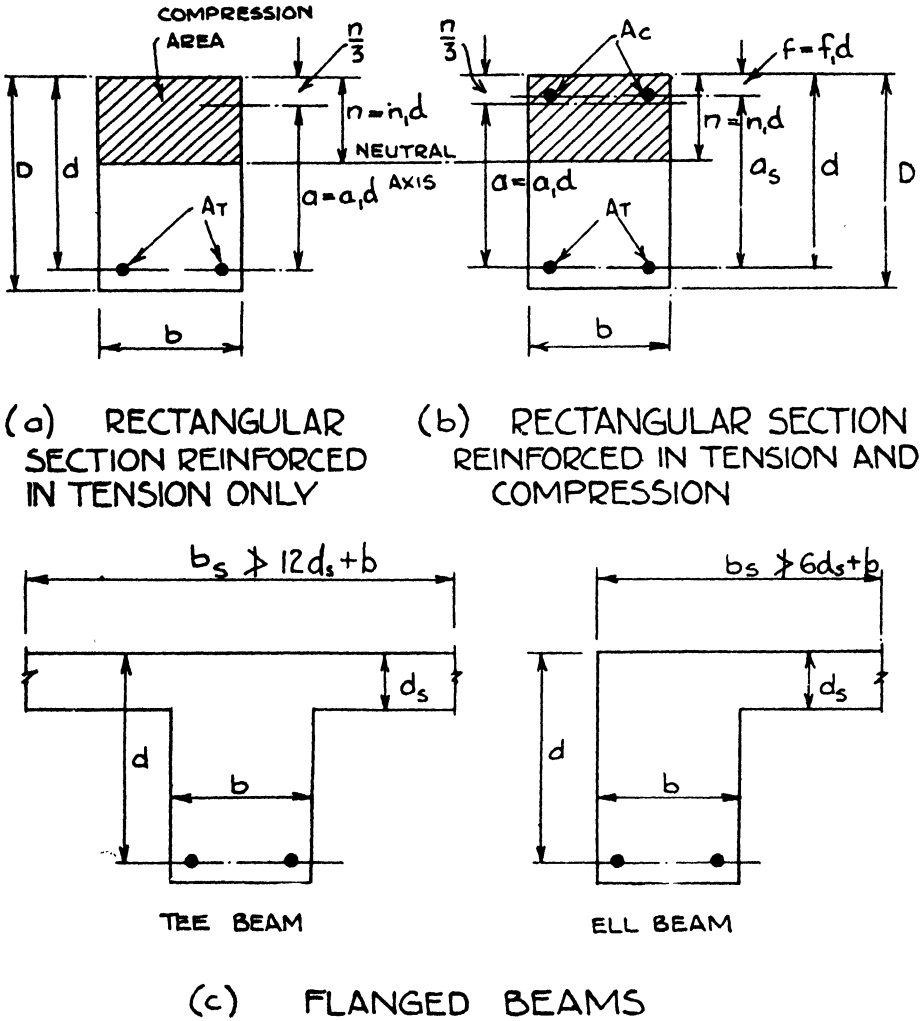


Fig. 37.—Rectangular and Flanged Beams.

The internal resisting couple $= Ta = Ca = R.M.$, which should not be less than the applied bending moment :

$$R.M. = A_T t a_1 d$$
$$\text{or} = 0{\cdot}5 b n_1 c a_1 d^2 = Q b d^2$$
$$\text{where } Q = 0{\cdot}5 n_1 c a_1.$$

Tabulated Factors.—Much arithmetical labour can be saved by the use of tables giving the relative values of the factors involved in the design of beams and slabs, and to this end *Tables* Nos. 27 to 32 (pages 271 to 281) inclusive have been compiled. Thus *Table* No. 27 gives values of n_1, a_1, and p for various values of r together with values of Q for concrete stresses from 500 to 1,000 lb.

per square inch and for steel stresses from 12,000 to 22,400 lb. per square inch with $m = 15$. On *Tables* Nos. 28 and 29 the resistance moments of a range of rectangular beams 1 in. wide for various combinations of maximum stresses are given, and *Table* No. 31 is a similar table for slabs 12 in. wide.

Table No 30 is also applicable to slabs and gives factors that when combined with the applied bending moment give the requisite depth of slab and the amount of reinforcement necessary. Miscellaneous data relative to the proportions, etc., of rectangular beams and tee beams are included on *Table* No. 32. On this table also are data for determining the points along a beam at which bars can be stopped off or bent up consistent with providing adequate tensile resistance to bending.

Proportions of Beams.—Apart from considerations of shear (see next chapter) the dimensions of a rectangular beam should be in accordance with the following rule: the effective depth should not be less than one-twentieth of the span when the maximum stress is 16,000 lb. per square inch in the tensile reinforcement. With tensile stresses of more or less than 16,000 in the steel, the minimum effective depth should be increased or decreased to a value represented by

$$\frac{\text{span} \times t}{320{,}000} \text{ (see } \textit{Table} \text{ No. 32).}$$

A good working rule is that the total depth of a rectangular or tee beam in inches should be equal to the clear span in feet. The breadth of rectangular beams and the breadth of the web of tee beams usually vary from one-third to once the total depth; for rectangular beams in building work, an average figure is one-half to two-thirds of the total depth, while in industrial structures beams having proportions of breadth to depth of $\frac{1}{2}$ to $\frac{1}{3}$ are often convenient. The lower ratio in each case would apply principally to tee-beams. Much, however, depends upon the particular circumstances controlling a given design, more especially such factors as headroom and the area required for shear. The breadth of beams should also be determined to conform with commercial widths of timber, and therefore 4 in., 5 in., 7 in., 8 in., and 11 in., or widths made up by combining these values, should be employed wherever possible, although in buildings, the widths may have to comply with the thicknesses of brick walls, i.e. 9 in., $13\frac{1}{2}$ in. or 18 in. The width should also bear a relation to the span of the beam, and the ratio of the length between lateral supports of a beam to the breadth of the compression flange should not be more than

$$20\left\{3 - 2\left(\frac{\text{calculated compressive stress}}{\text{permissible compressive stress}}\right)\right\}$$

Design of Rectangular Beams.—The various processes of stress determination and design are as follows:

To Design a Section for a given Bending Moment and for given Stresses.

Method (*a*).—For the given stresses find Q from *Table* No. 27 and find bd^2 from $\frac{M}{Q}$. Select suitable values of b and d (possibly from consideration of shear)

to give the required value of bd^2. For the ratio of the given stresses find a_1 from *Table* No. 27 and the amount of tensile steel required from

$$A_T = \frac{M}{a_1 dt}.$$

Method (*b*).—If the stresses correspond with those for which *Tables* Nos. 28 and 29 are prepared, select a suitable effective depth and read off from *Table* No. 28 or 29 the resistance moment corresponding to this depth. The applied bending moment divided by this resistance moment gives the breadth of beam required. If the relative values of d and b thus derived are unsuitable, select another value of d and repeat. From the same table read off the area of steel for the appropriate value of d and multiply this area by b to give A_T.

TO DETERMINE THE STRESSES ON A GIVEN SECTION SUBJECT TO A GIVEN BENDING MOMENT.

Determine p for the given section and from *Table* No. 27 find the corresponding values of a_1 and r. Then the maximum stresses are

$$t = \frac{M}{a_1 d A_T}$$

$$\text{and } c = \frac{t}{r}.$$

TO DETERMINE THE RESISTANCE MOMENT OF A GIVEN SECTION WITH GIVEN MAXIMUM STRESSES.

Determine p, and from *Table* No. 27 find the corresponding values of a_1 and n_1. The resistance moment based on the steel is given by $A_T t a_1 d_1$, and based on the concrete by $\frac{n_1 a_1 b d^2 c}{2}$. The maximum safe resistance moment of the section is then the smaller of the two calculated resistances.

2.—Beams with Compressive Reinforcement.

Concrete not Neglected.—When neither sufficient depth nor breadth can be obtained to provide sufficient concrete compressive resistance in a section, compressive reinforcement has to be provided. The provision of this extra reinforcement does not generally lead to economy, although a certain amount of concrete is saved thereby, but in some cases, such as support sections of continuous beams, convenience of steel arrangement allows double reinforcement to be provided cheaply.

The moment of resistance of a section doubly reinforced is the sum of the moments of resistance of the concrete and compression steel. The concrete resistance moment would be calculated as for singly-reinforced beams, and the steel resistance moment will equal $A_C f_c a_s$,

where A_C = area of compressive reinforcement
f_c = equivalent stress in the compressive reinforcement
a_s = distance from centroid of tension steel to centroid of compressive steel.

If f is the distance from the compressed edge of the section to the centroid of the compression steel (*see Fig.* 37(*b*)), then

$$a_s = d - f$$

$$f_c = \left(\frac{n - f}{n}\right)(m - 1)c.$$

The method of designing a rectangular section with given maximum stresses and limiting values of b and d, to take a specified bending moment, is as follows: Find Q from *Table* No. 27 for the appropriate maximum stresses, and find from the same table the value of n_1 for the stress ratio. Then the required moment of resistance of the compression steel is given by $R_S = M - Qbd^2$. Determine $n = n_1 d$ and evaluate a_s and f_c from the formulæ already given and substitute in

$$A_C = \frac{R_S}{f_c a_s}$$

to find the area of compression steel required. Next evaluate the resultant lever arm from

$$a_r = \frac{M}{A_C f_c + 0{\cdot}5bnc} \text{ or, approximately, } a_r = \frac{a_s + a_1 d}{2}$$

in which $a_1 d$ is the lever arm corresponding to the appropriate value of n_1, that is,

$$a_1 = 1 - \frac{n_1}{3}.$$

and find the area of tensile steel required from

$$A_T = \frac{M}{a_r t}.$$

If A_T is greater than A_C binders should be provided at a spacing not exceeding twelve times the diameter of the compression bars.

The limit of the rational application of this method of calculation is when $A_C = A_T$, and for this condition the value of $Q = \frac{M}{bd^2}$ and the corresponding percentage (p) of reinforcement (A_C or A_T) are as follows:

Steel stress (Tension)	Concrete stress (Compression)	m	$A_C = 0$		$A_C = A_T$	
			Q	p	Q	p
16,000	600	15	95	0·67	155	1·00
18,000	750	15	126	0·80	226	1·42
18,000	750	17·8	137	0·89	315	2·00
18,000	950	15	179	1·16	434	2·75

"Steel-Beam Theory."—If A_T is less than A_C the "steel-beam theory" (see later) can be applied and both A_T and A_C should be proportioned thus:

$$A_T = A_C = \frac{M}{a_s t}.$$

When this method is adopted binders should be provided at spacings not exceeding 8 times the diameter of the compression bars.

Some engineers are not in favour of the indiscriminate application of the "steel-beam theory" to reinforced concrete sections, and at first sight it would seem that a beam of any size can be designed to resist almost any bending moment irrespective of the compressive stress in the concrete. In fact with a theoretical steel stress of 18,000 lb. per square inch the maximum fibre stress in the concrete, in accordance with the usual assumptions, exceeds 1,200 lb. per square inch, and, although this gives a factor of safety of at about two on the ultimate strength of average concretes, there is little margin for such factors as accidental overloading, differences between theoretical and actual moments and stresses, poor workmanship, etc., such factors being those usually covered by the generally accepted factor of safety of four. Automatic partial safeguards against unreasonable use of the "steel-beam theory" include the provision of sufficient concrete area for shear, the available space for the number of top and bottom bars required, and the reduction of lever arm consequent upon large numbers requiring more than one layer.

In cases where excessively high stressing of the concrete is particularly undesirable, a rational compromise is to provide a section with equal tensile and compressive reinforcement, the area of which is determined so as to limit the concrete stress to 25 to $33\frac{1}{3}$ per cent. above the normal maximum working stress. Thus for a beam normally designed for 16,000 and 600, sections requiring A_C in excess of A_T to maintain the stresses within these limits could be designed with $A_C = A_T$ and the concrete stress not exceeding, say, 800 lb. per square inch. For normal stresses of 18,000 and 750 the increased concrete stress might be 950 lb. per square inch. On *Tables* Nos. 28 and 29 the resistance moments and area of steel per inch width (with $A_C = A_T$) for the normal stresses of 16,000 and 600 and 18,000 and 750, and for increased stresses of 16,000 and 800 and 18,000 and 950, are tabulated, from which suitable beam sections to resist given moments can be conveniently selected. It will be observed that 18,000 and 750 are the London By-law stresses for 1 : 2 : 4 ordinary concrete in buildings, while 18,000 and 950 are the stresses for Quality A 1 : 2 : 4 concrete.

3.—Design of Slabs.

Resistance of Slabs.—It is usual to design slabs for a strip one foot in width; hence a slab is equivalent to a rectangular beam with $b = 12$ in., with the bending moment expressed per foot width. In general the formulæ developed for rectangular beams apply also to slabs, but they can be modified to facilitate computation; for example, the effective depth required

$$= d = k_1\sqrt{M}$$

and the area of steel $$= A_T = k_2\sqrt{M}.$$

Values of k_1 and k_2 are given on *Table* No. 30 for various stresses, and when using these values it should be remembered that M should in this instance be in foot pounds. The method of using this table is to find the value of k_1 for the given permissible stresses and find the effective depth from the formula already given. If this depth is adopted then the value of k_2 applicable to the permissible stresses should be used to determine the area of reinforcement required per foot

width of slab. If any depth greater than that determined as before is adopted, find the value of

$$k_1 = \frac{\text{effective depth adopted}}{\sqrt{M}}$$

and with the given steel stress find the corresponding value of k_2. This is the value that should be multiplied by $\sqrt{M}$ to give A_T. If any depth less than that first calculated is adopted, find the value of

$$k_1 = \frac{\text{effective depth adopted}}{\sqrt{M}}$$

and with the given concrete stress find the corresponding value of k_2, which should then be multiplied by $\sqrt{M}$ to find the required steel area.

It is not common to reinforce slabs in compression unless a convenient arrangement of reinforcement is obtained thereby, but where it is necessary to do so the method of calculation would be identical with that for rectangular beams, and links should be provided to prevent the compression bars from buckling.

When the maximum prescribed stresses are 18,000 and 750 (ordinary building work) or 12,000 and 880 (impermeable construction) or in the same ratio as either of these pairs of stresses, suitable sections can be readily selected from *Table* No. 31, which gives the moments of resistance of slabs 12 in. wide without compression steel.

Slab Details.—Alternative arrangements of slab reinforcement are illustrated in *Fig.* 38, normal floors being in accordance with the detail at (*a*). Where the live load is so large compared with the dead load that reinforcement against negative bending moments are required, the detail at (*b*) is suitable.

Independently of the amount of tensile reinforcement required by calculation to resist the bending moment, the area of steel should not be less than 0·3 per cent. of the effective concrete section, and slabs spanning in one direction are normally provided with distribution steel placed at right angles to the main steel. The Code recommends that the area of such distribution steel should not be less than 20 per cent. of the area of the main steel; the London By-laws require 10 per cent.; for bridge deck slabs, the Ministry of Transport requires the amounts shown in *Fig.* 1 (p. 14). Attention is drawn to the fact that fabricated meshes do not always have sufficient cross wires to provide the specified amount of distribution bars and the deficiency has to be made up by an additional sheet or by wiring loose bars to the main fabric reinforcement.

For slabs designed with reinforcement in two directions the reinforcement spanning across the shorter equivalent span should be placed at midspan under, and at the supports, over the reinforcement lying normal to it.

Shear reinforcement is not usually necessary in slabs, and generally shear need not be considered unless the load exceeds one ton per square foot.

Although slabs as thin as 2¼ in. can be satisfactorily constructed given a rich mix and constructional care, for general work 3 in. should be considered as the minimum thickness. The minimum effective depth of the slab should not be

less than one-twentieth of the span for a slab reinforced in one direction, nor less than one-thirtieth of the span for slabs designed to span in two directions.

Where slabs span in one direction, there will generally be a negative bending moment over any main beams or ribs running parallel to the span of the slab. Thus reinforcement in the top of the slab and normal to the main reinforcement will be required. In the absence of precise calculations of the quantity of steel required for this purpose, reinforcement equal in area to 0·3 per cent. of the total cross-sectional area of the slab should be inserted. As this reinforcement is in the nature of principal reinforcement it should be spaced at centres not exceeding twice the effective depth. The lengths of the bars should be such that they extend on each side of the beam an average distance of about six times

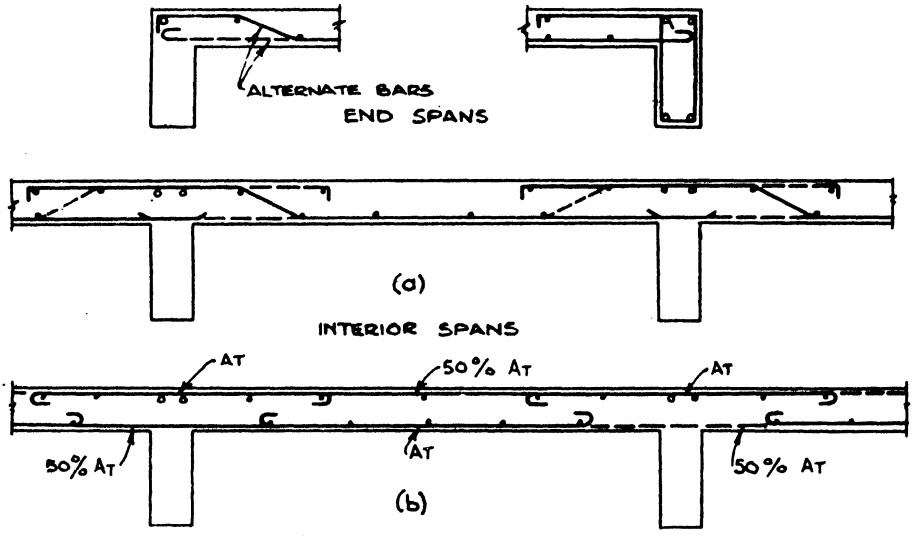


Fig. 38.—Typical Slab Details.

the slab thickness, and the ends of the bars should be staggered. Thus in a 5-in. slab the area of reinforcement required would be

$$\frac{0{\cdot}3 \times 5 \times 12}{100} = 0{\cdot}18 \text{ sq. in.}$$

with spacing not exceeding $2 \times 4{\cdot}25 = 8\frac{1}{2}$ in. Three-eighth inch bars at 7-in. centres are suitable, and if the beam is 8 in. wide the average length of the bars would be

$$8 + (2 \times 6 \times 5) = 5 \text{ ft. } 8 \text{ in.}$$

Such reinforcement is also sufficient as flange reinforcement for tee- and ell-beams.

4.—Flanged Beams.

Proportions.—Where beams are constructed monolithic with a slab, the slab can be considered as forming the compression flange of the beam so long as

the moment is such as to produce compression on the face of the beam nearer the slab. The treatment of such beams is similar to that for rectangular beams, there being, however, two cases to consider:

(*a*) When the neutral axis falls within the slab.
(*b*) When the neutral axis falls below the slab.

Referring to *Fig.* 37(*c*) the additional notation is

d_s = total depth of slab
b_s = effective breadth of flange.

The breadth of slab forming the effective flange of the beam should not exceed the least of the following:

(*a*) The distance between centres of adjacent beams, or the extreme width of the available slab.
(*b*) One-third the span of the beam.
(*c*) Twelve times the thickness of the slab plus the width of the rib of the beam.

If the slab only extends on one side of the rib and the effective section is an ell section instead of a tee section, the allowable width of the flange should not be greater than that given in (*a*) or (*b*), nor greater than four times the slab thickness plus the rib breadth.

The depth of the rib is determined by the effective depth required and is controlled by the same considerations as hold good for rectangular beams. The breadth of the rib is usually controlled by the area required for shear (see next Chapter) but consideration should also be given to the accommodation of the steel; unless circumstances dictate otherwise a rough rule for determining the minimum rib breadth in inches is $2\frac{1}{2}$ times the number of bars in one layer.

Design.—When the neutral axis falls within the slab thickness the design of a tee or ell beam is identical with the design of a rectangular beam in which b_s is substituted for b.

When conditions are such that the neutral axis falls below the slab—that is, when $n > d_s$—it is usual to neglect the small amount of compressive resistance afforded by the concrete lying between the neutral axis and the underside of the slab, and the lever arm is considered as

$$a = d - \frac{d_s}{2}.$$

This is only approximately correct, but the approximation is on the safe side.

The moment of resistance to compression is given by $0{\cdot}5b_s d_s ca$ approximately, although if d_s is very much less than n it is more economical to take the less approximate value of $b_s d_s c_s a$, where c_s is equal to the mean stress in the slab and is given by

$$c_s = \frac{c}{2}\left(1 + \frac{n - d_s}{n}\right).$$

The moment of resistance to tension is given by $A_T ta$, or the area of reinforcement required is found from

$$A_T = \frac{M}{ta}.$$

It is not common for tee-beams to require compression reinforcement, but where this is unavoidable the same principles will apply as for rectangular beams.

Typical details of the arrangement of reinforcement in beams and slabs and the method of designing a floor panel, beam and slab construction, are given in the "Additional Examples" following *Table* No. 40 (page 301).

5.—Plastic Theory.

Study has recently been given to the analysis of reinforced concrete sections on a basis other than the conventional theory considered in this chapter. The alternative methods are variously described as plastic theories, or balanced design, or the failure-load method, the principle being that failure occurs when the concrete attains its ultimate compressive strength simultaneously with steel reaching its yield stress. The modular ratio is thereby not involved, and the working load is assessed by applying an appropriate factor of safety to the calculated ultimate load or resistance moment. The method is applicable to columns with concentric or eccentric load (see pages 175 and 189), while the formulæ given below apply to beams, and have been modified by the author to embody terms familiar in British practice and in certain other respects. The notation is as used elsewhere in this chapter with the following additions :

M_U = ultimate resistance moment (in. lb.)
c_u = ultimate 28-day strength of 6-in. concrete cubes (lb. per square inch)
t_y = yield stress of reinforcement (lb. per square inch).

Rectangular beams :

If compression controls : $$M_U = c_u\left[0{\cdot}4bd^2 + A_c a_s\left(\frac{t_y}{c_u} - 1\right)\right] \quad . \quad . \quad . \quad . \quad (a)$$

Tension controls if $$A_T < \frac{c_u}{t_y}\left[0{\cdot}54bd + A_c\left(\frac{t_y}{c_u} - 1\right)\right],$$ in which case

$$M_U = c_u\left[bd^2 d_o\left(1 - \frac{d_o}{2}\right) + A_c a_s\left(\frac{t_y}{c_u} - 1\right)\right] \quad . \quad (b)$$

where $$d_o = \frac{1}{bd}\left[A_T\frac{t_y}{c_u} - A_c\left(\frac{t_y}{c_u} - 1\right)\right].$$

Tee beams : If $d_s < 0{\cdot}54d$ use formula (a) or (b) ; otherwise (c) or (d) apply :

If compression controls : $$M_U = b_s d_s c_u\left(d - \frac{d_s}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

Tension controls if $$A_T < b_s d_s \frac{c_u}{t_y},$$ in which case

$$M_U = A_T t_y\left(d - \frac{d_s}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

CHAPTER XII

SHEAR AND TORSION

1.—Shear Resistance.

Shear Stresses.—The shear force on a member subjected to bending moment produces diagonal tensile stresses in the concrete and, if the stresses exceed the safe tensile stress on the concrete, reinforcement should be provided either in the form of diagonal bars or binders to augment the shear resistance of the section.

The shear stress on a section is determined from the expression

$$s = \frac{S \pm \frac{M}{d} \tan \alpha}{ba}$$

where S = the total shear force across the section (lb.)
a = the lever arm (in.)
b = the breadth of a rectangular beam or the breadth of the rib of a flange beam (in.)
M = the bending moment at the section (in.-lb.)
d = the effective depth (in.)
α = the angle between the top and bottom edges of the beam.

The negative sign in the expression applies when the bending moment increases with the increase in d, as occurs at haunches adjacent to the interior supports of continuous beams. The positive sign applies when the bending moment decreases with the increase in d, as occurs at haunches at the ends of a freely supported beam.

When the beam has parallel edges (that is, the usual case of a beam of uniform depth) the shear stress on the section is given by the expression

$$s = \frac{S}{ab}.$$

Without assistance from shear reinforcement the concrete alone can resist the total shear when s is not greater than the safe shear stress $\left(= \frac{c}{10}\right)$ given on *Table* No. 23 (page 263) for the appropriate mix of concrete. The value of s may exceed the safe value if reinforcement is provided, and a limit of $0{\cdot}4c$ in which c is the maximum permissible compressive stress on the concrete should not be exceeded. In general cases, however, extreme values of s should not be used and a limit of $s = 0{\cdot}3c$ is preferable for secondary beams, although the higher limit might be used for main beams in structures not of the warehouse class or in similar cases where the full design load is probable.

In general practice it is considered that if the value of s exceeds $\frac{c}{10}$ the concrete must be neglected in calculating the shear resistance of a section. Some designers go to the other extreme and, irrespective of the value of s, always consider the concrete to be capable of taking $\frac{c}{10}$ lb. per square inch. Rational design would appear to lie between these two extremes and, instead of disregarding the concrete altogether when s is greater than $\frac{c}{10}$, a diminishing effective value should be taken as s increases, so that the value of the concrete is $\frac{c}{10}$ lb. per square inch when s is equal to $\frac{c}{10}$ and is zero when s is equal to or greater than $\frac{c}{5}$. This method is no doubt reasonable when the shear stress on the concrete lies between $s = \frac{c}{10} = 60$ and $s = \frac{c}{5} = 120$ lb. per square inch and the shear reinforcement is stressed to not more than 12,000 lb. or 14,000 lb. per square inch. With the higher intensities of shear stress now in common use and with a stress of 18,000 lb. per square inch on the shear reinforcement, it is preferable to take the whole shear on the reinforcement when s exceeds $\frac{c}{10}$.

Shear Value of Binders.—The shear taken by vertical binders is given by

$$S = \frac{Ata}{p} = Va$$

where A = cross-sectional area of binder taking into account the number of vertical arms (sq. in.)
t = allowable tensile stress in binders (lb. per sq. in.)
a = lever arm of section (in.)
p = pitch or spacing of binders (in.)
V = shear value of binders (taking into account the diameter of the bars from which the binders are made, the allowable stress, and the pitch) $= \frac{At}{p}$.

Values of V for t = 18,000 lb. per square inch and for various spacings and sizes of bar for two-armed binders are given on *Table* No. 33 (page 283), and the shear resistance of any system of binders can be found by multiplying the appropriate value of V by the lever arm of a given section.

In certain cases, such as beams subjected to vibration and impact, the stresses in the shear reinforcement should be less than 18,000 lb. per square inch and should be of the order of 14,000 lb. per square inch. For such beams, also, small diameter binders at close pitches are preferable to larger diameter bars at wider spacings. In impermeable construction, the stress in the binders should not exceed 12,000 lb. per square inch, that is, the values of V are two-thirds of those tabulated on *Table* No. 33.

The spacing of binders to take any given shear force S is given by

$$p = \frac{Ata}{S}.$$

To be effective as shear reinforcement the value of p should not exceed the lever arm, and if the calculated value is greater than a the diameter of the binders can be reduced until a suitable pitch is attained. Although binders may not be required for shear they should always be provided in a beam (except in simple lintel beams), and the maximum spacing should not exceed the effective depth. The minimum diameter of bar suitable for binders is $\frac{1}{4}$ in., and bars over $\frac{1}{2}$ in. diameter are usually difficult and costly to provide satisfactorily. When compression steel equal in area to the tension steel is provided in a beam and the calculation for resistance moment is based on the "steel-beam theory," the pitch of the binders should not exceed eight times the diameter of the compression

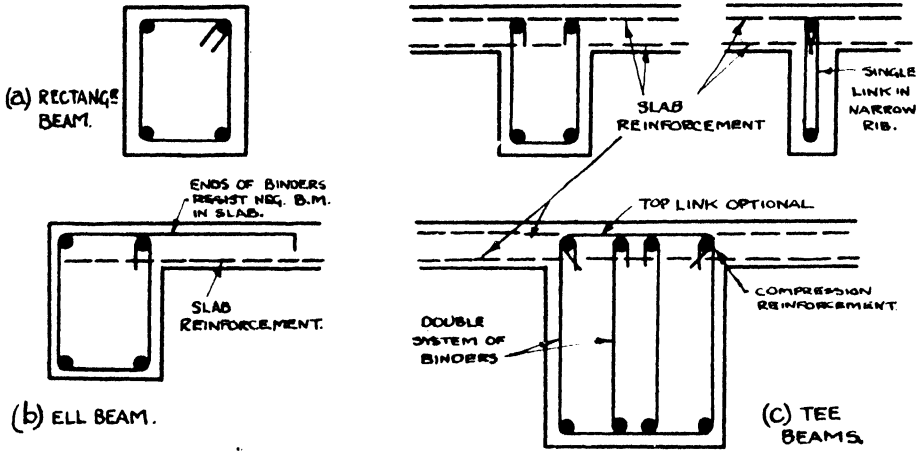


Fig. 39.—Beam Binders.

bar, and the diameter should be arranged accordingly. If the concrete is not ignored in calculating the compressive resistance, the binders should be spaced at distances not exceeding twelve times the diameter of the compression bar.

All binding should be effectively anchored to the top bars in a beam; *Fig* 39 indicates a number of approved shapes commonly adopted. In the beam with the double system of binders the top portion is an independent bar; this arrangement allows the binders to be open at the top until the beam reinforcement has been fixed, and is particularly suitable for heavily reinforced rectangular beams.

Shear Value of Diagonal Bars.—Since the principle assumed in evaluating the shear resistance of diagonal bars is that the latter form the tension members of a lattice girder for which the inclined compression in the concrete provides the corresponding compression member, it follows that the shear resistance at any section is the sum of the vertical components of all the diagonal bars and compression struts cut by the section. Thus, with a single system of diagonal bars

arranged as in *Fig.* 40(*a*) the shear resistance at any vertical section in the length L is given by

$$S = At \sin \theta \text{ (for single system),}$$

or when $\theta = 45$ deg.

$$S = 0{\cdot}707At$$

where A = the normal cross-section area of diagonal bars (sq. in.)
t = the maximum allowable tensile stress (lb. per sq. in.)
θ = the angle made by the bar with the horizontal.

If a double system, as in *Fig.* 40(*b*), is provided, double the resistance is obtained, and the shear resistance of any vertical section within the length L_1 is given by

$$S = 2At \sin \theta$$

and when $\theta = 45$ deg.

$$S = 1{\cdot}414At$$

The same resistance can be considered as provided for the length L_2 as long as the unit shear stress on the concrete at section AB does not exceed $\frac{c}{10}$.

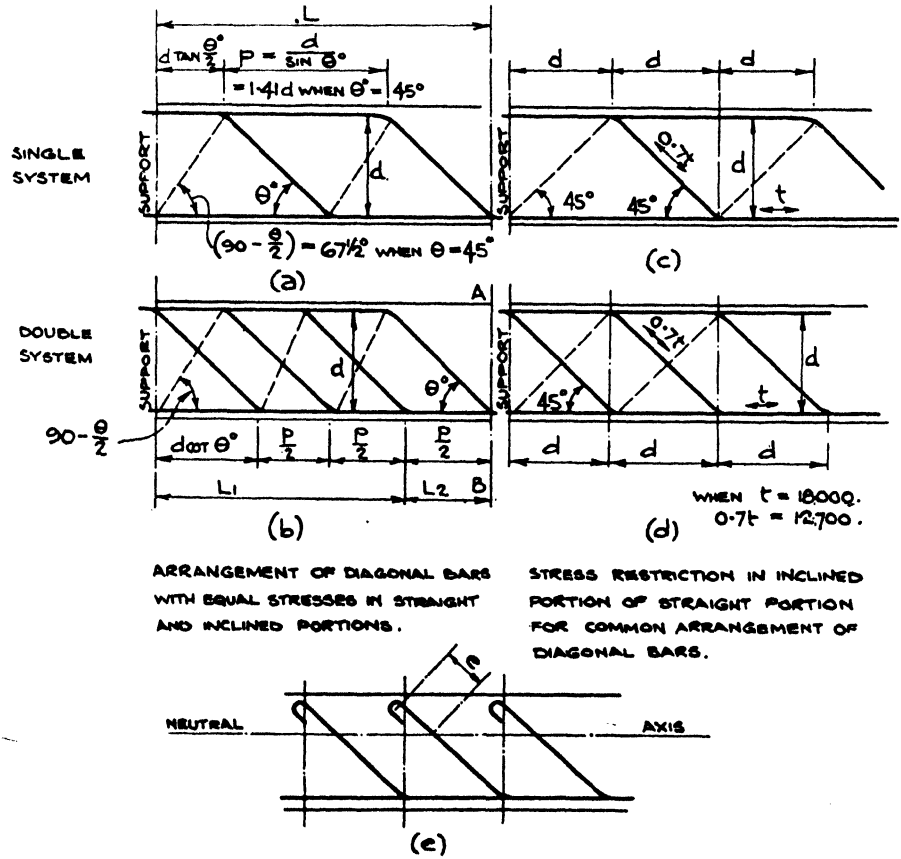


Fig. 40.—Diagonal Shear Bars.

If the diagonal bars are spaced in either of the two ways shown in *Fig.* 40(*a*) and (*b*), the stress (*t*) in the bent-up portion of the bar can be taken as the same as in the bottom straight portion of the bar, but if the common arrangement shown in *Fig.* 40(*c*) and (*d*) for bars bent up at 45 deg. is adopted, the stress in the bent-up portion cannot exceed the stated proportion of the stress in the bottom straight length.

Diagonal shear bars must be anchored sufficiently beyond the intersection with the neutral axis to develop the stress adopted. If a bar is simply stopped at the top of the bend as in *Fig.* 40(*e*), instead of being bent horizontally, the allowable stress should not exceed that represented by the bond on the length *e*. This detail is not advised for any members except those of little importance, and in any case the bar is practically valueless except for very deep beams.

On *Table* No. 33 are tabulated the shear values of bars from ½ in. to 1½ in. in diameter bent up at 45 deg. and 30 deg. for both single and double systems at a stress of 18,000 lb. per square inch. This stress is usual for normal building construction, but for impermeable construction at 12,000 lb. per square inch, two-thirds of the tabulated values should be taken. If other stresses are adopted the shear value will be proportional.

Diagonal shear bars are usually provided by simply bending up the main tensile reinforcement, but in so doing inspection must be made to ensure that beyond the bending-up point the bar is not required to assist in providing resistance moment. The limiting points at which bars can be dispensed with as regards bending are tabulated on *Table* No. 32 (page 281), which applies to beams having up to eight bars in the principal tensile reinforcement. (Although bars can be bent up at the points indicated it is not implied that if they are not bent up they can necessarily be stopped off at these points, since they may not have sufficient bond length from their point of critical stresses. This aspect depends on the rate of change of bending moment, and should be investigated for any particular beam.)

Design.—Examples of the calculation of the shear resistance at any section of a beam and the design of a section to take a certain shear force are given on the page facing *Table* No. 33. In general the procedure would be as follows:

To calculate the shear resistance at any section, the factor V for the binders is read from *Table* No. 33 and by multiplying by the lever arm the shear resistance of the binders is obtained. The resistance of any diagonal bars will be the sum of the resistances of each bar as read off the table. An inspection of the arrangement of the diagonal bars will indicate whether a single, double, or more complex system is provided. The total resistance due to the steel is the sum of the resistance of the binders and the diagonal bars. The resistance of the concrete is given by the product of the lever arm and the breadth multiplied by the safe shear stress $\left(\frac{c}{10}\right)$. The higher of the two resistances (that is, reinforcement or concrete) represents the resistance of the section.

In designing a section the first step is to determine s from $s = \frac{S}{ab}$ and then allocate to one of the two cases:

Case 1, s less than $\frac{c}{10}$,

Case 2, s greater than $\frac{c}{10}$.

In the first case no shear reinforcement is required but nominal binders should be provided. If the problem falls in Case 2, from inspection, decide whether the shear reinforcement shall consist of binders or diagonal bars, or both. If binders alone, then $\frac{S}{a}$ gives the value of V required, and a suitable pitch and diameter can be selected from the *Table* No. 33. If diagonal bars form the principal reinforcement there would generally be some binders, if only a nominal amount, and the value of V for these binders can be found from the table. Diagonal bars should then be provided to take the shear $(S - Va)$. If both diagonal bars and binders are provided, the procedure is to combine and adjust the values of Va and $(S - Va)$ as best suits the problem in hand.

In important beams it is advisable to plot the shear resistance diagram for the whole beam on the same base and to the same scale as the shear force diagram, and be assured that the latter is amply covered by the former. For normal beams it is usually sufficient to determine the point at which no shear reinforcement is required and calculate the reinforcement required at the point of maximum shear. Between these two points the intermediate shear reinforcement can generally be allocated by judgment.

2.—Torsion.

The design of reinforced concrete sections subject to twisting moments is not susceptible to accurate mathematical analysis, and calculations must therefore be based largely on the results of tests. Among the most recent series of published tests, the results of which can be applied to practical design, are those made by Mr. Leslie Turner and Mr. V. C. Davies.* The data on *Table* No. 34 (page 285) have been calculated from the results of these tests.

Sections subject to twisting moments should be well shaped and be free from re-entrant angles. In designing for the normal shear and bending moments, lower stresses than usual should be adopted, thereby leaving a margin for the secondary torsional stresses. The moment of resistance to torsion of a reinforced section can be expressed as

$$T_R = RD^3s$$

where D = equivalent diameter (in.)
s = safe shear stress (lb. per sq. in.)
R = section coefficient.

The value of R depends upon the percentage of reinforcement and the shape of the section; suitable values are given on *Table* No. 34 for the shapes most commonly met with in practice. The equivalent diameter D can be described as the diameter of the largest circle that can be inscribed within the solid section. If the section is subjected to a bending moment or a shear force that will cause cracking of the concrete below the neutral axis, the area of the cracked concrete

* Institution of Civil Engineers, Selected Paper No. 165.

should be ignored when assessing the shape of the section and the value of D. In this respect it should be remembered that for continuous beams, whose shape in plan is a circular arc, the maximum twisting moment occurs at the points of contraflexure; hence the full section can be considered when investigating the torsional stresses.

The percentage of reinforcement is equal to the sum of the longitudinal reinforcement and binding inserted solely to resist torsion. Only longitudinal bars near the outer faces of the section can be considered effective, and it is usual to provide equal amounts of binding and longitudinal bars, since lateral binding is of utmost value for resisting torsion.

The method to be adopted in designing a section for torsion is to design first for the normal bending moments and shears (using reduced stresses) and to determine D for this section. For the concrete mix adopted take the value of s $\left(=\frac{c}{10}\right)$ given on *Table* No. 23. If the applied twisting moment is T in.-lb., the section coefficient required is

$$R = \frac{T}{D^3 s}.$$

From *Table* No. 34 the percentage of reinforcement required for the appropriate shape can be determined. This amount of reinforcement should be divided equally between lateral binders and longitudinal bars. An example is given on the page facing *Table* No. 34.

The combination of torsional shear stress with the shear stress due to normal bending is a complex process and, failing the adoption of a more accurate analysis, the designer should assure himself that, at any section reinforced for shear, the numerical sum of the two stresses is restricted to a reasonable value (say, not in excess of twice the normal allowable shear stress). Consider the example on p. 284, in which at the point of contra flexure (fifth-point of span approximately) the stress due to normal shear is 52 lb. per square inch and the torsional shear stress does not exceed 75 lb. per square inch. Thus the sum of the two shear stresses is 127 lb. per square inch, which is less than $2 \times 75 = 150$ lb. per square inch. The design is therefore satisfactory, since the binders provided give ample shear reinforcement.

CHAPTER XIII

COLUMNS

1.—Column Loads.

Design Data.—The superimposed loads for which columns in buildings should be designed are the same as those for beams given on *Table* No. 2 (page 213), except that the minimum alternative load need not be considered. Where the superimposed load does not exceed 100 lb. per square foot, the column loads can be reduced in accordance with the schedule given on *Table* No. 2 for multi-story buildings. External columns in buildings (and internal columns under certain conditions) should be designed to take bending moments due to the restraint at the ends of beams framing into the columns. Formulæ for the calculation of these bending moments and expressions for the determination of bending moments in columns due to wind pressure on buildings, water towers, silos, and similar structures are given in Chapter V.

The working stresses for which columns should be designed are the direct compressive stresses given on *Table* No. 23 (page 263) when the columns are subjected to direct load only. When the column is subjected to a bending moment in addition to a direct load, the stresses should be determined in accordance with the methods described in Chapter XIV, and these stresses should not exceed the compressive stresses given for bending on *Table* No. 23. Columns of this kind should, however, be checked to ensure that the direct compressive stress is not exceeded when only direct load occurs. The tabulated stresses can, however, be exceeded by not more than one-third for any forces or moments due to wind pressure alone.

The general modifications to the tabulated working stresses as set out in Chapter IX should be considered when assessing safe working stresses for columns. When the loads on the columns are approximately ascertained the concrete should not be stressed up to the limit, but when the column loads are carefully calculated from the elastic reactions from the beams the stresses can be increased up to the limit.

The reduction of stresses in slender columns is discussed later.

Reinforced concrete columns are usually either rectangular columns with independent binders or octagonal columns with helically-bound corners.

Columns with Independent Binders.—The effective area of a column section is the area of the concrete plus m times the area (A_c) of the longitudinal reinforcement; for a rectangular column

$$A = BD + (m - 1)A_c$$

where B and D are the breadth and width of either the core or of the whole

section (see *Fig.* 41) depending on which is considered as the effective load-carrying section. The London By-laws accept the effective area calculated on the whole section. The value of m depends upon the mix, and it is usual to accept a value of 15 for ordinary calculations notwithstanding the fact that such values underestimate the modulus for normally made concrete and give values of m in excess of the true values. The Code of Practice adopts a stress of 13,500 lb. per square inch in the compressive reinforcement (ordinary mild steel) in columns without reference to the concrete mix or modular ratio.

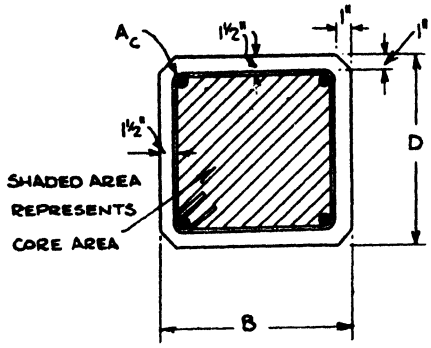


Fig. 41.—Column Section.

The safe axial load carried by a short column is

$$W = c_0 A$$

and thus the concrete stress in a short column carrying a concentric load is given by $c_0 = \frac{W}{A}$.

In a normal column the amount of reinforcement should be reasonable compared with the size of the column, and the following limits are generally suitable:

Minimum area of longitudinal reinforcement	= 0·8 per cent. of whole concrete area.
Maximum ,, ,,	= 8 per cent. of core area.
Minimum volume of binders	= 0·2 per cent. of the whole concrete volume.
Maximum ,, ,, ,,	= 5 per cent. of core volume.
Maximum spacing ,, ,,	= Twelve times the diameter of the smallest longitudinal bar or the width of the column, whichever is smaller.

The largest convenient diameter of binder is ¼ in., but larger diameters can be used; usually a much smaller bar is all that is necessary, and $\frac{5}{16}$ in. is a very convenient size. The arrangement, and therefore the diameter and spacing, of independent binders depend on the number of longitudinal bars in the column;

Fig. 42 illustrates a variety of arrangements. Those arrangements in which the binder is in the form of links provide almost twice the length per layer (and therefore almost twice the volume for a given diameter) given by the simpler types.

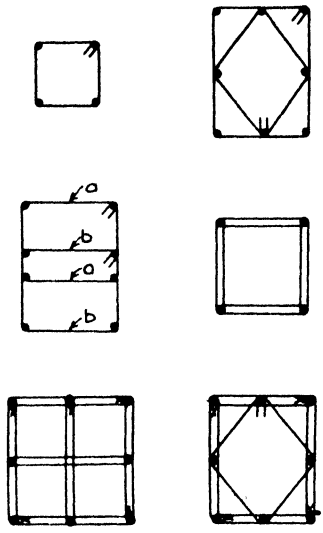


Fig. 42.
Typical Column Binders.

Immediately above and below a joint in the longitudinal reinforcement the binders are sometimes spaced at closer centres than elsewhere required in the column. Such joints should only be made at floor levels or beam intersections, unless unavoidably otherwise, as in that position a more secure and more conveniently constructed junction is obtained. Two types of column splices are indicated in *Fig.* 43.

The range of maximum safe loads carried by square columns of various mixes and with maximum and minimum percentages of longitudinal reinforcement is tabulated on *Table* No. 35 (page 287). These loads have been calculated with the standard stresses for the specified mixes, and if these stresses are modified a proportionate modification should be made to the safe load. From this table a suitable design for a square column can readily be selected to carry a given load.

Helically-bound Columns.—By making the binding in a column in a continuous helical form instead of in independent layers the stress to which the

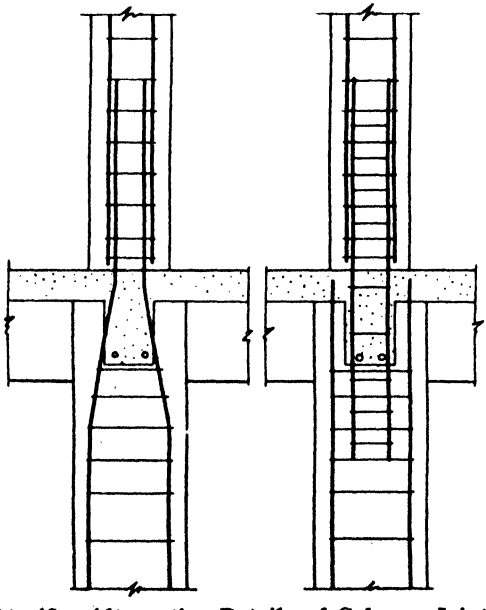


Fig. 43.—Alternative Details of Column Joints.

bound core can be safely subjected is much increased. Most building regulations give methods of calculating the increase in allowable load, and the following conditions conform to the London By-laws.

The form of the binding must be circular, or practically so, on plan, and the area of concrete effectively stressed must be limited to that contained within the bound core.

The maximum safe load which a helically-bound column can carry is given by

$$P = P_C + P_T + P_B$$

where P_C = load carried on the concrete core
P_T = load carried on the longitudinal reinforcement
P_B = increase in load due to helical binding.

Values of these components are as follows:

$$P_C + P_T = [0{\cdot}7854D^2 + A_C(m - 1)]c_o$$

where D = core diameter (in.)
m = the modular ratio (= 15)
c_o = permissible direct compressive stress (lb. per square inch)
A_C = area of longitudinal reinforcement which should not be less than 0·8 per cent. of the total area nor more than 8 per cent.

The value of c_o must be reduced when, because of slenderness or other reasons, the maximum stress cannot be permitted.

$$P_B = 7{,}080DA_b$$

where A_b = cross-sectional area of helical binding per foot length of column.

This expression is based on a stress of 13,500 lb. per square inch in the helical binding.

The spacing of helical binding should be not less than 1 in. and not greater than 3 in. An important limitation is that

$$P_C + P_B \not> 1{\cdot}875Ac,$$

where $P_C = 0{\cdot}7854D^2c$
and A = total cross-sectional area of the concrete (sq. in.)

Consider as an example, the calculation of the safe direct load on an 18-in. octagonal column, 1 : 1 : 2 mix, Quality A concrete, reinforced with eight $1\frac{1}{2}$-in. vertical bars and $\frac{3}{8}$-in. helical binding at $2\frac{1}{2}$-in. centres.

From *Table* No. 23, the safe direct compressive stress = 1,000 lb. per square inch.

The core diameter can be taken as 16 in. and the area of longitudinal steel = 14·14 sq. in.

Hence $P_C + P_T = [(0{\cdot}7854 \times 16^2) + (14{\cdot}14 \times 14)]1000 = 399{,}500$ lb.

The cross-sectional area of the helical binding per foot length of column = 0·53 sq. in. = A_b.

Hence $P_B = 7{,}080 \times 16 \times 0{\cdot}53 = 60{,}000$ lb.

Therefore the total safe load = 399,500 + 60,000 = 459,500 lb. (assuming that it is a "short column.")

Economical Column Design.—As so many variants enter into the

problem of designing a column, it is not easy at first sight to decide which combination leads to the most economical member. Given a constant slenderness factor, and any specified load over 100 tons, consideration of varying richness of mix, of maximum and minimum percentages of binders and longitudinal reinforcement, and of independent and helical binders, would lead to the following general conclusions.

Other things being equal the richer the mix the more economical the column becomes. In square columns the minimum amount of longitudinal reinforcement produces the cheapest design for a given mix. Also for any mix a square column is generally less costly than an octagonal column with a helically-bound core.

Taking eight alternative methods of design for columns to carry loads from 100 tons to 500 tons, the economical order is usually as given below, the most efficient type heading the list.

Rich (1 : 1 : 2) mix, square column with minimum vertical steel.
Rich mix, octagonal, with maximum volume of helical binders and minimum area of vertical steel.
Intermediate mix, square, with minimum area of vertical steel.
Intermediate mix, octagonal, with maximum volume of helical binders and minimum area of vertical steel.
Lean (1 : 2 : 4) mix, square, with minimum area of vertical steel.
Lean mix, octagonal, with maximum volume of helical binders and minimum area of vertical steel.
Lean mix, octagonal, with maximum volume of helical binders and maximum area of vertical steel.
Lean mix, square with maximum area of vertical steel.

Columns with helically-bound cores are either square, octagonal, or circular; usually an octagonal section is the most economical of these, since the shuttering is less costly than for a circular column and there is less ineffective concrete in the corners than in a square column. The minimum outside size of the column should be 2 in. more than the diameter of the bound core.

Although helically-bound columns are not necessarily the most economical form of column construction, the extra cost is usually offset by the advantages arising from the extra available floor space, especially in multifloor buildings of the warehouse class.

2.—Long Columns.

If the ratio of the effective length of a column to the least radius of gyration exceeds 50 the safe load on the column is reduced, the reduction factor being given by the expression

$$R_L = 1{\cdot}5 - \frac{0{\cdot}12 L_E}{g}$$

where L_E = the effective length of column (ft.) and
g = the least radius of gyration (in.).

On *Table* No. 35 are given values of R_L for slenderness ratios up to the

usually accepted limit. The permissible load on a long column is given by the expression $P_L = R_L P$
in which P is the permissible load on a short column (that is, a column having a ratio of effective length to least radius of gyration of 50).

The effective length of a column can be taken as the actual length of the column if the end conditions are equivalent to "hinged both ends." If the column is "fixed at both ends" the effective length can be taken as three-quarters of the actual column length. If the column is "fixed at one end and free at the other" the effective length should be taken as twice the actual column length. For intermediate conditions of end restraint intermediate values would apply. By "hinged" is meant that the column is properly restrained in position but not in direction. By "fixed" is meant that the column is properly restrained both in position and direction. By "free" is meant that the end of the column is unrestrained in either position or direction. The actual length of the column should, in the case of buildings, be taken as the distance between floor levels

For columns with independent binders the least radius of gyration should be calculated on the gross section of the column, and on the core section for columns with helical bindings when the latter are taken into account in determining the load-carrying capacity. The radius of gyration should take into account the reinforcement at a modular ratio of 15.

For square and rectangular columns it is usually more convenient to base the slenderness ratio on the least lateral dimension (provided the section has no re-entrant angles) instead of on the radius of gyration. For a square column, for which the ratio of effective length to least lateral dimension exceeds 15, the reduction coefficient can be derived from the expression

$$R_L = 1{\cdot}5 - \frac{0{\cdot}4L_E}{D_L} \quad \text{where } D_L = \text{the lateral dimension (in.) of the column.}$$

On *Table* No. 35 values of R_L for square columns are tabulated for various actual slenderness ratios, in which the effective length and the least radius of gyration, or least lateral dimension, are expressed in the same units.

3.—Plastic Theory.

The assessment of the safe axial load on a short column adopted in the Code of Practice is based on the plastic theory (see pages 162 and 189). The ultimate load P_o on a rectangular column can be expressed as

$$P_o = 0{\cdot}8BDc_u + A_s(t_y - 0{\cdot}8c_u) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

and the safe axial load, with a factor of safety of three, is

$$P = c_o\left[BD + A_s\left(\frac{c_s}{c_o} - 1\right)\right].$$

B, D, and c_o have the signification used elsewhere in this chapter, and
c_u = 28-day strength of 6-in. cubes (lb. per square inch) $= 3c = 3{\cdot}75c_o$
t_y = yield stress of reinforcement $= 3c_s$ (lb. per square inch)
A_s = total area of reinforcement (square inch)
c_s = 13,500 lb. per square inch for mild steel, 15,000 lb. per square inch for medium tensile steel, and generally $= \frac{t_y}{3}$.

CHAPTER XIV

COMBINED STRESSES

1.—General Principles.

THE maximum stresses in certain structural members, such as arches, walls of rectangular silos and bunkers, tie beams, columns subject to eccentric loads, chimneys, etc., are due to the combined effect of bending and direct force. The latter may be either a pull or a thrust, and the method of determining the magnitude and distribution of the stress on the section depends on the nature of the direct force and the relative magnitude of the moment and the force. There are three principal cases to consider:

(i) When the direct force is a thrust and the resultant stresses on the section are wholly compressive.

(ii) When the direct force is a pull and the resultant stresses on the section are wholly tensile.

(iii) When the direct force is either a pull or a thrust and both tensile and compressive forces are produced on the section.

The effect of a moment M and a direct force N acting simultaneously on a section is equivalent to a direct force acting at a distance e from the centroid of the stressed area where $e = \frac{M}{N}$. For convenience it is usual to consider that this eccentricity e is measured from the centroid of the concrete area, and the error involved in this approximation is negligible. In some problems the actual eccentricity of the load on the member is given, in which case, if the resultant moment is required, it can be determined from the equation $M = eN$.

The value of e relative to the dimensions of the section determines into which of the three cases the problem falls. For problems coming within the limits of Case (i) the maximum and minimum stresses are calculated by adding and subtracting respectively the stresses due to the direct force alone and to the moment alone. The limit of applicability of this case is reached when the tensile stress that would be produced by the bending moment alone (considering the whole concrete section as well as the steel fully effective) is equal to, or slightly exceeds in some cases, the compressive stress due to a concentric load N. With rectangular sections containing normal percentages of reinforcement these limiting conditions are approached when the ratio of $\frac{e}{D}$ lies between 0·167 and 0·5, where D is the total depth of the section. For any section the limiting value of e is equal to $\frac{Z}{A}$, where A is the effective area of the section expressed in concrete

units and Z is the modulus of the effective section (also expressed in concrete units) measured about an axis passing through the centroid of the section.

When N is a pull the stresses on the section are entirely tensile and the problem falls under Case (ii) when the ratio $\frac{e}{f}$ is less than 0·5, where f is the distance between the centroids of the reinforcement on opposite faces of the section. In this case the tensile resistance of the concrete is neglected.

When Case (i) is applied to a particular problem with N a thrust and excessive tensile stress is produced in the concrete, or when Case (ii) is applied to a problem with N a pull and compressive stresses are produced, the problem should be considered as coming within the limits of Case (iii). Various methods of calculation of the stresses for this case have been devised. Any direct method of solution of such problems becomes complicated since an exact analysis involves the solution of a cumbersome cubic equation, and any ready means of com-

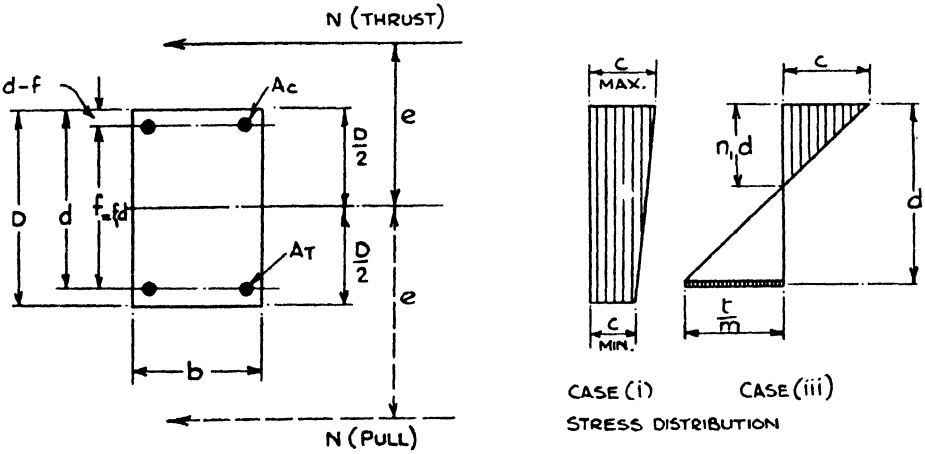


Fig. 44.—Rectangular Section.

putation would necessitate the possession of an impracticably large number of curves if all the probable variations of the components of the equation are to be allowed for. The comprehensive method advocated in this chapter involves the assumption of a trial factor that can be checked and adjusted, and in common with all other methods this method is based on the two conditions of equilibrium that

(*a*) the algebraic sum of all forces acting on the section is zero, and
(*b*) the algebraic sum of the moments of all forces acting on the section is zero.

For rectangular sections, or sections reducible to equivalent rectangles, the notation will be in accordance with that indicated in *Fig.* 44, and for the consideration of any irregular section the notation will be as shown in *Figs.* 45 and 46. When there is no reinforcement on the compression face of the section, the factor A_c in the formulæ will be written zero and consequent simplifications will follow.

For convenience an abstract of the methods of stress determination for any given rectangular section subject to combined bending and direct pulls or thrusts is given on *Table* No. 36 (page 289), together with values of certain of the factors involved in the calculation. For large values of $\frac{e}{D}$ (say exceeding 1·5) an approxi-

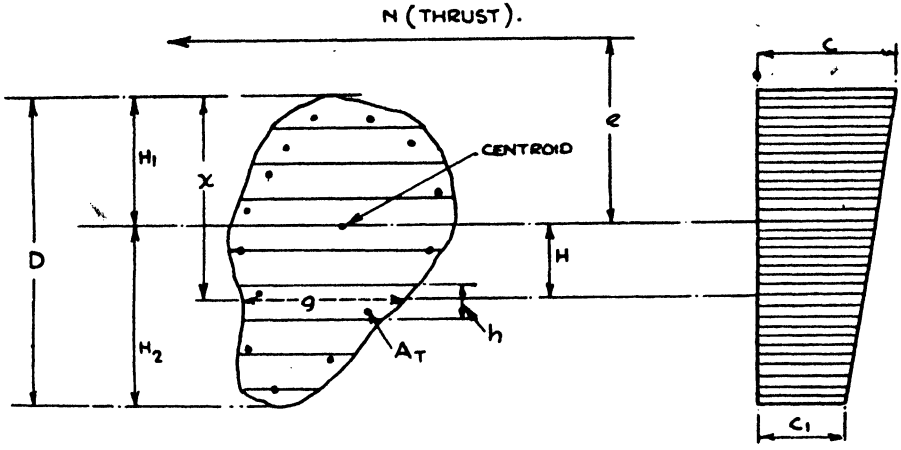


Fig. 45.—Irregular Section.

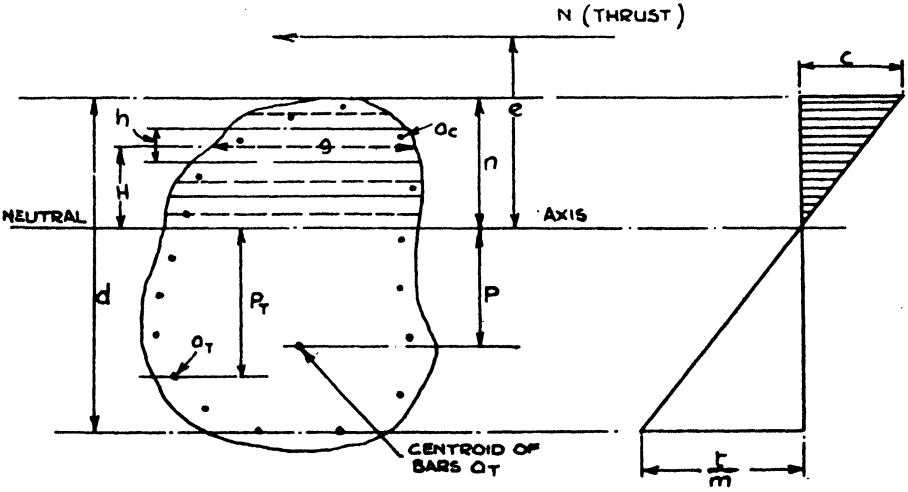


Fig. 46.—Irregular Section.

mate method can be followed that gives the resultant stresses on the section with a reasonable degree of accuracy.

2.—Bending and Direct Thrust.

Rectangular Section subject to Bending and Compression where *e* is less than 0·167*D*—In this case, with any proportion of reinforcement,

compressive stresses only are developed on the section and the maximum and minimum values are given by

$$c = \frac{N}{A} \pm \frac{M}{Z}.$$

For a rectangular section the values of A and Z are given by

$$A = A_1 + A_2$$

where $A_1 = (m - 1)(A_C + A_T)$

and $A_2 = bD.$

In this case $Z = \dfrac{2A_1(0{\cdot}5D - d + f)^2}{D} + \dfrac{A_2 D}{6}.$

The design of sections for this case involves the assumption of trial scantlings and reinforcement areas.

For the special case of $m = 15$ and $A_C = A_T$ the factors given on *Table* No. 36A can be used, the method to be adopted being given on the page facing this table, where examples of stress calculation are illustrated.

Rectangular Section subject to Bending and Compression where e is greater than 0·167D and less than 0·5d.—With no reinforcement (a plain concrete section) tension would be developed in one face of the section when e exceeds 0·167D, but with increase in the proportion of reinforcement the ratio of e to D can be considerably increased before tensile stresses are developed. The actual limiting value of $\frac{e}{D}$ depends on the amount of A_C and A_T and the relative values of f, d, and D. Cases where $\frac{e}{D}$ lies between 0·167 and 0·5 should be first treated as if $\frac{e}{D}$ were less than 0·167, and if no tensile stress is developed the stresses calculated by this method will be the actual stresses developed. Even if a small amount of tensile stress is developed, treatment as in the preceding section is justified so long as the tensile stress in the concrete for the worst combination of M and N does not exceed about one-sixth of the allowable compressive stress.

If the stress calculated in accordance with the method in the preceding paragraph exceeds this limit the tensional resistance of the concrete must be ignored, and the stresses calculated as in the next paragraph.

Determination of Stresses for a Rectangular Section subject to Bending and Compression where e is greater than 0·5d and less than 1·5d.—This is the general case, when tension on the concrete is ignored, and is applicable to sections with and without compression reinforcement and with any cover ratio or value of $\frac{d-f}{D}$ and any ratio of elastic moduli.

The first steps are to take a trial position for the neutral axis by assuming n_1 the neutral axis factor (see later observations) and calculate the maximum stresses c and t from the formulæ

$$c = \frac{NF}{Gbd + Kf_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$t = \frac{c(J + K) - N}{A_T} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the factors F, G, K, and J have the following signification

$$F = \frac{e - 0{\cdot}5D}{d} + 1$$

$$G = \frac{n_1}{2}\left(1 - \frac{n_1}{3}\right)$$

$$K = HA_C \text{ where } H = \frac{(m-1)(n_1 + f_1 - 1)}{n_1}$$

$$J = 0{\cdot}5bdn_1.$$

The ratio $\frac{t}{c}$ calculated from formulæ (1) and (2) should coincide with or be very nearly equal to the stress ratio corresponding to the trial value of n_1 and the appropriate modular ratio as determined from

$$n_1 = \frac{1}{1 + \frac{t}{cm}}.$$

In the first trial there may be some discrepancy between the two values of n_1, in which event the factors G, H, and J should be amended and a second calculation made for c and t; this should give a satisfactory value of n_1. On *Table* No. 36 values of n_1 for various values of $\frac{t}{c}$ for different modular ratios are tabulated, together with values of G and H. The value of F is constant for a given section with given ratio of $\frac{M}{N}$.

When the section is only reinforced in tension formulæ (1) and (2) become simplified to

$$c = \frac{NF}{Gbd}$$

$$t = \frac{cJ - N}{A_T}$$

since K is equal to zero.

For the special case of $m = 15$ and $A_C = A_T$, the stress can be obtained by adopting the method applicable to the factors given on *Table* No. 36A; a worked example is given on the page facing this table.

Rectangular Section subject to Bending and Compression where e is greater than $0{\cdot}5d$.—If the member being designed is a slab, mid-span section of beam, etc., and need not have compression steel, suitable concrete sizes and tension steel area can be determined by first assuming an appropriate value of d (and therefore D) and finding the minimum breadth of section required to keep the concrete stress within the specified limit from the relation

$$b = \frac{NF}{cdG}.$$

In this instance G is calculated (or read off *Table* No. 36) from the permissible values of t and c. The area of tension steel required is given by

$$A_T = \frac{cJ - N}{t}.$$

If the value of b thus obtained is unsatisfactory, an adjustment to the assumed value of d might give a reasonable section; but if a suitable value of b is not obtained within rational or permissible limits of d, then a convenient section can be found (*a*) by reducing the working stress in the tensile reinforcement, thereby increasing the area of concrete in compression; (*b*) by adding compression steel; or (*c*) by combining (*a*) and (*b*).

If reinforcement is added to aid the compressive resistance of the section, or if the member is such that conventional design or other considerations require the provision of compression steel (for example, columns, piles, support section of beams, sections subject to reversal of flexure), it is necessary to assume (or fix from other considerations) suitable values of b as well as d. With these values, and with the ratio of the allowable stresses in tensile steel and concrete, the factors F, H, J, and G can be calculated or read off *Table* No. 36. The amount of compression steel required is given by

$$A_C = \frac{P}{Hf_1}$$

$$\text{where } P = \frac{NF}{c} - bdG.$$

The area of tension steel can be found from

$$A_T = \frac{c(J + HA_C) - N}{t}.$$

If A_C should exceed A_T both values should be adjusted by reducing the value of tensile stress or by modifying the concrete dimensions.

Rectangular Section subject to Bending and Compression where e is greater than $1{\cdot}5d$.—When the eccentricity of the thrust is large compared with the dimensions of the sections the stresses will be primarily determined by the bending moment, the thrust only producing a secondary modification. In this case the stresses should first be calculated for the bending moment on the section alone as described in Chapter XI. The actual stresses can then be determined by adding c_1 to the maximum compressive stress in the concrete, and by deducting mc_1 from the maximum tensile steel stress, where

$$c_1 = \frac{N}{A_T m + bn + A_C(m - 1)}.$$

The design of sections that are within the limits of $\frac{e}{d}$ applicable to this case can be readily carried out by the method outlined for the preceding case.

Any Section subject to Bending and Compression where e is less than $\frac{Z}{A}$.—Given any reinforced concrete section subject to a direct force and a moment, the magnitude of $\frac{M}{N}$ being small, the first step in the determination of the stresses is to find the effective area of the section and the modulus Z of the section about an axis passing through the centroid of the section. When the section is symmetrically reinforced about the axis of bending, and is either rectangular (bending about a diagonal), circular, diagonal, or annular, etc., the

values of A and Z for the concrete section are readily obtained from the data given on *Table* No. 39 (page 297). The additional area A_A and additional modulus Z_A due to the reinforcement are given by

$$A_A = (m - 1)\Sigma A_T$$

$$Z_A = \frac{2(m-1)}{D}\Sigma A_T x^2$$

where A_T = area of a bar or group of bars placed at a distance x from the axis of bending.

When the section is irregular, as in *Fig.* 45, the values of A and Z can be determined by dividing the section into a number of narrow horizontal strips and by calculating the position of the centroid of the section, the effective area of the section, and the moment of inertia of the section. Thus, referring to *Fig.* 45

Effective area of each strip $= A_T(m - 1) + gh = a$

Total effective area of section $= \Sigma a = A$

Position of centroid: $H_1 = \dfrac{\Sigma ax}{\Sigma a}$

Moment of inertia: $I = \Sigma a(0{\cdot}083h^2 + H^2)$.

If h is small compared with g the term $0{\cdot}083h^2$ can be neglected.

The maximum compressive stress is given by

$$c = \frac{N}{A} + \frac{H_1 M}{I}$$

and the minimum compressive stress is given by

$$c_2 = \frac{N}{A} - \frac{H_2 M}{I}.$$

In extreme cases c_1 may be negative; this is permissible if the magnitude of this tensile stress on the concrete does not exceed, say, one-sixth of the allowable compression stress.

Any Section subject to Bending and Compression where e is greater than $\frac{Z}{A}$.—When the value of c_2 as determined in the preceding paragraph, exceeds the permissible negative value, or in those cases where e is so large compared with D that the simultaneous production of compressive and tensile stresses can be assumed at the outset, the tensile stress on the concrete should be neglected and the total tension should be resisted by reinforcement only. In this case it is necessary to select a position for the neutral axis, either after consideration of the maximum permissible stresses or otherwise, and to plot this axis parallel to the line of action of the direct force on a diagram of the given section drawn to scale, as indicated in *Fig.* 46. Next find the centre of gravity of the forces represented by the tension in the reinforcement below the neutral axis:

Total area of tensile reinforcement, $A_T = \Sigma a_t$

Position of centre of gravity, $p = \dfrac{\Sigma a_t p_t^2}{\Sigma a_t p_t}$

where a_t = sectional area of individual bars or groups of bars.

If all the bars are equal in diameter, then

$$p = \frac{\Sigma p_t^2}{\Sigma p_t}.$$

The next stage is to divide the compression area above the neutral axis into a number of narrow horizontal strips. The depth h of each of these strips need not be the same, as any regularity in the conformation of the section may suggest convenient subdivisions. When the strips are of equal depth, or when the section is symmetrical or is hollow, the necessary modifications and simplifications should be readily perceived.

For each strip determine the factors Q and S, where

$$Q = H[gh + (m - 1)a_c]$$
$$\text{and } S = H + p$$

a_c being the total area of reinforcement in each strip.

In previous cases the eccentricity of the direct force has been measured about the centroid of the concrete area; this method can also be adopted in the case under review, but the centroid calculation can be avoided without serious sacrifice of accuracy if e is taken to represent the distance from the neutral axis to the line of action of the direct force. With this approximation the calculated maximum stresses are given by

$$c_1 = \frac{Nn(e + p)}{\Sigma QS}$$

$$t_1 = \frac{d - n}{pA_T}\left(\frac{c_1}{n}\Sigma Q - N\right).$$

The value of n corresponding to these stresses should be compared with the assumed value, and adjusted if necessary (see p. 188).

The design of sections falling within the limits of this case is most readily determined by assuming (or otherwise determining) the dimensions of the section and a disposition for the reinforcement. The position of the neutral axis would generally be such as would correspond to the permissible maximum stresses, and c_1 and t_1 would be found by the method and formulæ already given. If c_1 is not appreciably less than the permissible concrete stress c, the maximum area of tension reinforcement, disposed as assumed in calculating p, would be given by

$$A_T = \frac{c(d - n)}{c_1 tp}\left(\frac{c_1}{n}\Sigma Q - N\right)$$

where t = permissible tensile stress.

The following example will indicate the application of the method to determine the stresses on the annular section illustrated in *Fig.* 47, which is subjected to a bending moment of 1,000,000 in.-lb. and a direct force of 50,000 lb. Here $e = 20$ in. and $\frac{e}{D} = 0{\cdot}56$. Assume $n = 13$ in. (about $0{\cdot}38d$), then

$$A_T = \text{nine } \tfrac{5}{8}\text{-in. bars below the neutral axis} = 2{\cdot}77 \text{ sq. in.}$$

$$p = \frac{2(5^2 + 11^2 + 16\tfrac{1}{2}^2 + 20^2) + 21^2}{2(5 + 11 + 16\tfrac{1}{2} + 20) + 21} = 16{\cdot}4 \text{ in.}$$

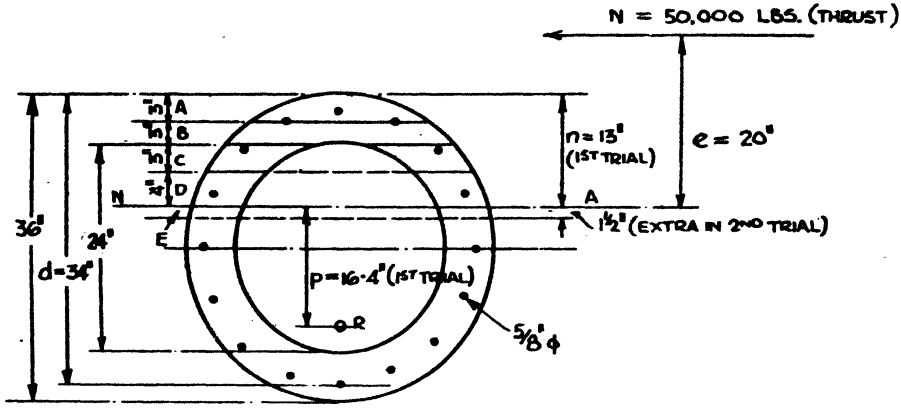


Fig. 47.—Moment and Thrust on an Annular Section.

Divide the area above the neutral axis into the strips A, B, C, and D, and with $m = 15$ determine the values of Q and S thus:

Strip	g	h	gh	$(m-1)a_c$	$\frac{Q}{H}$	H	Q	S	QS
A . .	14 in.	3 in.	42	8·6	51	11½ in.	586	27·9	16,400
B . .	24 in.	3 in.	72	4·3	76	8½ in.	645	24·9	16,080
C . .	16 in.	3 in.	48	8·6	57	5½ in.	314	21·9	6,870
D . .	12½ in.	4 in.	50	8·6	59	2 in.	118	18·4	2,170
							$\Sigma Q = 1{,}663$		$\Sigma QS = 41{,}520$

$$c_1 = \frac{50{,}000 \times 13 \times 36{\cdot}4}{41{,}520} = 569 \text{ lb. per square inch.}$$

$$t_1 = \frac{21}{16{\cdot}4 \times 2{\cdot}77}\left(\frac{569}{13} \times 1{,}663 - 50{,}000\right) = 10{,}600 \text{ lb. per square inch.}$$

The value of n corresponding to these stresses is

$$\frac{34}{1 + \dfrac{10{,}600}{15 \times 569}} = 15{\cdot}2 \text{ in.,}$$

compared with the trial value of 13 in., and considering the magnitude of the stresses a second trial should not be necessary since any reasonable increase in n will only slightly modify the concrete stress and will still less affect the steel stress. However, the problem will be re-worked with $n = 14{\cdot}5$ (that is, 1½ in. more than previously) to indicate a ready method of adjustment. As before

$$A_T = 2{\cdot}77 \text{ sq. in.}$$

$$\text{and } p = \frac{2(3\tfrac{1}{2}^2 + 9\tfrac{1}{2}^2 + 15^2 + 18{\cdot}5^2) + 19{\cdot}5^2}{2(3\tfrac{1}{2} + 9\tfrac{1}{2} + 15 + 18{\cdot}5) + 19{\cdot}5} = 15{\cdot}2 \text{ in.}$$

Add the additional strip E; the values of $\frac{Q}{H} = gh + (m-1)a_c$ for the

original strips A, B, C, and D will remain, and therefore the revised tabulation becomes

Strip	g	h	gh	$(m-1)a_s$	$\frac{Q}{H}$	H	Q	S	QS
A . .	As before				51	13	663	28·2	18,700
B . .					76	10	760	25·2	19,150
C . .					57	8	456	23·2	10,580
D . .					59	3½	207	18·7	2,870
E . .	12½ in.	1½ in.	13	nil	18	0·75	14	15·95	223
						$\Sigma Q = 2{,}100$		$\Sigma QS = 51{,}523$	

$$c_1 = \frac{50{,}000 \times 14{\cdot}5 \times 35{\cdot}2}{51{,}523} = 495 \text{ lb. per square inch.}$$

$$t_1 = \frac{19{\cdot}5}{15{\cdot}2 \times 2{\cdot}77}\left(\frac{495}{14{\cdot}5} \times 2{,}100 - 50{,}000\right) = 10{,}000 \text{ lb. per square inch.}$$

The value of n corresponding to these stresses is 14·5 in., the value assumed. Therefore these second values of c_1 and t_1 will be the stresses produced by the given moment and thrust.

In Chapter VIII some factors are given for the direct design of annular sections such as that just investigated.

3.—Bending and Direct Pull.

Any Section subject to Bending and Tension where e is less than 0·5f.—If f is the distance between the centroids of the reinforcement on opposite faces of a given section, and if e is measured about an axis midway between the two centroids, then if e is less than 0·5f the stresses are wholly tensile. The average stress in the reinforcement on the face nearer the line of action of N is given by

$$t_1 = \frac{N(e + 0{\cdot}5f)}{fA_T}$$

where A_T = total area of reinforcement on this face.

The average stress in the reinforcement on the face remote from the line of action of N is given by

$$t_2 = \frac{N(0{\cdot}5f - e)}{fA_C}$$

where A_C = total area of reinforcement on this face.

The maximum stress depends upon the distance of the farthest bars in any group from the centroid of that group.

These two expressions can be readily rearranged to give the area of reinforcement required in design problems.

Rectangular Section subject to Bending and Tension where e is greater than 0·5f and less than 1·5d.—This is the general case, and the method of treatment is similar to that given previously for combined bending and direct thrust, modifications being introduced to allow for the difference between a direct thrust and a direct pull.

In determining the stresses produced by given values of M and N on a

certain section, first assume a neutral axis factor (see p. 188) and calculate the maximum stresses from

$$c = \frac{NL}{bdG + Kf_1} \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

$$t = \frac{c(J + K) + N}{A_T} \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

where the factors L, G, K, and J have the following signification:

$$L = \frac{e + 0{\cdot}5D}{d} - 1$$

$$G = 0{\cdot}5n_1\left(1 - \frac{n_1}{3}\right)$$

$$K = HA_C \text{ where } H = \frac{(m - 1)(n_1 + f_1 - 1)}{n_1}$$

$$J = 0{\cdot}5bdn_1.$$

The ratio of $\frac{t}{c}$ as derived from (1*a*) and (2*a*) should be reasonably equal to that corresponding to the trial value of n_1, and in the event of appreciable discrepancy a second trial should be made and the factors G, H, and J recalculated. On *Table* No. 36 values of the various factors involved have been tabulated.

When the section is reinforced in tension only ($A_C = 0$), the formulæ (1*a*) and (2*a*) reduce to

$$c = \frac{NL}{bdG}$$

$$t = \frac{cJ + N}{A_T}.$$

When designing sections to take a moment and pull the procedure would be as follows: If the provision of compression steel is not anticipated, assume d (and D) and determine the minimum breadth from

$$b = \frac{NL}{cdG}$$

where G is calculated (or read off *Table* No. 36) from the permissible stresses. If the value of b is unsatisfactory, d should be adjusted or compression steel added. The area of the tension steel required is given by

$$A_T = \frac{cJ + N}{t}.$$

In the case of singly-reinforced slabs subject to a moment and direct pull (walls and bottoms of tanks and bunkers, etc.) a simple procedure is the following.

Determine the eccentricity $e = \frac{M}{N}$ and evaluate $e_s = e + \frac{D}{2} - d$.

Find $A_{T_1} = \frac{Ne_s}{at}$ and $A_{T_2} = \frac{N}{t}$. The total tensile reinforcement required

$= A_{T_1} + A_{T_2}$. The value of d (and D) is that required to resist M acting alone, and the value of the lever arm, a, is approximately that corresponding to the maximum permissible stresses.

In designing sections in which compressive reinforcement is required (or is usual) first assume or otherwise determine suitable values of b and d, and with these values and the allowable maximum stresses determine

$$A_C = \frac{Q}{Hf_1}$$

$$\text{where } Q = \frac{NL}{c} - bdG.$$

The tensile reinforcement is found from

$$A_T = \frac{c(J + HA_C) + N}{t}.$$

If it is necessary to reduce the amount of compression steel, this can usually be effected by reducing t and thus increasing n. Usually in problems of bending and direct pull the tension is the deciding factor, and a more economical section can be arrived at by decreasing the working stress on the concrete.

Rectangular Section subject to Bending and Tension where e is greater than 1·5d.—Stress determination for a given section falling within the limits of this case is similar to that given in the corresponding paragraph for combined bending and direct thrust, and the stresses are computed as if the section were subjected to the moment acting alone. Evaluate

$$c_1 = \frac{N}{A_T m + bn + A_C(m - 1)}$$

and deduct c_1 from the maximum concrete stress, and add mc_1 to the steel tension stress to find the actual maximum stresses in the concrete and steel.

Design problems are best treated by an adaption of the approximate method for singly reinforced slabs outlined in the preceding paragraph, in which the depth, breadth, and amount of compression steel (if any) are determined as if the moment were acting alone, and

$$e_s = e + 0{\cdot}5D - d$$

is evaluated and the area of tension steel required found from

$$A_T = \frac{N}{t}\left(\frac{e_s}{a} + 1\right)$$

where a = lever arm of section designed for pure bending.

Any Section subject to Bending and Tension where Compressive and Tensile stresses are simultaneously produced.—With modifications to allow for N being a pull instead of a thrust, the method given in the corresponding case for combined bending and direct thrust can be applied to problems of stress determination on any given section that cannot be treated as rectangular. A trial position for the neutral axis is taken and the part of the section above the neutral axis is divided into a number of narrow horizontal strips. The value

of A_T and p and the summations ΣQ and ΣQS are determined and substituted in the modified formulæ

$$c_1 = \frac{Nn(e - p)}{\Sigma QS}$$

$$t_1 = \frac{d - n}{pA_T}\left(\frac{c_1}{n}\Sigma Q + N\right).$$

If the value of n corresponding to the stresses c_1 and t_1 is approximately equal to that assumed, these stresses are approximately the maximum stresses produced by the applied moment and pull. If the discrepancy between the calculated and trial values of n is serious, a second trial value of n must be selected and the summations revised by taking in a greater or smaller number of strips to accord with the revised n.

4.—Position of Neutral Axis.

The accuracy of the results of certain of the methods advocated in the foregoing paragraphs and the labour entailed in arriving at these results depend upon the accuracy with which the position of the neutral axis is selected. From a preliminary consideration of the section and forces acting it is possible to assume a value for n very close to that corresponding to the calculated stresses. The maximum stresses for which the section has been ostensibly designed may indicate a reasonable value of n for the first trial, or consideration can be given to the stress ratio for pure bending as determined by the percentage of tension steel. The value of n selected should differ from the pure bending value thus:

Bending and compression: the selected value of n should be greater than the pure bending value, and will increase with decrease in the value of $\frac{e}{D}$.

Bending and tension: the selected value of n should be less than the pure bending value, and will decrease with increase in the value of $\frac{e}{D}$.

If there is such a discrepancy between the assumed value of n $(= n_t)$ and the value corresponding to the calculated stresses $(= n_c)$ that it is necessary to select another value $(= n_s)$ intermediate between n_t and n_c, the following considerations should be borne in mind:

Bending and compression: the value of n_s should be nearer n_c than it is to n_t.

Bending and tension: the value of n_s should be nearer n_t than it is to n_c.

If the values of n_t and n_c do not differ by more than 10 per cent. it is unnecessary to recalculate the stresses with a second trial value of n, since more direct adjustments can be made. If n_c is within 5 per cent. of n_t the calculated stresses are well within 5 per cent. of the actual stresses, and if n_c is between 5 per cent.

and 20 per cent. of n the actual concrete stress c can be safely approximated thus:

$$c = 0{\cdot}5c_1\left(1 + \frac{n_t}{n_c}\right)$$

where c_1 is the calculated concrete stress.

The stress in the steel is affected to a less degree by small variations in the value of n.

By way of illustration, applying this adjustment to the example worked on p. 183, we get

$n_t = 13$ in.; $n_c = 15{\cdot}2$ in. (= 117 per cent. of n_t);

$c_1 = 569$ lb. per square inch.

$$\therefore\ c = 0{\cdot}5 \times 569\left(1 + \frac{13}{15{\cdot}2}\right) = 528 \text{ lb. per square inch compared with}$$

495 lb. per square inch by the more involved calculation.

Applying this method of adjustment to example (*c*) given on the page facing *Table* No. 36, we get

$n_t = 0{\cdot}55 \times 16{\cdot}5 = 9{\cdot}1$ in.; $n_c = 0{\cdot}49 \times 16{\cdot}5 = 8{\cdot}1$ in. (= 89 per cent. of n_t);

$c_1 = 412$ lb. per square inch.

$$\therefore\ c = 0{\cdot}5 \times 412\left(1 + \frac{9{\cdot}1}{8{\cdot}1}\right) = 436 \text{ lb. per square inch compared with}$$

426 lb. per square inch by the second trial position calculation.

5.—Plastic Theory.

The plastic theories (see pages 162 and 175) can be applied to eccentrically-loaded columns and the formulæ given below have been developed to apply to symmetrically reinforced rectangular sections.

t_y = yield stress of reinforcement (lb. per square inch)

c_u = ultimate 28-day strength of 6-in. concrete cubes (lb. per square inch)

A_s = total area of reinforcement (square inch).

The ultimate load P_U for various degrees of eccentricity is:

$e > 0{\cdot}25D$—

Based on compression: $$P_U = \frac{c_u}{2e + f}\left[0{\cdot}8bd^2 + A_s f\left(\frac{t_y}{c_u} - 1\right)\right] \quad . \quad . \quad (f)$$

Based on tension: $$P_U = \frac{c_u}{2}\left[b(D - 2e) + \sqrt{b^2(D - 2e)^2 + 4A_s f b\frac{t_y}{c_u}}\right] \quad (g)$$

The smaller of (f) and (g) controls.

$e = 0{\cdot}25D$— $$P_U = P_{U1} = \frac{c_u}{\frac{D}{2} + f}\left[0{\cdot}8bd^2 + A_s f\left(\frac{t_y}{c_u} - 1\right)\right] \quad . \quad . \quad . \quad (h)$$

$e < 0{\cdot}25D, >$ zero—

$$P_U = P_o\left(1 - \frac{4e}{D}\right) + \frac{4e}{D} \cdot P_{U1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (j)$$

where P_{U1} is obtained from formula (h) and P_o from formula (e), p. 175.

$e = o$—

$P_U = P_o$ per formula (e), p. 175.

CHAPTER XV

SPECIFICATIONS AND QUANTITIES

1.—Specifications.

Conditions of Contract.—It is usual for the specification of materials and workmanship to be preceded by a series of clauses termed the Conditions of Contract. Since many of these clauses are of a legal nature, it is preferable to adopt one of the standard forms issued by such authorities as the Royal Institute of British Architects or the Association of Consulting Engineers.

The Conditions of Contract describe the extent of the contract, the requirements concerning drawings and bills of quantities, the method of measuring up, assessment of the cost of extra work, and the provision of facilities for inspection and co-ordination with other contractors. A clause should provide that the contractor at his own cost shall maintain in good order and repair the whole of the works executed under the contract up to the date of granting the completion certificate and for a further stated period, during which the contractor shall at his own cost restore and make good such portions of the work as may be considered defective.

The Conditions of Contract should also include clauses dealing with the provision of an office for the clerk of works, and details of the method of payment, the retention of money, and the contract time should also be included in terms suitable for the particular contract. Other additional matter in this document should include the payment of fees to local authorities, setting-out the works, provision of a competent foreman, conditions relating to sub-contractors, liability for damage to persons or property, fire insurance, penalties and bonuses, prime-cost items, arbitration in the case of disputes, and the supply of water and power.

Materials.—A general clause should be included in the specification to the effect that all materials should be new and of the best quality and description and in every respect subject to the engineer's approval. If necessary, or convenient, the materials should be of British production. They should be subjected to such tests as are specified and to such other tests as the engineer may deem advisable. Samples of all materials should be submitted to the engineer for his approval. Any materials delivered to the site and rejected by the engineer should be immediately removed from the site and replaced by the contractor free of charge.

Cement.—The clauses covering the quality of the cement should state that the material is to be normal setting Portland cement complying with the appropriate British Standard Specification and a normal or rapid-hardening brand as deemed suitable for the particular structure. Clauses should be inserted

covering the method of packing, the delivery of test results from the manufacturers, storage on the site, and prohibiting the use of damaged cement.

Aggregates.—The requirements with regard to the cleanness, hardness, and grading of the fine and coarse aggregates should be stated, together with the prohibition of certain materials and the absence of laminated particles and dust, loam, organic matter, etc. Particulars of these requirements for different types of structures are given in Chapter IX.

Water.—The source from which the water should be obtained is usually specified in the Conditions of Contract, and in the specification a clause should be included to the effect that the water must be clean and fresh.

Reinforcement.—The reinforcing materials should be one or more of the qualities given in the British Standard Specification, although in special cases other materials may be permissible. Clauses should be inserted requiring the material to be clean and free from oil. Other clauses should clearly specify in what manner the reinforcement shall be paid for, that is whether it shall be paid on net calculated weights in accordance with the bar-bending lists, or on the gross weight of material delivered to and used in the works. The gauge of tying wire and the requirement that this shall be supplied by the contractor and included in his general price for reinforcement should be specified.

Timber.—Timber is mainly required for joinery, piles and fenders, and in temporary work such as shuttering, timbering excavations, and shoring. The requirements regarding the quality and type of timber vary considerably with its different uses, and specifications for general building works should be consulted. Having regard to the permanence of concrete structures, the use of good quality timber for joinery and other work incorporated in the structure is self-evident. All timber should be sound and straight grown, free from sap, shakes, loose knots, worm-holes, waney edges, and other defects. Only well seasoned wood should be used and clauses should be inserted to cover the treatment of joints, nail-holes, and painting. Generally, Baltic red-wood is a satisfactory medium priced material for doors and partitions of non-residential buildings and for structures where decorative woodwork is not required. The most suitable timbers for fenders and rubbing-pieces on jetties and wharf-walls are greenheart, Oregon pine, and elm. Such members are attached to the concrete framework by wrought-iron straps and mild-steel bolts, with a piece of hardwood packing between the concrete and the softer timber. Longitudinal rubbing pieces are usually attached to the vertical fender piles by oak treenails, and all exposed corners should be rounded off.

Timber piles for wharf and jetty construction and for foundation work have certain advantages over reinforced concrete piles, but when they are in such positions that they are alternately wet and dry, as in structures in tidal water, these advantages are often offset by the rapid and serious deterioration that takes place. In the first instance timber piles of moderate lengths are cheaper than reinforced concrete piles, and, even for long piles, short lengths can be readily spliced together with a minimum of delay. The delay that occurs while reinforced concrete piles are maturing is also avoided, although with rapid-hardening cements and with a well-planned construction programme this cause of delay is almost eliminated in modern practice. Although more resilient when subjected to blows from vessels, a timber pile wears away more rapidly due to

abrasion by shingle, and the action of worms and rot render the life of timber short and uncertain. On the other hand a blow from a vessel may cause undetected cracks in a reinforced concrete pile and thus allow sea water to reach the reinforcement and cause corrosion.

Just as creosoting assists in retarding deterioration of timber, so the application of a coating of tar is sometimes added to concrete piles with the object of preservation. Timber piles must be bound at the head with an iron ring and, unless driven through soft mud only, the lower end should be pointed and shod. The timber dollies for fitting into pile helmets are usually of elm and should be so placed that the blows are received along the grain.

Brickwork and Masonry.—Brickwork may be incorporated in a reinforced concrete structure either for load-carrying purposes or to act merely as a facing or filling material. The nature of the facing and the importance of the ultimate appearance of the structure will determine the excellence of the materials and workmanship of brickwork filling for wall panels in a concrete framework; but when the brickwork is carrying loads or providing supports for floor slabs, beams, or stairs, it is essential that certain standards should be maintained. These standards, the purposes of which are fairly obvious, include the stipulation that the bricks should be of the best quality of the type specified, new, of good shape, and uniformly hard. They should be free from stones and flaws and should be well burnt. The texture should be such that the bricks do not absorb more than 10 per cent. of their weight of moisture when saturated, and immediately before laying they should be well soaked in water. It is essential that they should be set in Portland cement mortar; the joints should be ¼ in. thick and be either "struck," or, for external work, "raked and pointed." Old English or Flemish bonds are the most suitable, and the face of walls and piers should be plumb. The type of brick used depends upon its purpose. For sustaining high pressures engineering bricks are essential.

Natural stone used for constructing supports for reinforced concrete structures should be fine grained, hard, sound, free from excessive discoloration marks, rust veins, and from heads and shakes and other imperfections. Stones cut from the lower beds of quarries are superior. Successive courses of stones in the work should be well bedded and laid on their natural bed. Exterior work should be neatly pointed, and one-sixth of the area should be composed of "headers" that penetrate 3 ft. or right through the wall or pier being constructed. For purely facing work it is sufficient if the minimum thickness is 5 in. with one-third of the area 9 in. thick with sufficient ties into the body of the concrete backing.

Iron, Steel, and other Metals.—Apart from its use as reinforcement mild steel may be used in conjunction with concrete in composite structures. If in the form of joists, stanchions, plate or lattice girders, the quality of the material should be in accordance with the British Standard Specification for mild steel for girder bridges and for building works. The sections should be well and cleanly rolled and straight, and be free from scales, blisters, laminations, cracked edges, and defects of any sort. The edges of plates of girder flanges should be planed, and the ends of sections should be cut square and neatly dressed. All rivet holes should be drilled, although if the piece is less than ¾ in. thick punching and reamering is sometimes permitted. The B.S.S. provides for rivets to be made from "A" steel, and tension tests should show an ultimate strength of

25 to 30 tons per square inch and a tough and silky fracture. The rivets should be capable of having the head flattened out to 2½ diameters when hot, and should be able to be bent double when cold without fracture. It is usual to specify that structural steelwork is to be painted one coat before erection, one coat before leaving the works, and two or three coats after erection.

A use for cast steel in reinforced concrete work is found in hinges in arch bridges. The castings should be thoroughly annealed, and in order to prove freedom from cracks they should be hung in chains and hammered. Cast steel for any permanent work should generally be in accordance with the B.S.S. for this material, the tensile strength of which should be 26 tons per square inch and the elongation not less than 20 per cent. A cold bend test should be made on a 1-in. diameter piece 9 in. long which should be bent through 90 deg. to a radius of one in. without signs of fracture.

Wrought iron should be well rolled, tough, fibrous, and uniform in character. The use of this material in concrete work is limited to its employment for the straps of pile shoes, for the straps connecting timber fenders, etc., to concrete wharves and similar purposes. The B.S.S. stipulates a minimum tensile strength of 20 tons per square inch for Grade C material with an elongation of 13 per cent. The fracture should be fibrous, and the cold bend test provides for the bar to be bent through 90 deg. to a radius equal to one to one and a half diameters.

The quality of cast iron for pipes, pile shoes, pile forks, etc., should be in accordance with the appropriate B.S.S. Failure to comply with the specified tests condemns all castings made from the pouring represented by the test piece. The material should be tough grey metal cast from approved pig iron or from a mixture of pig and scrap. Castings should be clean and sound, free from air and sand holes and from lumps and flaws. Cast-iron pipes should be specified to be cast vertically with the socket downwards, and the allowable variation of thickness should be stated. Pipes should be coated with Dr. Angus Smith's solution, and if required for pressure work should be subjected to an hydraulic test of 200-ft. head of water for gas pipes and 400-ft. to 800-ft. head for water and sewage pipes.

Usually pipes that have to be built into reinforced concrete tanks are made of brass, phosphor bronze, or other copper alloy, since deterioration due to corrosion is much less than for ferrous metals and replacements that may affect the water-tightness of the structure are thus obviated. Pieces that have to be built into tank walls should have an additional intermediate flange cast in such a position that it will be buried in the thickness of the wall and thus form a type of waterstop.

Workmanship.—A general clause should be inserted in the specification to cover workmanship, and should state that the contractor's employees should be skilled men and that the engineer reserves the right to demand the substitution of any employee who appears to be incompetent or negligent. It is usual to specify that the contractor should be called upon to pay the standard rate of wages in force in the district in which the works or manufacture of parts is carried out.

Excavation.—The clauses covering the excavation should state whether the excavation is measured net in the solid or whether an allowance is made over

net sizes for timbering. They should also state that timbering and pumping should be included in the contractors' price, unless these items are separated in the bill of quantities. In addition, the rates for excavation should allow for digging, throwing, loading, wheeling, part return fill and ram, and the disposal of surplus spoil. It is usual to indicate the percentage of excavated material which will have to be removed from the site and to what distance it will have to be carried before it is dumped. Clauses should also be inserted to cover any particular requirements concerning the structure in hand.

Shuttering.—A general clause concerning shuttering should require the temporary falsework to be substantially and rigidly constructed, and efficiently propped and braced to prevent deformation due to depositing concrete or caused by other loads to which it may be reasonably subjected, and to prevent distortion due to the effect of weather, etc. The inner face of shuttering should be treated to obtain a fair surface and the joints should be tight enough to prevent leakage of cement grout. The shuttering should be constructed to facilitate easing and removal of the various parts in the recognised sequence without jarring the concrete, and no shuttering or strutting should be struck until the concrete has attained a satisfactory strength. Clauses should be inserted regarding the provision of bolts, nails, screws, wire-ties, and other connections, the formation of cambers on beam soffits, fillets, chamfers, arrises, etc., and cleaning out the forms. The responsibility of assessing the correct time for striking the shuttering is usually put on the contractor.

Bending and Fixing Reinforcement.—The clauses covering these operations should specify that the bending is to be carried out cold and with a gradual motion. Where necessary, conditions under which hot bending and welding are allowed should be given. The bars when bent should be fixed in the work in accordance with the detailed drawing, and intersections should be wired together with soft-iron binding wire or approved patent ties. Distance pieces made of metal or cement mortar should be used to maintain the specified cover between the bar and the outside face of the concrete, and spacer bars should be provided between layers of reinforcement where shown on the drawing. The specification should state whether the spacer bars are provided by the contractor or are included in the measured weight of reinforcement.

Concrete.—The clauses concerning concrete should cover the proportions in which the materials are to be mixed for various parts of the structure, the minimum strength at 7 and 28 days to be attained by concretes of various mixes, the method of measuring the materials by weight or volume, the amount of water to be used, and the slump or other consistency test to be complied with. The minimum time of mixing, the procedure to be adopted in hand mixing, the conditions under which the concrete shall be distributed to the site and deposited in the shuttering should also be covered, together with the method of forming and positions of construction joints and other stopping places, the conditions of curing, the protection of concrete in cold or extremely hot weather, and other factors concerning the particular structure. Full particulars of the manufacture and testing of concrete cubes should also be incorporated.

Miscellaneous Items.—The specification should also include clauses relating to the normal and special finishes required, particulars of piles, preparation of ground under foundations, the provision and erection of refractory linings

and other works peculiar to any particular type of structure, and the conditions under which tests on the completed structure should be performed.

2.—Quantities.

Methods of Measurement.

The bill of quantities for a reinforced concrete structure includes the following principal divisions, each of which is subdivided into items as explained in subsequent paragraphs.

(1) Excavation and filling.
(2) Plain concrete in foundations and elsewhere.
(3) Reinforced concrete.
(4) Shuttering.
(5) Reinforcement.
(6) Finishing processes, asphalting, fixing machinery, pipes and the like, glazing, plumbing, waterproofing, refractory linings, joinery, etc.

Bills of quantities should be prepared and expressed in accordance with one of the generally accepted methods in order to avoid ambiguities that may lead to disputes over measurements and payments for work done. In Great Britain there are three "standard" methods in vogue. If the contract is for a structure of the building class under the direction of an architect, the "Building Works" method of measurement (sometimes called the "London" method) should be adopted. This is the "Standard Method of Measurement of Building Works," revised in 1935 and authorised by the Surveyors' Institution, the Quantity Surveyors' Association, the National Federation of Building Trades Employers, and the Institute of Builders. For a civil engineering contract the method given in the "Report of the Committee on Engineering Quantities" issued by the Institution of Civil Engineers should be used. In Scotland the use of the "Scottish Mode for the Measurement of Reinforced Concrete Work" (printed by H.M. Stationery Office) is general. The principal requirements for each of these three methods are considered here, but reference should be made to the complete codes for further subdivisions dealing with details of special work.

For small contracts and in other special circumstances, a summarised bill of quantities, similar to that in the example on page 314, may be sufficient. Reinforced concrete of any mix is then given in one item in cubic yards; likewise, shuttering is in square yards in one item, and reinforcement is in hundredweights or tons and separated into bars of $\frac{5}{8}$ in. diameter and over and bars smaller than $\frac{5}{8}$ in.

Reinforced Concrete.

For civil engineering works it is sufficient if in-situ reinforced concrete is measured in cubic feet throughout with separate items for columns, beams, slabs, walls, or other distinctive parts of the structure. For building works the bill is usually made up in greater detail, providing items for vertical, horizontal, circular and raking work. Slabs should be measured in square yards and the thickness stated, the measurement extending over all beams and columns. Falls should be described, and the angle of pitch specified when this exceeds 15 deg. Holes should be enumerated and described, but no deduction should be made for holes of two square feet or less in area. For stairs, the steps should be expressed in

cubic feet or in lineal feet and the shape and size described, while landings should be given in square feet with the thickness specified. Finishes to stairs should be included in the concrete item. The foregoing are in accordance with both the Scottish and Building Works methods, which differ in respect to other parts of the structure.

The Building Works method requires columns and beams to be measured in cubic feet with sections within the following limits kept separate: sections ¼ sq. ft. or less, from ¼ to ½ sq. ft., from ½ to 1 sq. ft., and over 1 sq. ft. Projections on columns should be measured in cubic feet, numbered and described. The Scottish Mode requires columns and beams to be measured in lineal feet, and the shapes and net sizes specified. Projections on columns should be numbered and described.

Both codes require walls to be measured in square yards and the thickness stated, but walls thicker than 12 in. should be measured in cubic yards under the Building Works method. Horizontal projections are measured in lineal feet; vertical projections not greater than 18 in. are measured as columns, but if over 18 in. they are measured as walling of the appropriate thickness.

Foundations are measured in cubic feet under the Scottish Mode but in cubic yards under the Building Works method, footings being measured separately and described; when less than 12 inch thick measure in square yards of specified thickness.

Precast concrete units are measured by number and description in both the Building Works and Scottish methods, but in the Civil Engineering code precast work is measured in cubic feet, except slabs which are taken in square yards. Separate items are provided for all work of different descriptions and if 2 sq. ft. or less in cross-sectional area; separate items are required for different sizes. If hoisting, fixing, or other labour is involved, this should be specified. It is necessary for the moulds and finishes to be included in the concrete price for precast work.

Plain Concrete.

Generally plain concrete should be measured in cubic yards, or in square yards when in work 12 in. or less in thickness. No deduction should be made for small cavities or chamfers, or for any rails, joists, or other materials built into the concrete. Where holes are to be left for bolts, these should be enumerated and described if 12 in. deep or less, and measured in lineal feet if over 12 in. deep.

Concrete block work should be measured in cubic yards, the volume of the joints being included in the measurement and the provision of moulds included in the price. The size and approximate weight of the blocks should be given, and joggles and dowels should be enumerated and allowed extra over the block work.

Shuttering.

Shuttering is usually measured as the area of the finished concrete surface needing support, the price including for all props, boards, struts, cleats, battens, wedges and other items required to give adequate support and stiffness. Some-

times the measurement of the shuttering is omitted and the price for the concrete includes such temporary work as is necessary. On civil engineering work it is usual to subdivide the shuttering into items for vertical, horizontal, sloping, curved, and other special work, but on building work a larger number of detail items covering all contingent labours is given. The published codes should be referred to for full particulars of these sub-divisions and extras; the following are the principal requirements only.

The Building Works code requires the shuttering to be measured in square feet for small areas, or square yards for large, all raking and circular cutting, splays, rounded edges, and the like being given in lineal feet. Chamfers and fillets not exceeding 2 in. wide should be included in the description for beam shuttering and not measured separately. Slabs sloping at more than 15 deg. to the horizontal are measured on both faces.

The Scottish Mode requires shuttering for walls and slabs to be measured in square yards of surface to be supported, with no deductions for openings 12 sq. ft. or less in area. Shuttering for beams and columns under this code should be measured in lineal feet and the size and shape specified.

Reinforcement.

The quantity of reinforcement can be determined by a number of methods. At one extreme the weight can be roughly assessed from the type of structure and volume of concrete, or the precise weight can be calculated from the detailed reinforcement bending schedules. The amount of steel in a normal reinforced concrete structure can vary from 1 ton to 5 cub. yd. of concrete to 1 ton to 30 cub. yd., so that any estimate based solely on the quantities of concrete can be taken as a rough guide only. Application of the ratios taken from actual jobs, and set out on p. 198, gives closer approximations for different classes of structure.

If detail drawings or even typical details are available without bar lists or bar-bending schedules, a more accurate estimate can be made by investigation of the principal reinforcement to be provided. For this purpose the principal reinforcement in a beam or slab can be considered as the horizontal bars near the lower edge at mid-span. The whole of the longitudinal bars in a column, for the present purpose, comprise the principal reinforcement. The area of this steel (A square inches) can be computed from the data given in *Table* No. 25 (page 267) (areas of round bars of various numbers and spacings), and the approximate average weight of reinforcement (W lb.) per foot run of beam or column, or square foot of slab, can be calculated from

$$W = KA$$

where K has the following values:

Non-continuous beams	$K = 3$ to 6
Continuous beams	6 to $8\frac{1}{2}$
Cantilevers	4 to 6
Columns	5 to 6
Continuous slabs spanning in one direction	5 to $8\frac{1}{2}$
" " " " two directions	$4\frac{1}{2}$ to $7\frac{1}{2}$

CONCRETE-STEEL RATIOS.

Type of structure	Cubic yards of concrete per ton of reinforcement
Buildings: Warehouses	10 to 15
Factories	14 to 20
Domestic buildings	15 to 20
Retaining walls: Dry firm ground	18 to 23
Wet loose ground	14
do. with piling	12
Bridges: Girder bridges	10 to 15
Arch bridges	8 to 12
Culverts	15 to 18
Water containers: Pump wells, etc.	10
Open rectangular tanks	10 to 18
Covered rectangular tanks	10 to 15
Swimming baths	18 to 24
Open circular tanks	15 to 20
Covered circular tanks	10 to 20
Pyramidical tanks in ground	8
Water towers: 20,000 gall.	10 to 15
80,000 ,,	10
125,000 ,,	16
Industrial structures:	
Rectangular bunkers	8 to 10
do. landsale type	12 to 15
Circular bunkers	15 to 20
Elevator pits	10 to 14
Slurry towers	8
Pit headgears	8½ to 15
Silos	9 to 15

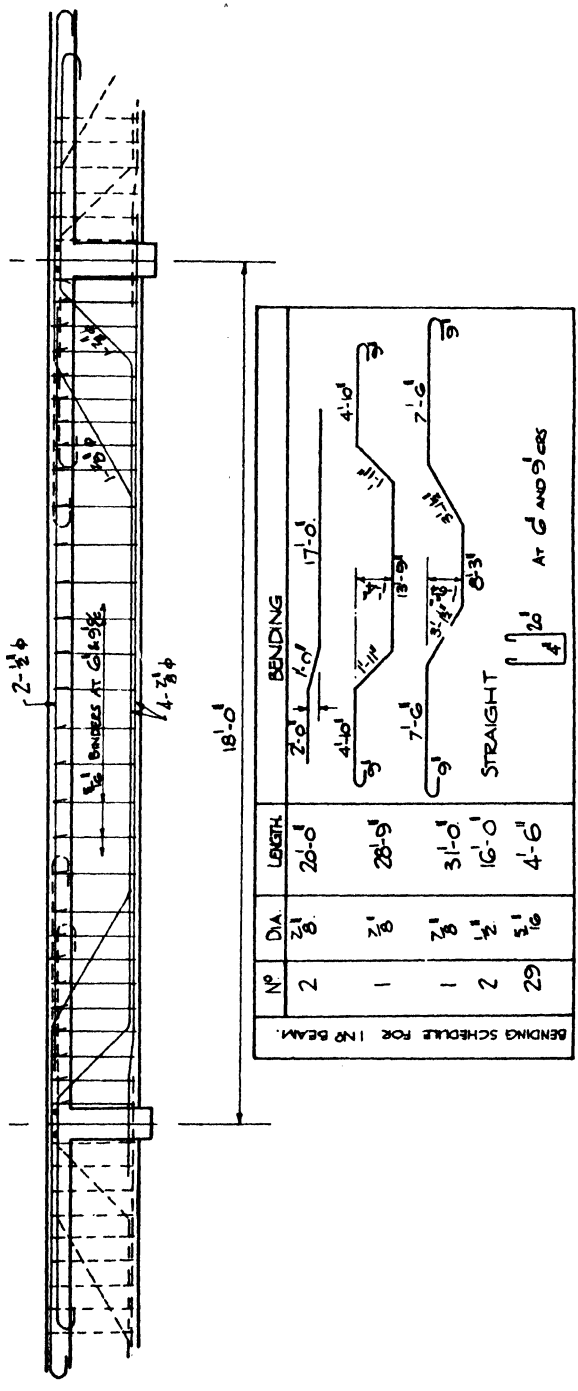
For slabs and beams the smaller figures apply to cases where there is no steel in compression or no special shear reinforcement; the higher figures apply where there is equal steel in compression and tension or where heavy shear steel is provided. The smaller figures for columns apply to minimum binding, and the higher for heavy binding or where loose splice bars are provided. For slabs spanning in two directions, A is the sum of the areas required in each direction.

The unit weight W multiplied by the length (ft.) of the beam or column or by the superficial area (sq. ft.) of slab, gives the approximate total weight (lb.) of reinforcement in the member. For special members such as important beams, arch ribs, etc., an approximate steel list should be made up and the weight computed by multiplying the total length of each diameter by the unit weights (*Table* No. 26, page 269). The weight of reinforcement in panel walls or other slabs with uniform reinforcement throughout can be found by multiplying the area (sq. yd.) of the wall or slab by the appropriate weight per square yard and adding sufficient for laps.

If a set of fully-detailed bar-bending schedules to be read in conjunction with the working drawings is available, the steel weights can be calculated precisely by using *Table* No. 26.

For the purposes of demonstrating the application of the two methods, a reinforcement detail of a beam is reproduced in *Fig.* 48. The weight of steel in this member could be calculated in the following manner.

(*a*) Approximately—Assuming that the bending particulars of each bar



BENDING SCHEDULE FOR 1 NR BEAM.			
Nº	DIA.	LENGTH	
2	⅞"	26'-0"	
1	⅞"	28'-9"	
1	⅞"	31'-0"	
2	½"	16'-0"	
29	5/16"	4'-6"	

Fig. 48.

were not given, an approximate weight could be estimated by considering the principal reinforcement. This is specified as four $\frac{7}{8}$-in. diameter bars. The cross-sectional area of these bars is 2·41 sq. in. In view of the fact that there is no compression steel at mid-span and inclined bars are provided for resistance to shear, a suitable value for K would be 6. The average weight of steel per linear foot of beam would therefore be

$$6 \times 2{\cdot}41 = 14 \text{ lb. per foot.}$$

The length of a single span of the beam between the support centres is 18 ft. Therefore the estimated total weight would be

$$18 \times 14 = 252 \text{ lb.} = 2\tfrac{1}{2} \text{ cwt. per span.}$$

(*b*) Accurately—Detailed particulars are given for every bar in *Fig.* 48. The total weight of reinforcement in one span can be calculated by tabulating the total length in each item and multiplying the summation of these lengths by the proper unit weight. It is necessary to adopt some system in taking out steel weights in order to avoid confusion and facilitate checking. Such a system is as follows:

Diameter in.	No.	Unit length ft. in.	Total length	Weight (lb.)
$\frac{7}{8}$	2	20 0	40	
	1	28 9	29	
	1	31 0	31	
			100 × 2·044	204
$\frac{1}{2}$	2	16 0	32 × 0·667	22
$\frac{5}{16}$	29	4 6	131 × 0·261	34
				260
				say 2½ cwt.

Reinforcement quantities for round bars are generally given by weight, (say) in hundredweights; bars $\frac{5}{8}$ in. and over are given in one item, while all diameters less than $\frac{5}{8}$ in. are given separately. Bent bars are not usually kept separate from straight bars unless there is a predominating weight of the latter, as in piles or panel walls. The unit rates should include for all labour, for tying wire, and for the mill's rolling margin (nominally 2½ per cent.). Some indication of the length of bars and type of work should be given, together with the basis of calculation, and bars over 40 ft. and less than 5 ft. should be separated. The method adopted for civil engineering works is more or less in agreement with the foregoing. A comprehensive clause for describing a reinforcement item to enable the contractor to give a keen quotation would read:

"Item: Supply, handle, cut, bend, crank and fix mild steel bars $\frac{5}{8}$-in. diameter and over, but not exceeding 1½ in. in diameter, in main reinforcement in beams and columns to all floors and roofs and in foundations, supplied in lengths averaging 25 ft. and not exceeding 40 ft. The price to include for tying wire, and no allowance in the weight paid for shall be made for rolling margin. The weight of any bar shall be computed from the length given on the drawings or descriptive lists, the weight in pounds per linear foot of bar being calculated as equal to 2·67 times the square of the specified diameter of the bar 2340 cwt.

"Item: All as in last item, but ⅜-in. diameter bars for column binders and links, beam stirrups, and reinforcement in slabs and walls . . . . . 360 cwt."

The standard methods for buildings conform to the Civil Engineering code in most requirements but differ in the following particulars. In the Building Works method of measurement bars over 30 ft. long should be kept separate in items for each 10 ft. advance. Separate items should be given for bars, whether bent or straight, stirrups, helicals, bars bent to large radius, special deformed bars and high carbon steel. Bends in the latter material should be enumerated and described as forged bends. Reinforcement for floors, roofs, walls, beams, and columns should be kept separate.

The Scottish mode requires each diameter, as well as bars under 6 ft. and over 30 ft. in length, to be kept separate. An allowance of 2½ per cent. is to be added to the weights to allow for the rolling margin. Separate items are required for stirrups, ties, and helicals.

Mesh reinforcement requires special treatment in bills of quantities. It is usually given in square yards, and the Building Works method and the Civil Engineering and Scottish modes require the measurement to represent the net area of work covered, and the amount of overlap stated. Waste and square cutting should be included in the price per square yard, but raking (or angle) and circular cutting and waste should be measured in linear feet and billed separately for extra labour. The make and grade of mesh should be stated, or the gauge of wire, weight per square yard, and other properties described. Labours in fixing, bending, etc., are included in the rates, unless the mesh is supplied by the manufacturers at a lump sum, when handling, cutting, bending (if any), and fixing should appear as a separate item.

When the supply of reinforcement or labour or both is subject to sub-contract, it should be clearly understood who does the cutting and hoisting and who supplies the tying wire.

Reinforced Concrete Piles.

Where piles occur in civil engineering works the concrete should be measured in cubic feet calculated on the cross-sectional area and overall length. Concrete in piles of different cross-sections should be given separate items and the price should include the moulds. The reinforcement should be given in hundredweights and should be sub-divided into bars, links, and helical binders. Forks (in cwt.), shoes (in number), and ferrules (in number) should be given separately. Whether the piles are driven to a given level or to a given set, the measurement can be either in number of piles or in linear feet of penetration, depending on the circumstances of the particular job. Grouped piles, isolated piles, sheet piles, raking, and trial piles should be given separate items.

The piles should be enumerated for pricing, handling, and pitching, separate items being given for pile lengths up to 30 ft., between 30 and 40 ft., and so on in intervals of 10 ft. For cutting off or stripping pile heads, or stripping for junctions, the piles should be enumerated and separate items given. For piles that are to be lengthened in position, items should be given for the concrete and reinforce-

ment and an additional item for the extra cost of lengthening including stripping the exposed end.

For piles driven in connection with building contracts, the methods of measurement stipulated by the Building Works code and the Scottish mode are practically identical and differ in detail from the Civil Engineering method. Such piles should be classified according to length and cross section, stating the number of each, and the measurement should be in lineal feet of length cast, keeping trial piles separate. Moulds should be priced separately, and the number required for each class of pile should be given. An item should be given for handling and driving each class of pile, stating the number and depth and specifying the weight and drop of the hammer. Cutting-off heads of piles should be numbered, and shoes should be numbered and the weight given. Reinforcement should be given separately. For lengthening piles in situ, quantities of concrete, reinforcement, and shuttering should be given.

Earthwork.

The quantities for earthwork, that is operations embracing excavation, dredging, and filling, are usually expressed in cubic yards, except for narrow trench excavation which is given in linear yards (of specified width and average depth), and shallow surface excavation and similar operations that are given in square yards. Generally, separate items are given for various depths of excavation, commencing at not exceeding 5 ft. and advancing in intervals of 5 ft. to the maximum depth required. These items should be further sub-divided to embrace the type of material to be excavated, for example, soft, hard, or rock, giving if possible some indication of the nature of the material, for example, gravel, clay, chalk, etc. Similarly, excavation in compressed air, between tides, and below low water should be kept separate from excavation in free air or above high water.

Unless otherwise stated excavation measurement should be net in the solid, no allowance being made for the space occupied by timbering, drainage trenches and sumps, or extra working space. Where working space is essential a separate item, in square or lineal yards, should be given, the width of such space being left to the contractor's discretion, unless the trade using the space requires a definite minimum width in which case this width should be stated.

Unless special items are inserted in the bill, the price is deemed to include timbering (if necessary), pumping, and maintaining a level and clear bottom. If timbering or other retaining materials are to be left in position, items should be given for these; timber should be expressed in cubic feet and steel sheet piling in tons.

The price would also be assumed to include the removal and disposal of part or the whole of the surplus spoil and for any part return back-filling, including its consolidation. The place of disposal may be specified or left to the contractor to provide.

Back-filling should be given a separate item if the material to be used has to be specially selected or placed in any particular way. Where filling is provided for separately from the excavation the quantity should be based on the net measurement as shown on the drawings, and where soft ground is involved

the price should include for making good any settlement below the specified level over a given period. If additional material in excess of that available from the excavations is required, the quantity involved should be given in a separate item stating the position of the borrow pit or other source.

Any existing brickwork, concrete, masonry, or similar materials demolished and removed in the course of the excavation should be included in a special item allowing for the measurement of such materials in cubic yards and for the price to be extra over the excavation. Boulders, up to a specified size (volume), should be considered as allowed for in the rate quoted for the class of material in which they occur, but boulders in excess of this size should be classed as rock.

The quantities involved in dredging can either be based upon hopper measurement or on soundings (the latter is usually recommended) and allowances made or specified to cover silting, erosion, or other special circumstances.

Other Operations.

Other materials sometimes associated with reinforced concrete are usually measured as next described on engineering works, but as considerable differences in practice occur on building works reference should also be made to the published codes of the various trades.

Finishes to floors, walls, and roofs and similar work should be fully described, the number of coats specified, and measurement made in square yards of finished surface. This would apply to mortar renderings, asphalt, waterproof sheeting, special roof coverings, granolithic, grout washing, and other surface treatments. Separate items should be given for horizontal and vertical work and for isolated strips 18 in. or less in width, the width being specified. Joints, laps, and laying of falls should be deemed to be included in the price, and no deductions in area should be made for small openings, such as for gratings and pipes. Angle plates should be given separately and measured in linear feet.

Finishes to stairs and pre-cast members are usually given in the concrete rates for these items.

Asphalt and other joint filling materials should be fully described and measured in linear feet. Joints in concrete work should be described and measured in linear feet.

Brickwork and masonry for engineering works are usually measured in cubic yards and the composition of the mortar specified. Special facing would be measured extra over the general masonry or brickwork and expressed in square yards, while copings, plinths, and similar special work would also be extra to the general work and expressed in linear yards. Different classes of work should be kept separate. Pitching and masonry facing to concrete should be measured in square yards, the average thickness being specified. All prices shall be deemed to include for pointing.

Permanent construction timberwork, including timber piles, should be measured in cubic feet; bolts and other connections should be expressed in hundredweights. Timber in joiner's work, for example, doors, floorings, etc., should be measured in accordance with the trade practice; this also applies to glazing, plumbing, ironmongery, and other similar items. Structural steelwork is expressed in tons and the rates usually allow for painting. Corrugated sheeting

is measured in square yards covered, the price allowing for laps and fitments. Steel sheet piling can be expressed in tons or fully described and expressed in square feet, extra linear measurement being provided for corner and junction piles.

3.—Taking-off Quantities.

In order to render the process of taking-off the quantities from the drawings as accurate as possible a definite system should be followed. The adoption of a system also facilitates checking. Many firms have their own systems, but in

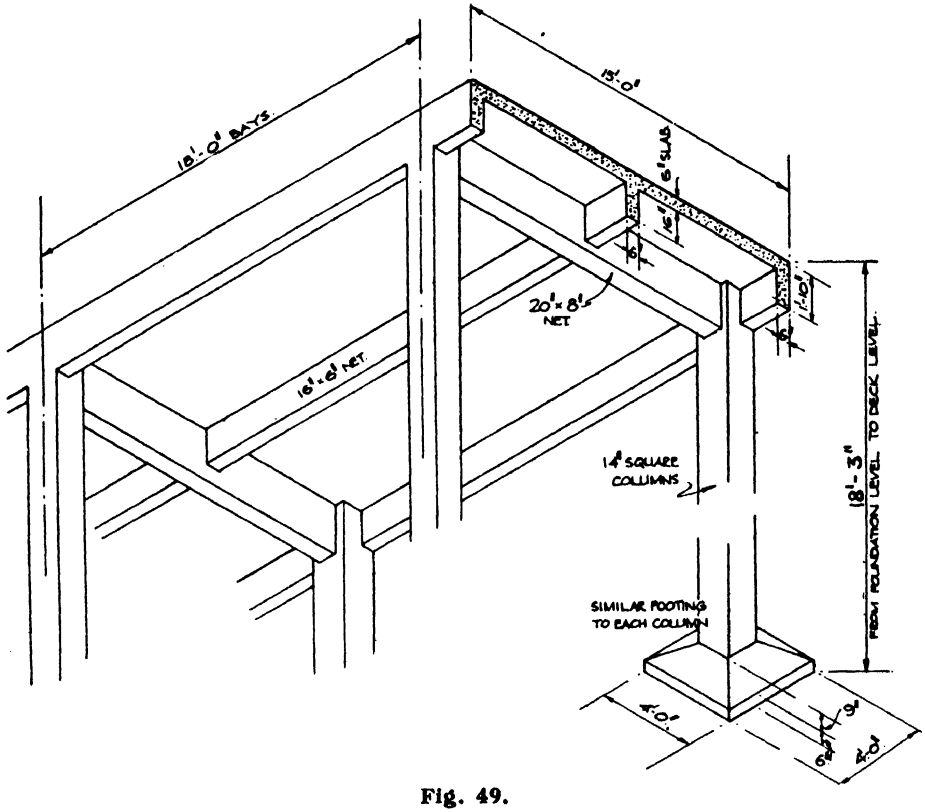


Fig. 49.

the absence of such the following method can be used for taking-off concrete and shuttering quantities. In this method the first column enumerates the number of identical items, the second column gives the three concrete dimensions in the order of length, breadth and depth; the third column gives the total volume represented by the product of the three dimensions. The shuttering dimension is given opposite the appropriate concrete item. The following example, illustrating the method of measuring a single 18-ft. length of the reinforced concrete gantry shown in *Fig.* 49, demonstrates the principal points in this method of taking off quantities of concrete shuttering. The items should be taken in order, either downwards from the top or upwards from the foundations.

In the following tabulation measurement is made from the top, commencing with the 6-in. slab. The longitudinal beams, the transverse beams, the columns and footings are then measured in this order.

	Reinforced Concrete		Shuttering
Slab	18′ 0″		18′ 0″
	15′ 0″		15′ 0″
	6″		270
	135		
		16″ × 6″	
Longitudinal beams	51′ 0″	2/16′ 10″ = 33′ 8″	51′ 0″
	6″	17′ 4″	2′ 8″
	1′ 4″		136
	34	51′ 0″	
Transverse beam	12′ 8″		12′ 8″
	8″		3′ 4″
	1′ 8″		42
	14.		
Columns	2/1′ 2″		2/4′ 8″
	1′ 2″		16′ 6″
	16′ 6″		154
	45		
Footings	2/4′ 0″		2/4′ 0″
	4′ 0″		4′ 0″
	11″ av.		32
	30		2/16′ 0″
			6″
			16
	258 cu. ft.		650 sq. ft.

These quantities would be converted to cubic yards and square yards respectively, if necessary, and added to similar items before transferring them to the bill.

4.—Cost Estimating.

A reinforced concrete engineer has often to provide an estimate of the cost of structure he has designed. If he has connections with a contracting organisation the necessary data and methods for making up the estimate would be available, but if he is not in the position to be able to call upon such expert estimating services he can adopt one of two courses. The first is to send his bill of quantities and drawings to one or more contractors asking for a competitive tender. Unless there is a reasonable chance of one of these tenders being accepted this is not a commendable procedure, and, further, such a procedure usually takes up more time than can be spared in the preliminary stages.

His second course is to make up an estimate himself, and it is generally sufficient for him to price out each item in his bill of quantities at the average prevailing rates for similar construction. Unless local rates are known, however, this may lead to misleading results. If a priced bill of quantities for previous work in the locality is not at hand, the rates given in technical periodicals can be adopted for the purpose of arriving at an approximate cost.

The preferable method to adopt is to calculate the probable rates, basing the computation on current wages and material costs. There are three factors to consider, (i) Labour charges, (ii) Material costs, and (iii) General costs.

Labour charges include all wages paid to workmen (exclusive of staff) on the site. The rate of wage payable to different classes of workers is given in most technical periodicals, and the man-hours required to perform any given

piece of work are given in the many text-books on this subject. Net labour costs may vary between the following limits:

Concrete—including unloading, storing, and handling materials, mixing, transporting, placing, ramming, screeding, etc., 4*s*. to 12*s*. 6*d*. per cubic yard.

Reinforcement—including unloading, stacking, sorting, bending, placing, etc., 2*s*. to 7*s*. per cwt.

Shuttering—including making, erecting, striking, or including cleaning, drawing nails, repairing, re-erecting, striking, etc., 2*s*. to 5*s*. per square yard.

Excavating—for each time the material is handled (but excluding cost of transport of surplus, timbering, pumping or ramming) measured in solid (does not allow for rock excavation), 1*s*. to 2*s*. per cubic yard.

The factors that cause the labour costs to tend towards the higher limits are small quantity of concrete in job; necessity of washing materials on the site; long distance from point of unloading to position in work, whether horizontally or vertically; exceptionally dry mix of concrete; concrete placed in small quantities, in narrow widths, or under water; slab work where large amount of screeding per cubic yard of concrete required; small or short bars; lack of repetition of bar bending or of shuttering; curved work for shuttering; hard ground to be excavated; excavation within confined spaces; deep excavation; waterlogged ground; tidal work; etc.

Similarly the absence or the opposite of the foregoing factors will tend to bring the net labour costs towards the lower limit.

The material costs can be obtained from current quotations, and the amount of each kind of material required can be determined from the schedule of quantities. On *Table* No. 23 (page 263) are given the quantities of stone, sand, and cement required for a cubic yard of concrete of any mix. To these net quantities must be added an allowance for waste; an increase of 10 per cent. on the net quantities of sand and stone is sufficient for measurement tolerances and waste, and 5 per cent. added to the net quantities of cement will usually allow for waste and for material used in making cement grout, etc. If applicable the cost of cleaning and returning cement bags should be allowed. If material is not quoted "delivered on site," the transport costs from the point of supply to the site must be included.

An exact estimate of the general costs involved in any contract depends principally upon the contract time, the plant it is proposed to employ, and the general planning of the work. These general costs include overhead charges, head-office costs, drawings, site staff (foreman, engineers, surveyors, clerks, watchmen, etc.), lighting and hoardings, insurance, licences, legal charges, loss of interest on retention money, contingencies, profits, and plant charges. The plant charges include the first cost (or depreciation) of all mixers, cranes, hoists, chutes, bending machines, circular saws, lorries or wagons and tracks, etc., that may be brought to the site, in addition to the costs of carriage, erection, dismantling, removing, spares, etc., that may be applicable to these pieces of plants. Such items as small tools, consumable stores, fuels, oils, timber, scaffolding, barrows, ropes, bolts, offices, stores, etc., are also included in this item. In detailed estimating it is usual to allocate the charges of each particular piece of plant to those items in the schedule to which it applies, but for an approximate estimate of a normal type of reinforced concrete structure in which profits do

not exceed 10 per cent. a most convenient and fairly accurate method of allowing for all the charges under the heading of general costs is to increase the net material costs by 15 per cent. and the net labour cost by 40 per cent. For jobs under, say, £1,000 or for very large jobs this method is not dependable, and such work should be estimated on a more precise basis.

The method of making up cost estimates from published unit rates is given in the pages following *Table* No. 40 (p. 301).

PART II

TABLES AND EXAMPLES

EXAMPLES ON USE OF TABLES Nos. 1 AND 2.

(*a*) To find the intensity of total load on a $4\frac{1}{2}$-in. flat roof slab, with an average of $1\frac{1}{2}$ in. of screeding and $\frac{1}{2}$ in. of asphalt:

Superload (for slab design, see *Table* No. 2)	= 50 lb. per sq. ft.
Asphalt = 0·5 × 12 (*Table* No. 1)	= 6 ,, ,, ,,
Screeding = 1·5 × 10 (ditto)	= 15 ,, ,, ,,
Slab = 12 × $4\frac{1}{2}$ (ditto)	= 54 ,, ,, ,,
	125 ,, ,, ,,

(*b*) To find the intensity of total load for a $4\frac{1}{2}$-in. floor slab in residential construction which spans 9 ft. and is surfaced with 1-in. boarding.

Since the span is less than 11 ft. 2 in. (see *Table* No. 2) the minimum live load will control.

Hence superload = $\frac{1}{4}$ ton/9 ft. = 62 lb. per sq. ft.

$4\frac{1}{2}$-in. slab	= 54 ,, ,, ,,
1-in. boarding	= 3 ,, ,, ,,
Allow for runners	= 1 ,, ,, ,,
Plaster on soffit	= 5 ,, ,, ,,
	125 ,, ,, ,,

(*c*) To find the equivalent intensity of load due to a 3-in. hollow block partition 9 ft. high plastered on both faces and supported on a floor slab spanning 10 ft., the partition being placed at midspan and running normal to the span:

Weight per foot run of partition = $12\frac{1}{2}$ lb. × 9 ft.	= 113 lb. per ft. run.
Plaster = 2 × $\frac{1}{2}$ × 9 lb. × 9 ft.	= 81 ,, ,, ,,
	194 ,, ,, ,,

Equivalent distributed load (see Chapter II) $w_p = 2p$.

$p = \frac{194}{10} = 19{\cdot}4$, hence $w_p = 2 \times 19{\cdot}4 = 39$ lb. per sq. ft.

(*d*) To find the total load per foot run of secondary beam in office ground floor construction; 5-in. concrete slab surfaced with $\frac{3}{4}$-in. granolithic and plastered $\frac{1}{2}$ in. thick on underside. Beams 10 in. net depth, by 6 in. wide, span 20 ft., at 10-ft. centres.

Superload from 10 ft. width of slab = 10 ft. × 80 lb. = 800 lb. per ft. run of beam.
(Check total superload = 800 × 20 = 16,000 lb. which exceeds the minimum of 2 tons—see *Table* No. 2.)

Dead load per sq. ft. of slab:

5-in. slab	= 60 lb.
Partitions (min.)	= 20 ,,
$\frac{3}{4}$-in. granolithic	= 9 ,,
$\frac{1}{2}$-in. plaster	= 5 ,,

Dead load on beam from slab = 10 ft. × 94 ,,	= 940 lb. per ft. run.
Beam rib 10 × 6	= 60 ,, ,, ,, ,,
Plaster on sides of rib = 2 × 0·83 × $\frac{1}{2}$ × 9	= 8 ,, ,, ,, ,,
Total	= 1008 ,, ,, ,, ,,

(*e*) To find the total load on a 9-in. by 9-in. lintel spanning a clear opening of 7 ft. and supporting a 9-in. brick wall (built in cement); height of wall exceeds 6 ft.

Height of 60 deg. triangle on 7 ft. base = $\frac{7}{2}\sqrt{3}$ = 6·04 ft.	
Weight of wall at 10 lb. per inch thickness	= 90 lb. per sq. ft.
Do. carried on lintel = $\frac{6{\cdot}04 \times 7}{2} \times 90$	= 1,900 lb.
Weight of lintel = 9 × 9 × 7	= 567 ,,
Total load	= 2,467 ,,

LOADINGS. DEAD LOADS TABLE Nº 1.

WEIGHTS OF MATERIALS (SEE ALSO B.S. Nº 648)

LB. PER CUBIC FOOT.

REINFORCED CONCRETE :-	
DESIGN LOAD.	144
WITH 1% STEEL	148
" 2% "	151
" 5% "	161
" 10% "	174
PLAIN CONCRETE (1:2:4)	145
WITH BALLAST AGG.	140
" BRICK "	115
" BREEZE "	72 TO 90
" PUMICE "	56
GRANITE	165
YORKSTONE	140
LIMESTONE	165
SANDSTONE	140
SLATE	180
TERRACOTTA	112
HOLLOWCLAY TILE	40 TO 50
FIREBRICK (SPECIAL)	56
PACKED STONE RUBBLE	140
SAND	90 TO 110
BALLAST	100 TO 120
MACADAM	150
HARDCORE	120
DRY EARTH FILLING	100
PUDDLED CLAY FILLING	135
GRANITE SETTS	160
WOODBLOCK PAVING	50 TO 60
TIMBER - GENERAL	40 TO 50
YELLOW PINE	30
RED PINE	30 TO 45
SPRUCE	30 TO 45
LARCH	35
ELM	35
TEAK	40 TO 55
PITCH PINE	41 TO 45
ENGLISH OAK	45 TO 60
JARRAH	56 TO 60
GREENHEART	75 TO 65
CORK (SOLID)	15
QUEBRACHO	80
SNOW - FRESH	5 TO 12
SNOW - COMPACTED	15 TO 50

FOR WEIGHTS OF LIQUIDS STORED MATERIALS, ETC., SEE TABLE Nº 4.

LB. PER SQUARE FOOT.

CONCRETE SLABS :-

TOTAL THICKNESS	SOLID R. C.	HOLLOW TILE
4"	48	-
5"	60	43
6"	72	48
7½"	90	54
9"	108	66
10½"	126	75
12"	144	84

WALLS :-	
9" BRICKWORK	90
3" BREEZE PARTITIONS	20
2" HOLLOW BLOCK	9
3" " "	12½
4" " "	15
LIGHT STEEL FRAMED WALLS WITH 4½" BRICK PANELS, WINDOWS, ETC.	50
CORRUGATED SHEETING :-	
GALVD IRON (WITH BOLT)	3
ASBESTOS-CEMENT	3·4
LATH AND PLASTER	8
WINDOWS	5
DOORS	8
SLATES	8
EARTHENWARE TILES	14
PAN TILES	10

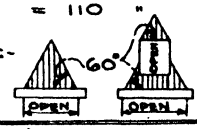
LB. PER SQUARE FOOT ONE INCH THICK.

LEAD SHEET	59½
COPPER SHEET	47
BRONZE, GUNMETAL	46
STEEL, ROLLED OR CAST	40¾
PLASTERBOARD	1½
BALLAST	7½ TO 9½
WROUGHT IRON	40
CAST IRON	38½
TIN	38
SPELTER	36½
ZINC	36½
ALUMINIUM	14
TIMBER BOARDING	3
ASPHALT	12
MORTAR SCREEDING	10
LIME PLASTER	9
GRANOLITHIC	12
TERRAZZO	12
REINFORCED CONCRETE	12
GLASS	14
COMMON BRICK IN R.C.	10
" " IN LIME	9
HARDCORE	10
FIREBRICK (SPECIAL)	5
WOOD PAVING	4½
RUBBER PAVING	5
WATERBOUND MACADAM	13½
TARMACADAM	12

BRICKWORK :-

PRESSED BRICKWORK	IN CEMENT	MORTAR	=	140 LB/SQ. FT.
COMMON "	"	"	=	120 "
" "	LIME	"	=	110 "

LINTELS SUPPORTING BRICK WALLS :-
EXTENT OF WALL CARRIED SHOWN BY SHADED AREA.



ROOFING :- (SEE ALSO "ROOF TRUSSES" BELOW)

	LB. PER SQ. FT PLAN AREA.
PITCHED ROOF INCLUDING TRUSSES, PURLINS, ASBESTOS SHEETING, WIND AND LIVE LOADS	= 30
CORRUGATED IRON SHEETING AND STEEL PURLINS	= 7 TO 9
SLATES, BATTENS AND STEEL PURLINS.	= 12 TO 16
DITTO WITH BOARDING, FELT, ETC.	= 15 TO 20
ROOF GLAZING, ASTRAGALS, PURLINS, ETC. INCLUDING ALL CONNECTIONS.	= 8 LB/SQ. FT. SLOPING SURFACE

STRUCTURAL STEELWORK.

AVERAGE WEIGHT = NOMINAL WEIGHT OF SECTION PLUS 10% FOR CLEATS, RIVETS, BOLTS, ETC (EXTRA FOR STANCHION CAPS AND BASES)

DITTO BEAMS	= WEIGHT OF SECTION INCLUDING CLEATS ETC. PLUS	2½% FOR RIVETS, BOLTS ETC
DO. STANCHIONS	= " " " " " " "	5% " " " "
DITTO PLATE GIRDERS	= " " " " " " "	10% " " " "
WEIGHT OF STEEL STAIRS (3 FT. WIDE INDUSTRIAL TYPE)		= ½ CWT. PER FT. RUN.
STEEL FRAMED WALLS TO TAKE 4½" BRICK PANELLING.		= 7 LB. PER SQ. FT. OF WALL.

STEEL ROOF TRUSSES

SPAN OF TRUSS IN FEET.	25	30	40	50	60	80	SPACING OF TRUSSES
APPROXIMATE WEIGHT IN LB. PER SQ. FT. OF HORIZONTAL PROJECTION.	2	2½	2¾	3	4¼	5	10 FEET
	1½	1½	1¾	2¼	3	3½	15 FEET

SMALL ROOF TRUSSES, PURLINS, ETC. = 6½ LB. PER SQ. FT. OF PLAN AREA.

RAIL TRACKS

BULL HEAD RAILS INCLUDING SLEEPERS (EXCLUDING BALLAST)	= 180 LB/FT. RUN OF TRACK
BRIDGE RAILS INCLUDING LONGITUDINAL SLEEPERS.	= 110 " "
RAIL TRACKS COMPLETE WITH BALLASTING.	= 150 TO 200 LB/SQ. FT.

EXAMPLES ON USE OF TABLE No. 2.

(See also page facing *Table* No. 1.)

(*a*) Find the total load on a column supporting a flat roof and ten floors including ground floor of office building. Total dead weight of roof can be taken as 60 lb. per sq. ft., and each floor 120 lb. per sq. ft. including partitions. Story heights are 10 ft. and the average size of column is 18 in. square. The column supports a panel of floor and roof 20 ft. by 16 ft.

Loads per sq. ft.

Dead load: Roof		=	60 lb. per sq. ft.
9 No. upper floors		=	1,080 ,, ,, ,,
Ground floor		=	120 ,, ,, ,,
Live load: Roof		=	30 ,, ,, ,,
9th = Top floor		=	50 ,, ,, ,,
8th floor	= 0·9		
7th ,,	= 0·8		
6th ,,	= 0·7		
5th ,,	= 0·6		
4th ,,	= 0·5		
3rd, 2nd and 1st = 3 × 0·5	= 1·5		
	5·0 × 50	=	250 ,, ,, ,,
Ground floor = 0·5 × 80		=	40 ,, ,, ,,
			1,630 ,, ,, ,,

Load on column = 1,630 × 20 × 16 = 521,600 lb.
Wt. of column = 18^2 × 10 × 10 ft. = 32,400 ,,

Total load = 564,000 ,,

(*b*) To find the overturning moment due to wind on an exposed water tower consisting of a 20-ft. diameter tank 12 ft. deep, supported on an enclosed square tower, average width 14 ft. The distance from ground level to the underside of the tank is 50 ft., and 3 ft. from ground level to foundation level:

Wind pressure on whole of exposed tower (from *Table* No. 2) = 29 lb. per sq. ft. of projected area, for structure 100 ft. high; for less than 100 ft. take 25 lb. per sq. ft.

Wind on tank: Total pressure = 20 × 12 × 25 × 0·6 = 3,600 lb.
(0·6 = reduction factor due to circular shape.)

$$\text{Centre of pressure} = 3 + 50 + \frac{12}{2} = 59 \text{ ft. above foundation level.}$$

Wind on sub-structure:

Normal to face: = 14 × 50 × 25 = 17,500 lb.

Normal to diagonal: = 14 $\sqrt{2}$ × 50 × 25 × 0·80 = 19,800 lb. approx.
(0·80 = reduction factor for wind normal to diagonal.)

$$\text{Centre of pressure} = 3 + \frac{50}{2} = 28 \text{ ft. above foundation level.}$$

Overturning moment (about axis parallel to face of tower) at foundation level: = (3,600 × 59) + (17,500 × 28) = 700,000 ft.-lb.

Overturning moment (about diagonal) = (3,600 × 59) + (19,800 × 28) = 766,000 ft.-lb.

LOADINGS. SUPERIMPOSED LOADS. TABLE Nº 2.

	DESCRIPTION (x – REQUIREMENTS OF LONDON BUILDING BY-LAWS.)	SLABS: NORMAL LOAD LB. PER SQ. FT.	SLABS: NORMAL LOAD MINᴹ SPAN	SLABS: MINᴹ TOTAL LOAD (TON)	BEAMS: NORMAL LB. PER SQ. FT. FLOOR	BEAMS: MINᴹ TOTAL LOAD (TON)	REMARKS. MINIMUM TOTAL LOADS ARE UNIFORMLY DISTRIBUTED AND FOR SLABS APPLY TO STRIP ONE FOOT WIDE.
FLOORS	RESIDENTIAL ROOMS (x)	50	11'-2"	1/4	40	1	COLUMNS, WALLS, AND FOUNDATIONS SHOULD BE DESIGNED FOR SAME SUPERIMPOSED LOADS AS BEAMS.
	HOTELS AND HOSPITALS :- BEDROOMS, WARDS, ETC.	50	11'-2"	1/4	40	1	
	PUBLIC SPACES	100	8'-5"	3/8	100	2	
	OFFICE FLOORS :- ENTRANCE FLOOR AND BELOW. (x)	80	10'-6"	3/8	80	2	FOR OFFICE FLOORS ALLOW A MINIMUM ADDITIONAL DEAD LOAD OF 20 LB. PER SQ. FT. FOR INTERNAL PARTITIONS.
	ABOVE ENTRANCE FLOOR (x)	80	10'-6"	3/8	50	2	
	RETAIL SHOPS, (x) CHURCHES AND SCHOOLS, READING ROOMS, ART GALLERIES.	80	10'-6"	3/8	80	2	FOR BUILDINGS MORE THAN TWO STORIES HIGH :- LOAD REDUCTION FOR DESIGN OF COLUMNS, WALLS AND FOUNDATIONS :-
	ASSEMBLY AND DANCE HALLS RESTAURANTS AND CAFÉS, GRANDSTANDS, GYMNASIA AND DRILL HALLS, THEATRE AND CINEMAS.	100	8'-5"	3/8	100	2	ROOF AND TOP FLOOR :- 100% LIVE LOAD. 2ND FLOOR FROM TOP :- 90% 3RD " " " :- 80% 4TH " " " :- 70% 5TH " " " :- 60% 6TH " " " :- 50% AND LOWER FLOORS. (THIS REDUCTION ONLY APPLIES TO FLOORS FOR WHICH LIVE LOAD = OR < 100 LB/FT²)
	WAREHOUSES (x) BOOK AND STATIONERY STORES (x)	200 (MIN.)	4'-2"	3/8	200 (MIN.)	2	
	GARAGES.	-	-	-	-	-	SEE TABLE Nº 4
STAIRS	STAIRS INCLUDING CORRIDORS AND LANDINGS. (x)	100 (MIN.)	8'-5"	3/8	100 (MIN.)	2	
	DITTO WITHIN CURTILAGE OF RESIDENCE. (x)	50	11'-2"	1/4	40	1	
ROOFS	FLAT (> 20° TO HORIZONTAL) (x)	50	-	-	30	-	LOADS PER SQ. FT. PLAN AREA.
	INCLINED (> 20° TO HORIZONTAL) (x)	15	DOWNWARDS NORMAL TO WINDWARD SLOPE				FOR SUPPORTING STRUCTURE AND FOUNDATIONS :- 10 LB. PER SQ. FT. PLAN AREA.
		10	UPWARDS NORMAL TO LEEWARD SLOPE.				
BRIDGES	ROADWAY.	224 (WITH IMPACT)	INCREASE FOR SPANS < 10 FT. DECREASE FOR SPANS > 75'. M·O·T· LOADING SEE CHAP. II. B.S.S. LOADING SEE TABLE 3.				FOR WEIGHTS OF ROAD AND RAIL VEHICLES, IMPACT AND DISPERSION, ETC. SEE TABLES Nᵒˢ 3 & 4.
	FOOTPATHS.	84	B.S.S. MINIMUM LOAD.				ALTERNATIVE :- 3 TO 5 TONS POINT LOAD.
WIND PRESSURE	BUILDINGS: UPPER TWO-THIRDS (x)	15	NEGLECT WIND WHEN HEIGHT IS LESS THAN TWICE WIDTH IN DIRECTION OF WIND.				WIND PRESSURE = 0·003 V²
	BUILDINGS: PROJECTIONS (x) ABOVE ROOF	25					
	BRIDGES: UNLOADED	50	B.S.S. PRESSURES. FOR VALUE OF EFFECTIVE AREA SEE CHAPT. II.				
	BRIDGES: LOADED :- ROAD BRIDGES	20					
	BRIDGES: RLY. BRIDGES	30					
	EXPOSED STRUCTURES: HEIGHT (FEET)	100	150	200	300	400	
	EXPOSED STRUCTURES: PRESSURE (LB/FT²)	29	34	38	44	48	
	REDUCTION FACTORS (x)	SEE ALSO CHAPTER II. 0·6 FOR CYLINDRICAL SURFACES, 0·8 FOR OTHER SHAPES.					

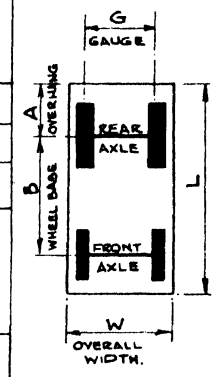
VELOCITY V M.P.H.	DESCRIPTION	PRESSURE LB/FT² FLAT SURFACE
40	HIGH WIND	5
50	GALE	8
60	STORM	11
90	HURRICANE	25
100	VIOLENT HURRICANE	30

STORED MATERIALS.	FOR WEIGHTS OF LIQUIDS AND OTHER STORED MATERIALS SEE TABLE Nº 4. FOR LOADS ON BOTTOM OF CONTAINERS SEE CHAPTER III

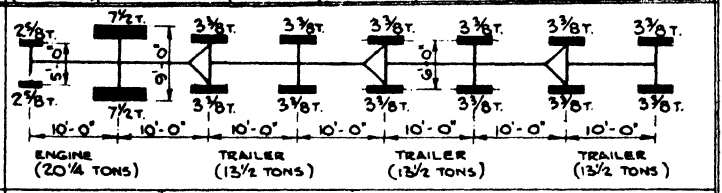
NOTES.

LOADINGS. VEHICLES. TABLE Nº 3.

	DESCRIPTION (M.O.T = MINISTRY OF TRANSPORT)	LOAD (TONS) AXLE (MAX)	LOAD (TONS) TOTAL LADEN	OVERALL LENGTH L	OVERALL WIDTH W	WHEEL BASE B	REAR OVERHANG A	GAUGE G
RAIL VEHICLES	LOCOMOTIVE	20 TO 25	-	-	-	-	-	-
	RLY. WAGON: 20 T.	15	30	24'-6"	8'-6"	12'-0"	6'-3"	4'-8½"
	DITTO 40 T.	14¾	59	-	-	6' 12' 6'	-	4'-8½"
	TRAM: 4 WHEELED	7	14	30'-0"	8'-0"	7'-0"	11'-6"	4'-8½"
	8 WHEELED	7½ DRIVING 3¾ PONY	22½	-	-	-	-	4'-8½"
	COLLIERY TUBS	⅜ TO ¾	¾ TO 1½	- HEIGHT =	3'-6" 3'-0" TO 3'-3"	1'-3" MIN^M	- TURNING RAD.	2'-0" TO 2'-3" = 12'-0"
STEAM ROAD VEHICLES	ROAD LOCO (M.O.T.)	⩾ 15¾	⩾ 21	-	⩾ 9'-0"	-	-	-
	DITTO. 16 TON.	5 (FRONT) 11 (REAR)	16	20'-0"	7'-0"	10'-0"	-	6'-8"
	STEAM ROLLER 20 T.	8 (FRONT) 12 (REAR)	20	-	9'-0"	10'-0"	-	5'-9"
	STEAM WAGON (M.O.T)	⩾ 9	⩾ 14 (4 WHEELS) ⩾ 20 (6 DO)	-	⩾ 7'-6"	-	⩾ B/2	-
	DITTO 5 TON.	4 (FRONT) 8 (REAR)	12	26'-0"	7'-0"	15'-3"	8'-3"	6'-0"
MOTOR VEHICLES	PRIVATE CARS 20 H.P.	⅞ (FRONT) 1 (REAR)	1⅞	15'-0"	5'-0"	8'-0	-	4'-0"
	DITTO 40 H.P.	1¼ (FRONT) 1½ (REAR)	2¾					
	MOTOR CARS AND HEAVY CARS (M.O.T) EXCLUDING STREET CLEANING AND SIMILAR VEHICLES.	⩾ 8	⩾ 12 (4 WHEELS) ⩾ 19 (6 DO.) ⩾ 22 (> 6 DO)	-	⩾ 7'-6" (⩾ 7' 2" PRIVATE CARS ⩾ 6'-0" TRACTORS)	-	⩾ B/2 (WHEN L ⩾ 20' A ⩾ B/2 + 9" OR ⩾ 7/24 L.)	-
	TRAILERS (M.O.T.)	⩾ 6½	⩾ 24 SINGLE TRAILER AND TRACTOR. ETC. (ALL TRAILERS ⩾ 40 T. INC^T LOCO)	⩾ 22'-0" EXCLUDING DRAW-BAR	⩾ 7'-6" EXCLUDING LAND IMPLEMENTS	-		
	TRAILER TRANSPORTING ROUND TIMBER (M.O.T)	⩾ 8	⩾ 13	-	-	-		
	ARTICULATED VEHICLE (M.O.T)	⩾ 8 TRAILER ONLY	⩾ 19	-	-	-		
	MOTOR COACH	3¼ (FRONT) 4¾ (REAR)	8	27'-6"	7'-0"	16'-0"		
	SINGLE-DECK BUS (LONDON)	⩾ 6	⩾ 9 (4 WHEEL) ⩾ 12½ (6 DO.)					
	DOUBLE-DECK BUS (LONDON)	⩾ 7	⩾ 10½ (4 WHEEL) ⩾ 12½ (6 DO.)					
		4 9/20 (FRONT) 6 1/20 (REAR)	10½					
	TROLLEY BUS 6 WHEELED (LONDON)	5⅜ (⩾ 6) FRONT 4 (⩾ 4½) (2 Nº REAR)	13⅜ (⩾ 13½)					



BRITISH STANDARD SPECIFICATION LOADING FOR ROAD BRIDGES (B.S.S. Nº 153) TYPE A LOADS WITH LOAD MULTIPLE OF 15 UNITS. (IMPACT EXCLUDED)



M.O.T. LOADING

SEE FIG. 1 CHAPTER II. FOR LOADING ON BRIDGES.
MAX. LOAD ON TRANSVERSE STRIP 2 FT. WIDE = 10 TONS (ROADS)

NOTES.

B.S. LOADING TYPE A (WITH 15 UNITS) ACCEPTED BY M.O.T. IN LIEU OF STANDARD LOADING (p. 14)
SEE PAGE 12 FOR IMPACT FACTORS FOR B.S. LOADING FOR ROAD BRIDGES.
SEE B.S. Nº 153 FOR LOADING AND IMPACT FACTORS FOR RAILWAY BRIDGES.

EXAMPLES OF USE OF TABLE No. 4.

(*a*) To find width of deck slab of a road bridge assumed to carry a tractor wheel load; slab 10 in. thick with 4-in. road metalling; span of slab 7 ft. between longitudinal beams (i.e. $L = 7$ ft.):

Longitudinal dispersion:

a = say, 12 in.; D = 4 in. + say, 8½ in. = 1 ft. 0½ in.
$A = a + 2D$ = 1 ft. + 2 ft. 1 in. = 3 ft. 1 in.

Transverse dispersion:

b = say, 1 ft. 6 in.; B = 1 ft. 6 in. + 2 ft. 1 in. = 3 ft. 7 in.

According to an American rule: $S = 0{\cdot}67\ (L + B) + A$
$= 0{\cdot}67$ (7 ft. + 3 ft. 7 in.) + 3 ft. 1 in.
= 10 ft. 2 in.

(*b*) To find the maximum total load on a 12-in. concrete slab supporting a single line of standard railway over which can pass at slow speeds any size of locomotive:

Load from ballast (12 in. thick), sleepers, chairs, rails, etc. (*Table* No. 1) = 100 + 180	= 280 lb. per sq. ft.
Concrete slab = 12 × 12	= 144 ,, ,, ,,
Total dead load	= 424 ,, ,, ,,

Dispersion of wheel loads (*Table* No. 4):

D = say, 12-in. ballast + 10½-in. effective depth of slab = 1 ft. 10½ in.

$A = 2D$ + distance over two sleepers (= min. at joints) = 3 ft. 9 in. + 3 ft. = 6 ft. 9 in. (This dimension should not exceed distance between axles of locomotive.)

$B = 2D$ + length of sleeper = 3 ft. 9 in. + 9 ft. = 12 ft. 9 in.

Axle load of 20 tons (*Table* No. 3) = $\dfrac{20 \times 2{,}240}{12{\cdot}75 \times 6{\cdot}75}$ = 521 lb. per sq. ft.

For infrequent passage and slow speed of such a heavy axle load, allow 25 per cent. impact.

Total load = 424 + (1·25 × 521) = 1,075 lb. per sq. ft.

Alternative: Total static load (= 424 + 521) plus 15 per cent. for vibration = 1,086 lb. per sq. ft.

(In normal main line railway bridge construction, longitudinal rail beams would be provided, thereby relieving slab of traffic load.)

(*c*) To find the equivalent uniform load at the support section of a beam 12 in. wide, spacing 10 ft., span 20 ft., fully continuous, in garage floor construction; max. wheel load = 3 tons. From *Table* No. 4, $f = 0{\cdot}965$ and $w = 330$ lb. per ton of wheel load. With $P = 3$ tons, equivalent uniform live load = 1·5 × 330 × 0·965 × 3 = 1,430 lb. per ft. run; compare with normal load of 120 lb. per sq. ft. = 10 × 120 = 1,200 lb. per ft. run. Hence point load will control; to the load of 1,430 lb. per ft. must be added the dead load of the slab and beams. This combined load, say 2,000 lb. per ft. run, will only affect the bending moment calculations, and should be incorporated in the expression $\dfrac{2{,}000 \times 20^2 \times 12}{12}$ in.-lb. The shear force will be based on 1·5 × 3 tons, plus the dead load shear.

LOADINGS. LIVE LOADS. TABLE Nº 4.

STORED MATERIALS. LB. PER CU. FT.

ALCOHOL	53	COAL - ANTHRACITE	52	PARAFFIN	50
AMMONIA	56	DUST	56	PETROL	44
ASHES	40	COKE	35	PETROLEUM OIL	51
BARLEY	35 TO 40	COLLIERY DIRT	70	SEWAGE	62 TO 75
BEER (FERMENTING)	64	COTTON IN BALES	15 TO 40	SHALE	70
BENZINE	43	FLOUR	40 TO 45	SLAG (BASIC)	112
BITUMEN	53	GRAVEL AND SAND	90 TO 110	STONE (CRUSHED)	100
BREEZE	35	HAY AND STRAW	15 TO 20	SUGAR	50
CEMENT	74 TO 114	IRON ORE - SPANISH	150	SULPHURIC ACID	115
NORMAL VALUE	90	" - SWEDISH	230	TAR	64
LOOSE	80	LINSEED OIL	56	TEA	28
IN BAGS	84	MILK	65	TURPENTINE	62
IN BARRELS	65 TO 84	MINERAL OIL	58	WATER - FRESH	62·4
COAL - SOLID	80	NAPHTHA	47	" - SEA	64
CRUSHED - WASHED	56	OATS	25 TO 30	WHEAT	40 TO 45
" - UNWASHED	58	PAPER	60	WINE	62

FOR WEIGHTS AND PRESSURES DUE TO MATERIALS IN WATER, SEE CHAPTER III.

GARAGE FLOORS.

NET WEIGHT OF CAR	NORMAL LOAD PER SQ. FT. OF FLOOR AREA	MINIMUM LOAD (THIS LOADING CONFORMS TO LONDON BUILDING BY-LAWS)
≯ 2¼ TONS	SLABS (≮ 10'-6" SPAN) BEAMS = 80 LB.	SLABS :- ⅜ TON UNIFORMLY DISTRIBUTED ON STRIP ONE FT. WIDE BEAMS :- 2 TONS UNIFORMLY DISTRIBUTED.
> 2¼ TONS	SLABS = 150 LB. BEAMS = 120 LB.	≮ 1·5 X MAX. COMBINATION OF WHEEL LOADS. MINM WHEEL LOAD = ONE TON.

EQUIVALENT UNIFORMLY DISTRIBUTED LOAD (ω) FOR B.M. CALCULATION DUE TO ONE TON DISTRIBUTED OVER AREA 2'-6" SQUARE.

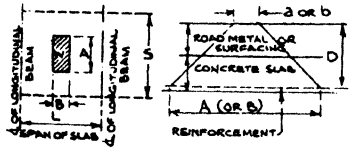
	SPAN OF SLAB	2'-6"	4'-0"	6'-0"	8'-0"	10'-0"	12'-0"	-
SLABS	MIDSPAN	360	310	235	190	160	(135)	LB. PER SQUARE FOOT OF SLAB
	MIDSPAN	360	320	270	230	200	175	
	FIXED SUPPORT	360	300	220	170	(140)	(115)	
	MIDSPAN	360	340	285	240	205	180	
	SUPPORT	360	295	270	160	(130)	(110)	

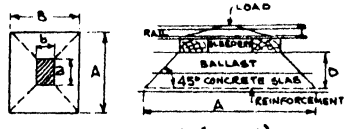
	SPAN OF BEAM	5'-0"	7'-6"	10'-0"	12'-6"	15'-0"	20'-0"	-
BEAMS	MIDSPAN	680	500	390	320	270	205	LB. PER FOOT RUN OF BEAM
	MIDSPAN	760	605	500	430	360	280	
	FIXED SUPPORT	620	470	340	280	240	170	
	MIDSPAN	790	630	510	440	370	290	
	SUPPORT	600	440	330	270	230	160	

REDUCTION FACTORS f.	SPACING OF BEAMS	3'-0"	5'-0"	10'-0"	15'-0"	20'-0"
WIDTH OF BEAM	6"	0·835	0·900	0·950	0·970	0·975
	12"	0·875	0·925	0·965	0·975	0·985
	18"	0·920	0·950	0·975	0·985	0·990

EQUIVT UNIFORM LB./FT. RUN OF BEAM = 1·5 ω f P
P = MAX WHEEL LOAD (TON)

LOAD DISPERSION





FOR WHOLE AXLE LOAD B = 2D + LENGTH OF SLEEPER

DISPERSION OF ROAD WHEELS :- (SEE ALSO p. 17)

A = a + 2D

B = b + 2D

a = CONTACT-LENGTH (0" TO 12")

b = TYRE WIDTH (6" TO 18")

FOR GARAGE FLOORS A = B = 2'-6"

WIDTH OF SLAB CARRYING LOAD = $S = \frac{2}{3}[L+B] + A$.

EXAMPLES OF USE OF TABLE No. 5.

(*a*) To find total horizontal pressure on the back of a vertical wall 15 ft. high retaining ordinary filled earth ; level fill subject to a surcharge of 2 cwt. per sq. ft. $= W$) :

Assume for earth : $w = 100$ lb. per cb. ft. Angle of repose $= \theta = 35$ deg.
From *Table* No. 5 : $k_2 = 0{\cdot}271$.
Intensity of pressure at base of wall due to earth backing and surcharge (Case 3) : $p_2 = k_2(wh + W) = 0{\cdot}271(100 \times 15 + 224) = 467$ lb. per sq. ft.
Intensity of pressure at top of wall due to surcharge : $p_2 = 0{\cdot}271 \times 224 = 60{\cdot}6$ lb. per sq. ft.

Total pressure $= \frac{h}{2}(467 + 60{\cdot}6) = 0{\cdot}5 \times 15 \times 528 = 3{,}960$ lb.

(*b*) A vertical wall 20 ft. high retains a heap of dry coal, the top surface of which is sloped downwards from the wall at the natural slope. Find the intensity of horizontal pressure at the base of wall, (*a*) neglecting friction on the back of wall and (*b*) allowing friction :

From *Table* No. 5, $w = 50$; $\theta = 35$ deg.

(*a*) Neglecting friction : $k_3 = 0{\cdot}205$
$p_3 = k_3wh = 0{\cdot}205 \times 50 \times 20 = 205$ lb. per sq. ft.

(*b*) Allowing for friction : $K_3 = 7$
$p_3 = K_3h = 7 \times 20 = 140$ lb. per sq. ft.

NOTES.

Pressure due to Clay.—The formula on page 26 gives the intensity of horizontal pressure (p lb. per sq. ft.) at any depth h ft. for clay having a weight of w lb. per cu. ft. The formula can be rewritten

$$p = whk_2 - 2k\sqrt{k_2}$$

where k_2 (from *Table* No. 5) applies to the angle a ; k and a have the values given on page 26.

There is no pressure against the wall down to a critical depth h_1 where

$$h_1 = \frac{2k}{w\sqrt{k_2}}$$

if the nature of the clay is prevented from changing.

The maximum intensity of horizontal passive resistance at depth h is given by

$$p_r = \frac{wh}{k_2} + \frac{2k}{\sqrt{k_2}} \quad \text{(see also page 28).}$$

PRESSURE OF CONTAINED MATERIALS. TABI

MATERIAL.	ANGLE OF REPOSE DEGREE	ANGLE OF REPOSE GRADIENT	FACTORS FOR VERTICAL WALL. k_1	k_2	k_3
PERFECT FLUIDS.	0°	–	1·000	1·000	1·000
	5°	1 IN 11·4	·992	·840	·786
FINE CEMENT. (MIN. VALUE) (MAX. = 17°)	10°	1 IN 5·7	·970	·702	·625
	11° 20′	1 IN 5	·962	·675	·591
VERY WET PEAT.	14° 0′	1 IN 4	·942	·610	·522
(WET CLAY.)	15°	1 IN 3·7	·934	·589	·500
VERY WET EARTH.	18° 30′	1 IN 3	·900	·520	·428
	20°	1 IN 2·7	·884	·490	·403
WET SAND; GRAIN.	25°	1 IN 2·1	·821	·406	·320
SANDY GRAVEL.	26° 35′	1 IN 2	·800	·382	·301
(DRY CLAY); DRY EARTH.	30°	1 IN 1·7	·750	·333	·256
AVE DAMP SAND.	33° 40′	1 IN 1·5	·692	·286	·216
FINE DRY SAND; COAL,	35°	1 IN 1·4	·670	·271	·205
SHINGLE; ASHES; COKE.	40°	1 IN 1·2	·586	·218	·161
(DAMP WELL DRAINED CLAY), MOIST EARTH.	45°	1 IN 1	·500	·172	·125
CLEAN GRAVEL (NATURAL BED)	50°	1 IN ·84	·414	·132	·095
DRY COMPACT PEAT.	55°	1 IN ·7	·329	·100	·071
	60°	1 IN ·58	·250	·072	·050
PUNNED EARTH.	65°	1 IN ·47	·179	·049	·035
	70°	1 IN ·36	·117	·031	·022
WELL PUNNED EARTH.	75°	1 IN ·27	·067	·017	·012
	80°	1 IN ·18	·030	·008	·005
	85°	1 IN ·09	·008	·002	·003

NOTES:–

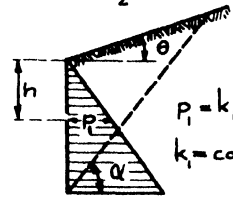
FRICTION NEGLECTED.

p = INTENSITY OF HORIZ^L. PRESS.

ω = UNIT WT. OF MATERIAL – SEE TABLES No^S 1 & 4 FOR VALUE

θ = ANGLE OF REPOSE.

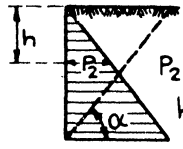
$\alpha = 45° + \frac{\theta°}{2}$ (APPROX.)



$p_1 = k_1 \omega h$

$k_1 = \cos^2\theta$

MAXIMUM POSITIVE SURCHARGE.

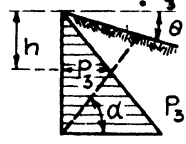


$p_2 = k_2 \omega h$

$k_2 = \frac{1-\sin\theta}{1+\sin\theta}$

LEVEL FILL.

$k_3 = \left(\frac{\cos\theta}{1+\sin\theta\sqrt{2}}\right)^2$



$p_3 = k_3 \omega h$

MAXIMUM NEGATIVE SURCHARGE.

DRY MATERIAL.	ω.	θ	μ	K_1	K_2	K_3
CEMENT	90	17°	29°	82	37	29
WHEAT.	48	25°	24°	39	15·5	12
ANTHRACITE.	52	27°	27°	41	15·4	12
SAND.	90	35°	30°	60	19·2	14
BITUMINOUS COAL.	50	35°	35°	33	9·7	7
ASHES.	40	40°	40°	23	6·4	4·5

ALLOWING FOR FRICTION ON VERTICAL WALL

μ = ANGLE OF FRICTION B'T'N MATERIAL & CONCRETE.

$p_1 = K_1 h$ $\quad p_2 = K_2 h$

$p_3 = K_3 h$

FOR FORMULÆ FOR ALL CONDITIONS OF VERTICAL AND INCLINED WALLS SEE CHAP. III.

SURCHARGE.

(APPROXIMATE TREATMENT)

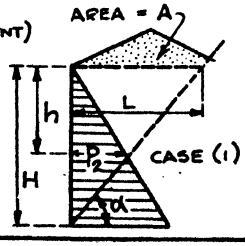
CASES (1) & (2):–

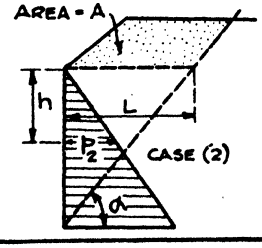
$p_2 = c k_2 \omega h$

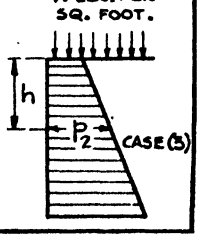
$c = 1 + \frac{4 \cdot 5 A}{HL}$

CASE (3)

$p_2 = k_2 (\omega h + W)$







EXAMPLES OF USE OF TABLE No. 6.

To find the horizontal pressures at various depths in a 20-ft. diameter bin 60 ft. deep containing grain ; and also the total load carried on the walls due to the filling :

For grain take $w = 48$ lb. per cb. ft.
$k = 0{\cdot}5$
$\tan \mu = 0{\cdot}444.$
Therefore $p(= wC) = 48C.$

The values of C are obtained from the table for

$$Q = 0{\cdot}25 \times 20 = 5{\cdot}0.$$

At 10 ft. depth :	$p =$	$4{\cdot}0 \times 48 = 192$	lb. per	sq. ft.
,, 20 ft. ,,	$=$	$6{\cdot}7 \times 48 = 322$	,,	,,
,, 30 ft. ,,	$=$	$8{\cdot}4 \times 48 = 403$	,,	,,
,, 40 ft. ,,	$=$	$9{\cdot}4 \times 48 = 452$	,,	,,
,, 60 ft. ,,	$=$	$10{\cdot}6 \times 48 = 498$	,,	,,

The load transferred to the walls of the bin by friction use formula given (on p. 28, Chapter III) for $w = 48$ lb. per cb. ft.

$= (12H - 0{\cdot}5p)D$
$= (12 \times 60 - 0{\cdot}5 \times 498)20$
$= 9{,}420$ lb. per lineal foot of circumference.

NOTES.

(*a*) The value of $k = 0{\cdot}5$ in the Janssen formula is commonly adopted for reinforced concrete bins, but Pleisner gives $k = 0{\cdot}30$ to $0{\cdot}35$. Thus $k = 0{\cdot}5$ is on the safe side for the calculation of the horizontal pressure and load carried by walls, but for vertical pressures on the bottom a value of $k = \frac{1}{3}$ is advisable.

(*b*) Grain flows at 30 to 32 deg., therefore the valley angle in pyramidical hopper bottoms should not be flatter than 30 deg.

DEEP SILOS. TABLE No. 6.

JANSSEN'S FORMULA.

$$p = \frac{\omega Q}{\tan \mu}\left[1 - \frac{1}{N}\right] = kV$$

WHERE

p = HORIZONTAL PRESSURE IN LBS. PER SQ. FT. AT DEPTH H FT.

ω = WEIGHT OF FILLING IN LBS. PER CU. FT. (SEE TABLE No 5)

Q = $\frac{\text{PLAN AREA OF BIN IN SQ. FT.}}{\text{PERIMETER OF BIN IN FT.}}$

μ = ANGLE OF FRICTION BETWEEN WALL AND FILLING (SEE TABLE No. 5)

V = VERTICAL PRESSURE IN LBS. PER SQ. FT. AT DEPTH H FT.

k = $\frac{\text{HORIZONTAL PRESSURE}}{\text{VERTICAL PRESSURE}}$ (= ·5 FOR GRANULAR MATERIALS)

N = NUMBER WHOSE COMMON LOG. = $\frac{H k \tan \mu}{2{\cdot}303 Q}$

H = DEPTH OF FILLING IN FEET ABOVE POINT BEING CONSIDERED.

VALUES OF $C = \frac{P}{\omega}$ FOR $\tan \mu = {\cdot}444$; $k = {\cdot}5$

VALUES OF H FEET	VALUES OF Q								
	10	7·5	5·0	4·0	3·0	2·5	2·0	1·5	1·0
10	4·5	4·0	4·0	3·9	3·5	3·3	3·1	2·5	2·2
15	6·5	5·9	5·5	5·1	4·5	4·1	3·6	3·0	2·3
20	8·1	7·5	6·7	6·0	5·2	4·7	4·0	3·2	2·4
25	9·5	8·8	7·7	6·7	5·7	5·0	4·2	3·3	-
30	10·9	9·9	8·4	7·1	6·0	5·3	4·4	3·4	-
35	12·2	10·9	8·9	7·7	6·3	5·4	4·4	-	-
40	13·2	11·8	9·4	8·1	6·5	5·5	4·5	-	-
45	14·2	12·3	9·8	8·3	6·6	5·6	-	-	-
50	15·1	13·0	10·1	8·6	6·7	5·6	-	-	-
55	16·0	13·5	10·3	8·7	6·7	-	-	-	-
60	16·5	14·0	10·6	8·8	6·7	-	-	-	-
65	17·2	14·3	10·7	8·9					
70	17·7	14·7	10·9	9·0					
75	18·2	15·0	11·0						
80	18·7	15·3	11·1						
90	19·4	15·7	11·3						
100	20·0	16·0	11·4						

VALUES OF Q.

SQUARE BIN; $D' \times D'$; $Q = {\cdot}25D$.

RECTANGULAR BIN; $B' \times D'$; $Q = \frac{BD}{2(B+D)}$

HEXAGONAL BIN; $Q = {\cdot}25$ (DISTANCE ACROSS FLATS)

OCTAGONAL BIN; $Q = {\cdot}25$ (DISTANCE ACROSS FLATS)

CIRCULAR BIN; $Q = {\cdot}25$ DIAMETER.

INCREASE IN LATERAL PRESSURE IS NEGLIGIBLE BELOW THE DEPTHS TABULATED ABOVE.

SINGLE SPAN FIXED-END BEAMS. TABLE Nº 7D

THE LOAD FACTORS C_{AB} AND C_{BA} CAN BE USED THUS:-

(i) B.M. AT SUPPORTS OF SINGLE SPAN BEAMS FIXED BOTH ENDS. (SEE ALSO TABLES Nºs 7 & 7B.)

$M_{AB} = -C_{AB}L_{AB}$ $M_{BA} = -C_{BA}L_{AB}$ WITH SYMMETRICAL LOAD $M_{AB} = M_{BA}$

(ii) CONTINUOUS BEAMS - MOMENT DISTRIBUTION METHOD - TABLE Nº 11A.

SPAN-LOAD FACTORS ARE $F_{AB} = C_{AB}L_{AB}$ $F_{BA} = C_{BA}L_{AB}$

WITH SYMMETRICAL LOAD $F_{AB} = F_{BA}$

(iii) FRAMED STRUCTURES - TABLE Nº 15.

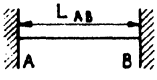
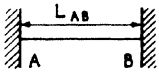
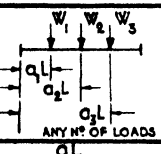
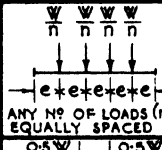
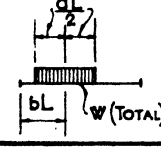
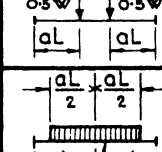
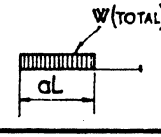
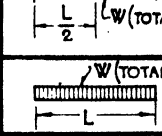
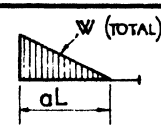
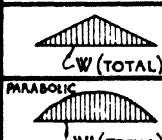
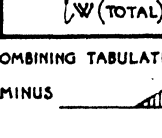
LOADING FACTORS P AND Q HAVE FOLLOWING VALUES:-

$P_{AB} = C_{AB}L_{AB}$ $Q_{BA} = C_{BA}L_{AB}$ WITH SYMMETRICAL LOAD $P_{AB} = Q_{BA} = \frac{A_{AB}}{L_{AB}} = C_{AB}L_{AB}$

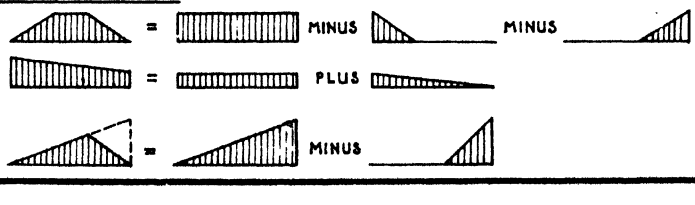
(iv) PORTAL FRAMES - TABLE Nº 16.

$$N = \frac{\text{AREA OF FREE B.M. DIAGRAM}}{\text{LOADED SPAN}}\left[=\frac{A}{L} \text{ OR } \frac{A}{0{\cdot}5L} \text{ OR } \frac{A}{H} \text{ OR } \frac{A}{fH}\right] = \frac{C_{AB}+C_{BA}}{2}\cdot L_{AB} \qquad Z = \frac{C_{AB}+2C_{BA}}{3(C_{AB}+C_{BA})}$$

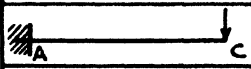
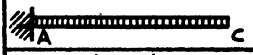
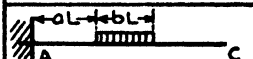
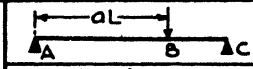
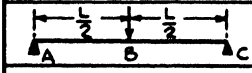
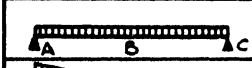
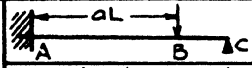
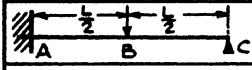
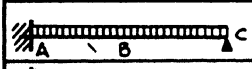
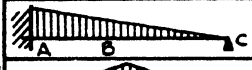
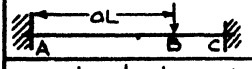
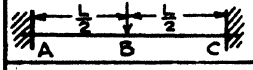
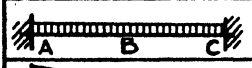
DISTANCE FROM LEFT HAND OR LOWER SUPPORT TO CENTROID OF FREE B.M. DIAG. $= Z_1 \times$ LOADED SPAN

UNSYMMETRICAL LOADING				SYMMETRICAL LOADING	
	LOAD FACTORS				LOAD FACTOR
	C_{AB}		C_{BA}		$C_{AB} = C_{BA}$
	$\Sigma a(1-a)^2 W$		$\Sigma a^2(1-a)W$		n=1 $0{\cdot}125W$ =2 $0{\cdot}111W$ =3 $0{\cdot}104W$ =4 $0{\cdot}100W$
	$\left[b(1-b)^2-a^2\left(\frac{1}{6}-\frac{b}{4}\right)\right]W$		$\left[b^2(1-b)-a^2\left(\frac{b}{4}-\frac{1}{12}\right)\right]W$		$\frac{a}{2}(1-a)W$
	$a(3a^2-8a+6)\frac{W}{12}$		$a^2(4-3a)\frac{W}{12}$		$0{\cdot}125\left(1-\frac{a^2}{3}\right)W$
	a=1·0	$0{\cdot}100W$	$0{\cdot}067W$		$0{\cdot}083W$
	=0·8	$0{\cdot}105W$	$0{\cdot}055W$		$0{\cdot}104W$
	=0·6	$0{\cdot}102W$	$0{\cdot}039W$	PARABOLIC	$0{\cdot}100W$
	=0·5	$0{\cdot}096W$	$0{\cdot}029W$		
	=0·4	$0{\cdot}086W$	$0{\cdot}020W$		

OTHER LOADINGS CAN GENERALLY BE CONSIDERED BY COMBINING TABULATED CASES, THUS:-



CANTILEVERS & SINGLE SPAN BEAMS. TABLE Nº 7.

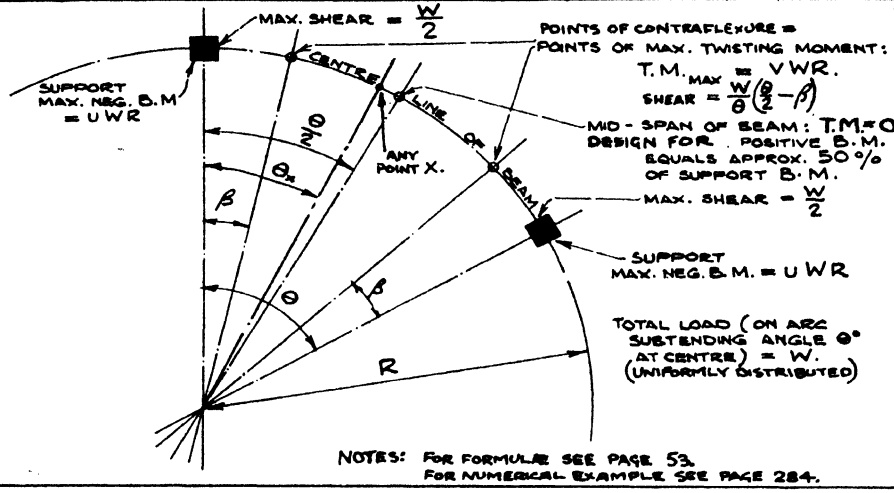
		BEAM TYPE	MOMENT COEFFICIENTS AT A	MOMENT COEFFICIENTS AT B	MOMENT COEFFICIENTS AT C	REACTION COEFFS. AT A	REACTION COEFFS. AT C	MAXIMUM DEFLECTION COEFFICIENT
CANTILEVERS			$-1\cdot0$	-	ZERO	$1\cdot0$	-	$\frac{1}{3}$
CANTILEVERS			$-\frac{1}{2}$	-	ZERO	$1\cdot0$	-	$\frac{1}{8}$
CANTILEVERS			$-(a+\frac{b}{2})$	-	ZERO	$1\cdot0$	-	-
CANTILEVERS			$-\frac{1}{3}$	-	ZERO	$1\cdot0$	-	$\frac{1}{15}$
CANTILEVERS			$-\frac{1}{2}$	-	ZERO	$1\cdot0$	-	$\frac{11}{96}$
SINGLE SPAN BEAMS	BOTH ENDS FREE		ZERO	$a(1-a)$	ZERO	$1-a$	a	$\frac{a^2}{3}(1-a)^2$
SINGLE SPAN BEAMS	BOTH ENDS FREE		ZERO	$\frac{1}{4}$	ZERO	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{48}$
SINGLE SPAN BEAMS	BOTH ENDS FREE		ZERO	$\frac{1}{8}$	ZERO	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{384}$
SINGLE SPAN BEAMS	BOTH ENDS FREE		ZERO	$\frac{1}{7\cdot81}$ MAX.	ZERO	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{76\cdot75}$
SINGLE SPAN BEAMS	BOTH ENDS FREE		ZERO	$\frac{1}{6}$	ZERO	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{60}$
SINGLE SPAN BEAMS	ONE END FIXED ONE END FREE		MINUS $\frac{a}{2}(1-a)\times(2-a)$	$\frac{a^2}{2}(1-a)\times(3-a)$	ZERO	$(1-a)\times(1+a-\frac{a^2}{2})$	$\frac{a^2}{2}(3-a)$	-
SINGLE SPAN BEAMS	ONE END FIXED ONE END FREE		$-\frac{3}{16}$	$\frac{5}{32}$	ZERO	$\frac{11}{16}$	$\frac{5}{16}$	$\frac{1}{107\cdot3}$
SINGLE SPAN BEAMS	ONE END FIXED ONE END FREE		$-\frac{1}{8}$	MAX. $\frac{9}{128}$	ZERO	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{185}$
SINGLE SPAN BEAMS	ONE END FIXED ONE END FREE		$-\frac{1}{7\cdot5}$	MAX. $\frac{1}{16\cdot7}$	ZERO	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{1}{210}$
SINGLE SPAN BEAMS	ONE END FIXED ONE END FREE		$-\frac{5}{32}$	$\frac{1}{10\cdot5}$	ZERO	$\frac{21}{32}$	$\frac{11}{32}$	$\frac{1}{139\cdot5}$
SINGLE SPAN BEAMS	BOTH ENDS FIXED		$-a(1-a)^2$	$2a^2(1-a)^2$	$-a^2(1-a)$	$(1-a)^2\times(1+2a)$	$a^2(3-2a)$	-
SINGLE SPAN BEAMS	BOTH ENDS FIXED		$-\frac{1}{8}$	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{192}$
SINGLE SPAN BEAMS	BOTH ENDS FIXED		$-\frac{1}{12}$	$\frac{1}{24}$	$-\frac{1}{12}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{384}$
SINGLE SPAN BEAMS	BOTH ENDS FIXED		$-\frac{1}{10}$	MAX. $\frac{1}{23}$	$-\frac{1}{15}$	$\frac{7}{10}$	$\frac{3}{10}$	$\frac{1}{382}$
SINGLE SPAN BEAMS	BOTH ENDS FIXED		$-\frac{5}{48}$	$\frac{1}{16}$	$-\frac{5}{48}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{1920}$

ALL MOMENTS = COEFFICIENT × TOTAL LOAD × SPAN = K_1WL

REACTIONS = COEFFICIENT × TOTAL LOAD = K_2W. DEFLECTION = COEFFICIENT × $\left(\frac{WL^3}{EI}\right)$

CIRCULAR BEAMS — TABLE Nº 7c.

APPLICABLE TO BEAMS FORMING PART OF COMPLETE CIRCULAR SYSTEM WITH UNIFORMLY SPACED SUPPORTS OR EQUIVALENT CONDITIONS I.E. FIXED ENDS.



NOTES: FOR FORMULÆ SEE PAGE 53.
FOR NUMERICAL EXAMPLE SEE PAGE 284.

MOMENT COEFFICIENTS.

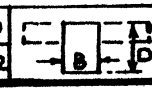
	ANGLE SUBTENDED AT CENTRE θ	c_1	NUMBER OF SUPPORTS IN COMPLETE CIRCLE	NEGATIVE B.M. OVER SUPPORT U.	T.M. AT SUPPORT. c_2	T.M. AT SUPPORT. c_3	TWISTING MOMENT (MAX.) AT POINTS OF CONTRAFLEXURE. V.	c_2	c_3	β.
CASE I. T.M. AT SUPPORT = 0	90°		4	0·137	-	-	0·021	-	-	19¼°
	72°		5	0·108	-	-	0·014	-	-	15¼°
	60°		6	0·089	-	-	0·009	-	-	12¾°
	51½°		7	0·077	-	-	0·007	-	-	10¾°
	45°		8	0·066	-	-	0·005	-	-	9½°
	40°		9	0·060	-	-	0·004	-	-	8½°
	36°		10	0·054	-	-	0·003	-	-	7½°
	30°		12	0·045	-	-	0·002	-	-	6¼°
CASE II. RIGID ANALYSIS.	180°	-	(2)	0·318	0·096WR		0·038	-	-	52°
	120°	2·41	3	0·215	0·27	0·09	-	0·06	0·03	36°
	90°	4·5	4	0·146	0·03	0·03	-	0·13	0·02	22°
	60°	10·4	6	0·091	0·02	0·02	-	0·08	ZERO	15°
	45°	-	8	0·068	negligible		0·005	-	-	10°

EXPRESSION FOR TWISTING MOMENT AT SUPPORT OR AT POINT OF CONTRAFLEXURE

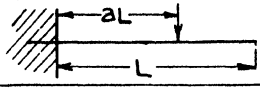
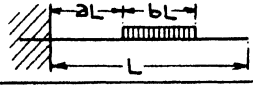
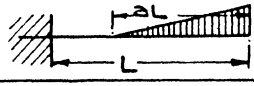
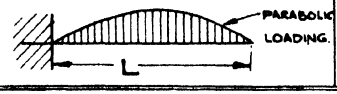
$$= \frac{c_2 G + c_3}{c_1 G + 1} \cdot \frac{WR}{\theta}$$ WHERE c_1, c_2 AND G AND c_3 ARE AS TABULATED.

$G = \frac{NJ}{EI}$ = TORSIONAL RIGIDITY FACTOR.
= APPROX. EQUAL FOR RECT., TEE AND ELL SECTIONS WITH SAME RATIO D : B.

D/B	1·0	1·5	2·0	2·5	3·0
G	1·12	0·70	0·47	0·33	0·22

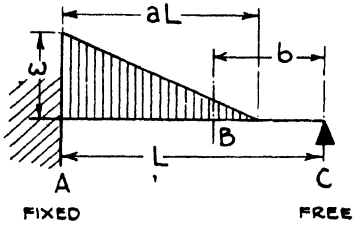
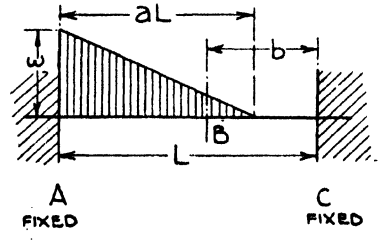


CANTILEVERS.
DEFLECTION COEFFICIENTS. (SEE ALSO TABLE Nº 7.) TABLE Nº 7A.

CASE	LOADING.	DEFLECTION COEFFICIENT. FOR FREE END OF CANTILEVER.
a.		$\frac{a^2(3-a)}{6}$
b.		$A + B(1-a-b)$ WHERE $A = \frac{1}{24}\left[8a^3+18a^2b+12ab^2+3b^3\right]$, $B = \frac{1}{6b}\left[(a+b)^3 - a^3\right]$
c.		$\frac{a^2}{60}(5-a)$
d.		$\frac{1}{60}\left[20-10a+a^3\right]$
e.		$\frac{7}{60}$

DEFLECTION = COEFFICIENT × $\frac{WL^3}{EI}$ W = TOTAL LOAD

SINGLE - SPAN BEAMS
WITH TRIANGULARLY DISTRIBUTED LOADING. TABLE Nº 7B.

BEAM TYPE	a	MOMENT COEFFICIENTS AT A (NEG.)	AT B (POS.)	AT C (NEG.)	REACTION COEFFTS. AT A	AT C
	1·0	0·133	0·060 b=·45	ZERO	0·80	0·20
	0·9	0·133	0·057 b=·47	ZERO	0·83	0·17
	0·8	0·132	0·053 b=·49	ZERO	0·87	0·13
	0·7	0·128	0·047 b=·53	ZERO	0·90	0·10
	0·6	0·121	0·041 b=·57	ZERO	0·92	0·08
	0·5	0·111	0·032 b=·62	ZERO	0·94	0·06
	0·4	0·096	0·024 b=·68	ZERO	0·96	0·04
	1·0	0·100	0·043 b=·55	0·067	0·70	0·30
	0·9	0·103	0·042 b=·56	0·062	0·74	0·26
	0·8	0·105	0·041 b=·57	0·055	0·78	0·22
	0·7	0·105	0·040 b=·59	0·047	0·82	0·18
	0·6	0·102	0·036 b=·62	0·039	0·86	0·14
	0·5	0·096	0·032 b=·66	0·029	0·90	0·10
	0·4	0·086	0·025 b=·70	0·020	0·93	0·07
TOTAL LOAD = W = $\frac{\omega \cdot aL}{2}$	-	BENDING MOMENT = COEFFICIENT × WL. B = POINT OF MAX. +VE B.M.			REACTION = COEFFT × W	

EXAMPLES OF USE OF TABLE No. 8.

(*a*) To find the maximum moments at the middles of the end span and centre span, and at the penultimate and interior supports of a five-span continuous beam; all spans = 15 ft.; dead load = 600 lb. per ft. run and live load = 1,200 lb. per ft. run.

(i) From *Table* No. 8.

Penultimate support:

Dead load: $0{\cdot}105 \times 600 \times 15^2 = 14{,}200$ ft.-lb.
Live load: $0{\cdot}120 \times 1{,}200 \times 15^2 = 32{,}300$,,

Total: $= 46{,}500$,, (neg.)

Interior support:

Dead load: $0{\cdot}080 \times 600 \times 15^2 = 10{,}400$ ft.-lb.
Live load: $0{\cdot}111 \times 1{,}200 \times 15^2 = 29{,}900$,,

Total: $= 40{,}300$,, (neg.)

Middle of end span:

Dead load: $0{\cdot}078 \times 600 \times 15^2 = 10{,}250$ ft.-lb.
Live load: $0{\cdot}100 \times 1{,}200 \times 15^2 = 27{,}000$,,

Total: $= 37{,}250$,, (pos.)

Middle of interior span:

Dead load: $0{\cdot}046 \times 600 \times 15^2 = 6{,}200$ ft.-lb.
Live load: $0{\cdot}086 \times 1{,}200 \times 15^2 = 23{,}200$,,

Total: $= 29{,}400$,, (pos.)

(ii) From *Table* No. 10.

Ratio of live to dead load $= \frac{1{,}200}{600} = 2$

Total load $= 1{,}200 + 600 = 1{,}800$ lb.

Penultimate support:	$0{\cdot}107 \times 1{,}800 \times 15^2 = 43{,}100$	ft.-lb.	(neg.)
Interior support:	$0{\cdot}102 \times 1{,}800 \times 15^2 = 41{,}100$	,,	(neg.)
Middle of end span:	$0{\cdot}094 \times 1{,}800 \times 15^2 = 38{,}100$	,,	(pos.)
Middle of centre span:	$0{\cdot}069 \times 1{,}800 \times 15^2 = 28{,}000$	,,	(pos.)

(*b*) Same problem as Example (*a*) opposite *Table* No. 9:

Penultimate support:

Dead load: $0{\cdot}107 \times 1{,}000 \times 15^2 = 24{,}100$ ft.-lb.
Live load: $0{\cdot}181 \times 10{,}000 \times 15 = 27{,}200$,,

$51{,}300$,,

EXAMPLE OF USE OF TABLE No. 10A.

If a negative moment of 200,000 in.-lb. is applied at the end support of the beam in Example (*a*) opposite *Table* No. 9, what is the resultant maximum bending moment at the penultimate support.

B.M. at penultimate support (see previous example) $= -51{,}075$ ft.-lb. $= -612{,}900$ in.-lb.

B.M. at support *B* (four equal spans) due to bending moment applied at one end only (see *Table* No. 10A) $= +0{\cdot}268 \times 200{,}000$ $= +53{,}600$,,

Resultant net B.M. $= -559{,}300$,,

Refer to p. 38 for explanation of difference between coefficients on *Table* No. 8 and those in accordance with the London Building By-laws.

CONTINUOUS BEAMS. EQUAL LOADING ON EQUAL SPANS. TABLE No 8.

	LOAD	ALL BEAMS FREELY SUPPORTED AT END SUPPORTS: ALL SPANS LOADED	INCIDENTAL LOADING
MAXIMUM BENDING MOMENTS	UNIFORMLY DISTRIBUTED	·125 ·071 ·071 ·100 ·100 ·080 ·025 ·080 ·107 ·072 ·107 ·077 ·036 ·036 ·077 ·105 ·080 ·080 ·105 ·078 ·053 ·046 ·053 ·078	·125 ·096 ·096 ·117 ·117 ·101 ·075 ·101 ·121 ·107 ·121 ·099 ·081 ·081 ·099 ·120 ·111 ·111 ·120 ·100 ·080 ·086 ·080 ·100
	CENTRE POINT LOAD	·188 ·156 ·156 ·150 ·150 ·175 ·100 ·175 ·161 ·107 ·161 ·169 ·116 ·116 ·169 ·158 ·119 ·119 ·158 ·171 ·110 ·130 ·110 ·171	·188 ·203 ·203 ·175 ·175 ·213 ·175 ·213 ·181 ·160 ·180 ·210 ·183 ·183 ·210 ·179 ·167 ·167 ·179 ·211 ·181 ·191 ·181 ·211
	POINT LOADS AT 1/3RD. POINTS	·167 ·111 ·111 ·133 ·133 ·123 ·084 ·123 ·143 ·095 ·143 ·119 ·056 ·056 ·119 ·141 ·106 ·108 ·141 ·120 ·050 ·061 ·050 ·120	·167 ·139 ·139 ·157 ·157 ·145 ·100 ·145 ·160 ·144 ·160 ·143 ·111 ·111 ·143 ·159 ·148 ·148 ·159 ·144 ·108 ·115 ·108 ·144
MAXIMUM SHEARS	UNIFORMLY DISTRIBUTED	·38 ·62 ·62 ·38 ·40 ·50 ·60 ·60 ·50 ·40 ·39 ·54 ·46 ·61 ·61 ·46 ·54 ·39 ·40 ·53 ·50 ·47 ·60 ·60 ·47 ·50 ·53 ·40	·44 ·62 ·62 ·44 ·45 ·58 ·62 ·62 ·58 ·45 ·45 ·60 ·57 ·62 ·62 ·57 ·60 ·45 ·45 ·60 ·59 ·58 ·62 ·62 ·58 ·59 ·60 ·45

B.M. COEFFICIENTS :– MULTIPLY BY (SPAN X TOTAL LOAD ON SPAN). COEFFICIENTS ABOVE THE LINE ARE FOR NEGATIVE B.Ms. AT SUPPORTS; THOSE UNDER LINE ARE FOR POSITIVE MID-SPAN B.Ms.
SHEAR COEFFICIENTS:– MULTIPLY BY (TOTAL LOAD ON SPAN).

EXAMPLES OF USE OF TABLE No. 9.

(*a*) To find the maximum moment at the penultimate support of a beam continuous over four equal spans of 15 ft. carrying a uniformly distributed load of 1,000 lb. per ft. run; a live point load of 10,000 lb. can occur at the middle of one or more of the spans simultaneously.

Maximum B.M. occurs when 1st, 2nd, and 4th spans are loaded with point load. Dividing the total loading into single-span loads:

Load = W.	*Span.*	*Load Factor = F.*	*Support Moment Coefficient = Q.*	*Product = FQW*
Uniformly distributed $W = 1{,}000 \times 15 = 15{,}000$ lb.	All spans	1·00	− 0·107	− 1,605
Point load $W = 10{,}000$ lb.	1st span	1·50	− 0·067	− 1,000
Ditto	2nd span	1·50	− 0·049	− 740
Ditto	4th span	1·50	− 0·004	− 60
			$\Sigma FQW =$	− 3,405

B.M. $= L \times \Sigma FQW = -3{,}405 \times 15 = -51{,}075$ ft.-lb.

It can be seen from the table that these results are the maximum possible, because if the third span were loaded a positive moment would be introduced that would reduce the total negative moment.

(*b*) To find the moment at the middle of the centre span of three continuous spans, 10 ft., 15 ft., and 10 ft. respectively. The load on the centre span is a uniform dead load of 7,000 lb. per ft. run and on each of the end spans the load is triangularly distributed, being 6,000 lb. per ft. at the inner supports and zero at the outer supports at which the beam is simply supported. (This is the type of loading that would be encountered in the design of a rectangular tank when wall counterforts—tied at their extremities—are continuous with the beams of the suspended bottom.)

First determine the B.M. at the inner supports by dividing the loading and evaluating the corresponding products. Owing to symmetry the moments at each interior support will be equal.

$$k_1 = k_2 = \frac{\text{End span}}{\text{Base span}} = \frac{10}{15} = 0{\cdot}67$$

$$x = y = 0{\cdot}67 + 1 = 1{\cdot}67$$

$$H = \frac{5}{(4 \times 1{\cdot}67 \times 1{\cdot}67) - 1} = 0{\cdot}493.$$

For 1st span loaded: $U_C = 3 \times 0{\cdot}67^2 \times 0{\cdot}493 = 0{\cdot}663$

$U_B = 0{\cdot}5 \times 1{\cdot}67 \times 0{\cdot}663 = 0{\cdot}555$

For 2nd span loaded: U_C (= U_B owing to symmetry) $= 0{\cdot}493\,(1{\cdot}67 + 0{\cdot}67) = 1{\cdot}15$

For 3rd span loaded: Owing to symmetry $U_C = 0{\cdot}555$; $U_B = 0{\cdot}663$.

Load = W.	*Span*	*(Load Factor) × (Support Mt. Coef.) = FQ.*	*Moment Multiplier = U.*	*Product = FQUWL.*
Triangularly distributed (apex at left-hand support) $W = 6{,}000 \times 10 \times 0{\cdot}5 = 30{,}000$ lb.	3rd	+ 0·018	$U_B = 0{\cdot}663$	+ 5,370 ft.-lb.
Uniformly distributed $W = 7{,}000 \times 15 = 105{,}000$ lb.	2nd $L = 15$ ft.	1·00 × (− 0·050)	$U_B = 1{\cdot}15$	− 90,500 ,,
Triangularly distributed (apex at right-hand support: therefore reverse system) $W = 30{,}000$ lb.	1st	− 0·071	U_C for 3rd span loaded = 0·555	− 17,740 ,,
			Net B.M. at inner support =	− 102,870 ft.-lb.

Free moment at mid-span $= \dfrac{7{,}000 \times 15^2}{8} = 197{,}000$ ft.-lb.

Less negative B.M. at mid-span $= 102{,}870$,,

Net positive B.M. at mid-span $= 94{,}130$,,

NOTE.

$$k = \frac{\text{length of span}}{\text{base span}}$$

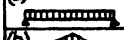
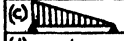
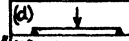
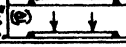
CONTINUOUS BEAMS — CONSTANT M. OF I. ANY SPAN AND LOAD — TABLE Nº 9.

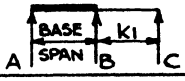
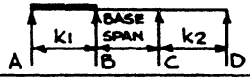
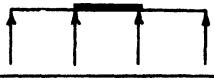
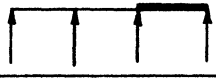
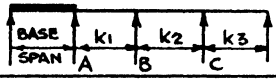
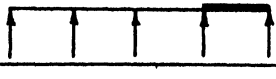
NOTES: DIVIDE GIVEN BEAM SYSTEM UP INTO A NUMBER OF SIMILAR SYSTEMS EACH HAVING ONE SPAN LOADED WITH A PARTICULAR TYPE OF LOAD. TO FIND THE B.M. AT ANY SUPPORT DUE TO ANY ONE OF THESE LOADS, EVALUATE THE FOLLOWING FACTORS FOR THE PARTICULAR SUPPORT AND LOAD TYPE :—

LOAD FACTOR = F (= UNITY FOR DISTRIBUTED LOAD).
SUPPORT MOMENT COEFFICIENT = Q.
MOMENT MULTIPLIER = U (= UNITY FOR EQUAL SPANS).

B.M. AT SUPPORT = FQU × W × BASE SPAN.

NOTE RE TRIANGULARLY DISTRIBUTED LOADS: FOR FQ USE THE VALUE GIVEN IN BRACKETS IN COLUMN HEADED SUPPORT MOMENT COEFFICIENTS WHEN APEX IS AT L.H. SUPPORT. REVERSE SYSTEM WHEN APEX IS AT R.H.S.

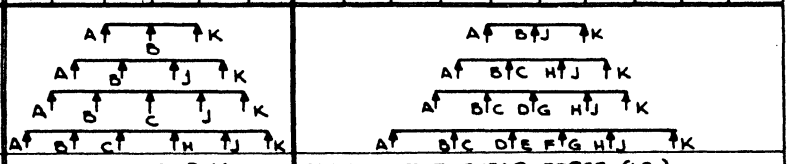
LOAD TYPE (W = TOTAL LOAD ON SPAN)	LOAD FACTOR = F	MAX. FREE B.M.
(a) 	1·00	·125 WL
(b)	1·25	·167 WL
(c) 	SEE NOTE	·128 WL
(d) 	1·50	·250 WL
(e) 	1·33	·167 WL

Nº OF SPANS	LOADED SPAN	EQUAL SPANS SUPPT. MT. COEFFTS Q_A	Q_B	Q_C	UNEQUAL SPANS. MOMENT MULTIPLIERS = U	
2.		-	−·063 (−·058)	-	$U_B = \frac{2}{1+k_1}$	-
		-	−·063 (−·067)	-	$U_B = \frac{2k_1^2}{1+k_1}$	
	BOTH SPANS LOADED WITH IDENTICAL LOAD.	-	−·125 (−·125)	-	SEE NOTE BELOW FOR UNEQUAL SPANS SIMULTANEOUSLY LOADED.	
3.		-	−·067 (−·056)	+·017 (+·016)	$U_B = ·5yU_C$ $U_C = 3k_1^2 H$	$x = k_1+1$ $y = k_2+1$ $H = \frac{5}{4xy-1}$
		-	−·050 (−·056)	−·050 (−·044)	$U_B = H(y+k_2)$ $U_C = H(x+k_1)$	
		-	+·017 (+·018)	−·067 (−·071)	$U_B = 3k_2^2 H$ $U_C = ·5 \times U_B$	
	ALL SPANS LOADED WITH IDENTICAL LOAD.	-	−·100 (−·092)	−·100 (−·101)	FOR TWO, THREE, OR FOUR UNEQUAL SPANS LOADED SIMULTANEOUSLY, DETERMINE B.Ms. FOR EACH SPAN LOADED SEPARATELY AND ADD.	
4.		−·067 (−·063)	+·018 (+·017)	−·004 (−·004)	$U_A = \frac{·133}{x}(14 + k_1H_1z)$ $U_B = zH_1$ $U_C = 2k_2H_1$	$x = k_1+1$ $y = k_1+k_2$ $z = k_2+k_3$ $H_1 = 14k_1Y$ $H_2 = 4·67k_1^2Y(x+1)$ $H_3 = 4·67k_2^2Y(k_3+z)$ $H_4 = 14k_3^2Y$
		−·049 (−·055)	−·054 (−·048)	+·013 (+·012)	$U_A = \frac{·543}{x}k_1(4·67k_1-U_B)$ $U_B = zH_2$ $U_C = 2k_2H_2$	
		+·013 (+·015)	−·054 (−·060)	−·049 (−·043)	$U_A = 2k_1H_3$ $U_B = xH_3$ $U_C = \frac{·543}{z}k_2(4·67k_2-U_B)$	
		−·004 (−·005)	+·018 (+·019)	−·067 (−·072)	$U_A = 2k_1H_4k_2$ $U_B = xH_4k_2$ $U_C = ·133H_4(4xy-k_1^2)$	
	ALL SPANS LOADED WITH IDENTICAL LOAD.	−·107 (−·107)	−·071 (−·072)	−·107 (−·107)	SEE NOTE ABOVE	$Y = \frac{1}{4xyz - k_1^2z - k_2^2x}$

CONTINUOUS BEAMS (EQUAL SPANS)
ADJUSTMENT DUE TO MOMENTS AT END SUPPORTS. TABLE Nº 10A.

-	Nº OF SPANS	BENDING MOMENT. A	B	C	H	J	K	SHEAR FORCE. A	B	C	D	E	F	G	H	J	K
BENDING MOMENT APPLIED AT A ONLY.	2	−1·00	+·250	-	-	-	NIL	+·104	−·104	-	-	-	-	-	-	−·021	+·021
	3	−1·00	+·267	-	-	−·067	NIL	+·106	−·106	−·028	-	-	-	-	+·028	+·006	−·006
	4	−1·00	+·268	−·071	-	+·018	NIL	+·106	−·106	−·028	+·028	-	-	+·007	−·007	−·002	+·002
	5	−1·00	+·268	−·072	+·019	−·005	NIL	+·106	−·106	−·028	+·028	+·008	−·008	−·002	+·002	+·0004	−·0004
EQUAL B.M. APPLIED AT A AND K	2	−1·00	+·500	-	-	-	−1·00	+·125	−·125	-	-	-	-	-	-	−·125	+·125
	3	−1·00	+·200	-	-	+·200	−1·00	+·100	−·100	NIL	-	-	-	-	NIL	−·100	+·100
	4	−1·00	+·286	−·142	-	+·286	−1·00	+·107	−·107	−·036	+·036	-	-	+·033	−·036	−·107	+·107
	5	−1·00	+·263	−·053	−·053	+·263	−1·00	+·105	−·105	−·026	+·026	NIL	NIL	+·026	−·026	−·105	+·105

KEY.



NOTES

ADJUSTMENT TO B.M. = (COEFFICIENT) × (APPLIED BENDING MOMENT)

ADJUSTMENT TO SHEAR FORCE (LB.) = COEFFICIENT × (APPLIED B.M. IN INCH-LB. / SPAN IN FEET)

SLABS SPANNING IN ONE DIRECTION TABLE Nº 10B.

BENDING MOMENT COEFFICIENTS (APPROX.) K.
(SEE TABLES Nºs 8 AND 10 FOR LESS APPROX. COEFF'S)

APPROXIMATELY EQUAL SPANS (L)
UNIFORMLY DISTRIBUTED LOAD
ω = DEAD + LIVE LOAD (LB/FT.²)

$$B.M. = K\omega L^2$$

FREELY SUPPORTED SPANS:– $K = \frac{1}{8}$ FOR MID-SPAN B.M.

IF CONSTRUCTED MONOLITHICALLY WITH SUPPORTS, PROVIDE RESISTANCE AT SUPPORT $= -\frac{\omega L^2}{24}$

CONTINUOUS SPANS:–

SECTION	END SPAN AT "FREE" SUPPORT.	END SPAN AT MID-SPAN	PEN-ULTIMATE SUPPORT	INTERIOR SPAN AT MID-SPAN	INTERIOR SUPPORT	NOTES
WITHOUT HAUNCHES	$-\frac{1}{24}$	$+\frac{1}{10}$ ×	$-\frac{1}{10}$ ×	$+\frac{1}{12}$ ×	$-\frac{1}{12}$ ×	X = CONFORMS TO LONDON BY-LAWS
WITH HAUNCHES (SEE SKETCH BELOW)	$-\frac{1}{18}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$+\frac{1}{16}$	$-\frac{1}{10}$	SEE SKETCH BELOW

NOTES:–

1. TABULATED COEFFICIENTS CAN BE ADOPTED IF LENGTHS OF ADJACENT SPANS DO NOT DIFFER BY MORE THAN 15 PER CENT OF THE LENGTH.
2. FOR ANY SYMMETRICAL LOAD:–

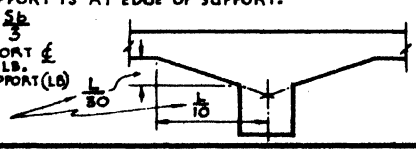
   B.M. COEFFICIENT = 8K × [COEFFICIENT FOR MIDSPAN B.M. FOR LOAD ON FREE SPAN BEAM. (SEE TABLE Nº 7)]

3. CRITICAL DESIGN SECTION FOR SUPPORT IS "AT EDGE OF SUPPORT."

   B.M. AT EDGE OF SUPPORT $= M_S - \frac{Sb}{3}$

   M_S = THEORETICAL B.M. AT SUPPORT ℄ (e.g. PER TABLE Nº 8) – IN. LB.
   S = SHEAR FORCE AT EDGE OF SUPPORT (LB)
   b = WIDTH OF SUPPORT (IN.)

4. MINIMUM PROPORTIONS OF HAUNCH



CONTINUOUS BEAMS — TABLE No 10.

BENDING MOMENT COEFFICIENTS (k) B.M. = WLk

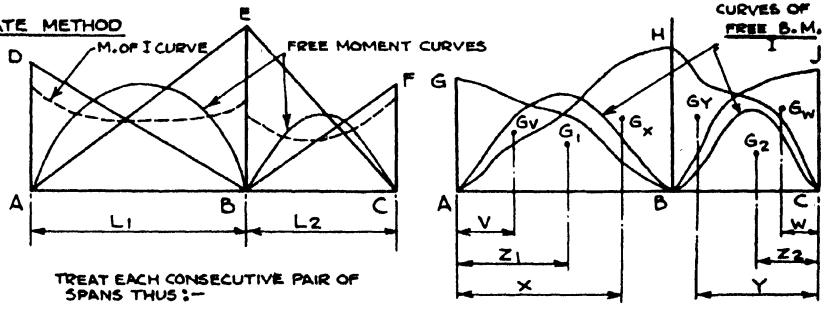
W = TOTAL UNIFORMLY DISTRIBUTED LOAD.

SECTION	DEAD LOAD	LIVE LOAD	TOTAL LOAD (APPROX)	TOTAL LOAD : RATIO OF LIVE TO DEAD LOAD.							
				·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0
NEAR MIDDLE OF END SPAN. (POSITIVE B.M)	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{10}$	·089	·092	·093	·094	·095	·096	·096	·097
PENULTIMATE. SUPPORT (NEGATIVE B.M.)	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{10}$	·104	·106	·107	·107	·108	·108	·109	·109
MIDSPAN INTERIOR SPANS. (POSITIVE B.M.)	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{12}$	·056	·063	·067	·069	·072	·073	·074	·075
INTERIOR. SUPPORTS. (NEGATIVE B.M.)	$\frac{1}{12}$	$\frac{1}{9}$,	$\frac{1}{10}\left(\text{OR } \frac{1}{12}\right)$	·093	·097	·100	·102	·103	·104	·105	·105

COEFFICIENTS APPLY TO APPROXIMATELY EQUAL SPANS i.e. VARIATION NOT EXCEEDING 15 PER CENT.
SEE CHAPTER IV FOR NEGATIVE B.M. AT MIDSPAN.

VARIABLE MOMENT OF INERTIA.

ACCURATE METHOD



TREAT EACH CONSECUTIVE PAIR OF SPANS THUS :—

ON THE SPANS DRAWN TO SCALE, CONSTRUCT THE FREE MOMENT CURVES AND THE MOMENT OF INERTIA CURVES. DIVIDE THE ORDINATES OF THE FORMER BY THOSE OF THE LATTER TO ENABLE THE CURVES OF $\frac{\text{FREE B.M}}{I}$ TO BE DRAWN.

FIND A_1 = AREA UNDER $\frac{\text{FREE B.M}}{I}$ CURVE FOR SPAN L_1 AND POSITION OF CENTROID G_1

AND A_2 = " " " " " " L_2 " " " " G_2

SET UP AD, BE, AND CF TO A SUITABLE SCALE TO REPRESENT ANY ASSUMED VALUES OF THE MOMENTS AT A, B, & C RESPECTIVELY. CONNECT DB, AE, EC, & BF.

LET $M_A = k_A AD$ $\quad M_B = k_B BE$ $\quad M_C = k_C CF$.

DIVIDE THE ORDINATES OF DBF AND AEC BY THE ORDINATES OF THE M. OF I. CURVE. TO GIVE THE CURVES AHC AND GBJ.

FIND A_V = AREA UNDER CURVE GB AND POSITION OF CENTROID G_V

A_X = " " " AH " " " " G_X

A_Y = " " " HC " " " " G_Y

AND A_W = " " " BJ " " " " G_W

SUBSTITUTE IN $$(A_1 Z_1 + k_A V A_V + k_B X A_X)\frac{1}{L_1} = -(A_2 Z_2 + k_C A_W W + k_B A_Y Y)\frac{1}{L_2}$$

UNKNOWNS ARE k_A, k_B, & k_C AND REQUISITE NUMBER OF EQUATIONS FOLLOWS FROM CONSECUTIVE PAIRS OF SPANS AND END SUPPORT CONDITIONS.

APPROXIMATE METHOD.

CALCULATE B.Ms. FOR CONSTANT MOMENT OF INERTIA AND INCREASE OR DECREASE BY THE APPROPRIATE PERCENTAGE TABULATED.

I_S = M. OF I. AT SUPPORT.
I_C = M. OF I. AT MIDSPAN.

RATIO OF $I_S : I_C$		·25	·50	·75	1·25	1·50
BOTH ENDS FIXED	MIDSPAN	+55%	+26%	+7%	−6%	−13%
	SUPPORT	−28%	−15%	−5%	+3%	+4%
ONE FIXED ONE FREE	MIDSPAN	+27%	+13%	+6%	−2½%	−10%
	SUPPORT	−40%	−20%	−8%	+5%	+13%

CONTINUOUS BEAMS		MOMENT DISTRIBUTION METHOD.	TABLE No. 11A
SUPPORT	APPLICATION	FORMULA FOR B.M. AT SPECIFIED SUPPORT.	
END SUPPORT A	TWO OR MORE SPANS	FIXED AT A	$-F_{AB} - \frac{D_{BA}}{2}\left(F_{BA} - F_{BC}\right)$
END SUPPORT C	TWO SPANS ONLY	FIXED AT C	$-F_{CB} - \frac{D_{BC}}{2}\left(F_{BC} - F_{BA}\right)$
CENTRAL SUPPORT B	TWO SPANS ONLY	FREE AT A & C	$-\frac{D_{BC}}{2}\left(2F_{BA} + F_{AB}\right) - \frac{D_{BA}}{2}\left(2F_{BC} + F_{CB}\right)$
		FIXED AT A FREE AT C	$-D_{BC}F_{BA} - \frac{D_{BA}}{2}\left(2F_{BC} + F_{CB}\right)$
		FIXED AT A & C	$-D_{BC}F_{BA} - D_{BA}F_{BC}$
PENULTIMATE SUPPORT B.	THREE OR MORE SPANS	FREE AT A	$-\frac{D_{BC}}{2}\left(2F_{BA} + F_{AB}\right) - \frac{D_{BA}}{2}\left[2F_{BC} - D_{CB}\left(F_{CD} - F_{CB}\right)\right]$
		FIXED AT A	$-D_{BC}F_{BA} - \frac{D_{BA}}{2}\left[2F_{BC} - D_{CB}\left(F_{CD} - F_{CB}\right)\right]$
PENULTIMATE SUPPORT C.	THREE SPANS ONLY	FREE AT D	$-\frac{D_{CB}}{2}\left(2F_{CB} + F_{DC}\right) - \frac{D_{CD}}{2}\left[2F_{CB} - D_{BC}\left(F_{BA} - F_{BC}\right)\right]$
		FIXED AT D	$-D_{CB}F_{CD} - \frac{D_{CD}}{2}\left[2F_{CB} - D_{BC}\left(F_{BA} - F_{BC}\right)\right]$
END SUPPORT D.	THREE SPANS ONLY	FIXED AT D	$-F_{DC} - \frac{D_{CD}}{2}\left(F_{CD} - F_{CB}\right)$
PENULTIMATE SUPPORT D.	FOUR SPANS ONLY	FREE AT E	$-\frac{D_{DC}}{2}\left(2F_{DC} + F_{ED}\right) - \frac{D_{DE}}{2}\left[2F_{DC} - D_{CD}\left(F_{CB} - F_{CD}\right)\right]$
		FIXED AT E	$-D_{DC}F_{DE} - \frac{D_{DE}}{2}\left[2F_{DC} - D_{CD}\left(F_{CB} - F_{CD}\right)\right]$
END SUPPORT E.	FOUR SPANS ONLY	FIXED AT E	$-F_{ED} - \frac{D_{DE}}{2}\left(F_{DE} - F_{DC}\right)$
INTERIOR SUPPORT C.	FOUR OR MORE SPANS	–	$-\frac{D_{CD}}{2}\left[2F_{CB} - D_{BC}\left(F_{BA} - F_{BC}\right)\right] - \frac{D_{CB}}{2}\left[2F_{CD} - D_{DC}\left(F_{DE} - F_{DC}\right)\right]$
PENULTIMATE SUPPORT B.	TWO OR MORE SPANS	WITH EXTERNALLY APPLIED NEG. B.M. M_P AT A.	$+M_P \cdot \frac{D_{BC}}{2}$ NOTE:- IF M_P IS POSITIVE, B.M. AT B $= -M_P \cdot \frac{D_{BC}}{2}$

SUPPORT REFERENCES:- A B C A B C D A B C D E

SYMBOLS:- L_{AB} = SPAN A-B, ETC. I_{AB}, ETC. = MOMENT OF INERTIA OF SECTION IN SPAN A·B, ETC.

D_{BA}, D_{BC}, ETC. = DISTRIBUTION FACTOR AT SUPPORT B FOR SPANS AB AND BC, ETC. $D_{BA} + D_{BC} = 1$.

F_{AB}, F_{BA}, ETC. = SPAN-LOAD FACTOR AT A & B FOR LOAD ON SPAN A·B ETC.
= NUMERICAL VALUE OF B.M. AT A & B DUE TO LOAD ON A·B ONLY ASSUMING FIXITY AT BOTH ENDS OF SPAN.
= $L_{AB} \cdot C_{AB}$, $L_{AB} \cdot C_{BA}$ WHERE C_{AB}, ETC. = LOAD FACTOR (TABLE 7D)

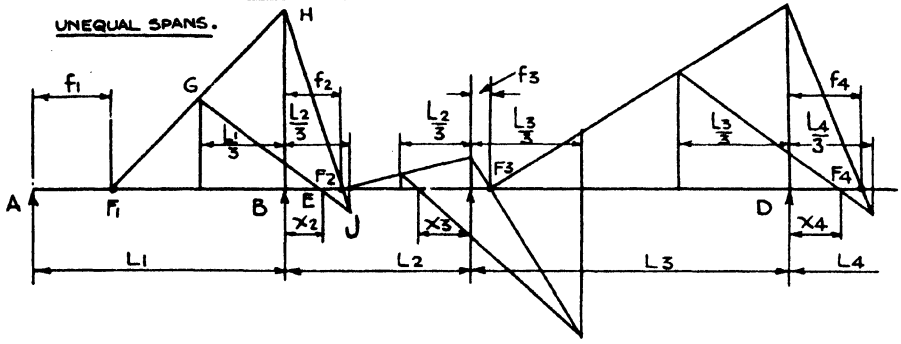
$$D_{BA} = \frac{1}{1 + \frac{I_{AB} L_{BC}}{I_{BC} L_{AB}}}$$

CONTINUOUS BEAMS — CONSTANT MOM. OF I. — TABLE Nº 11.

"FIXED POINTS" — GRAPHICAL SOLUTION.

(1) TO DETERMINE FIXED POINTS

UNEQUAL SPANS.



ON THE GIVEN BEAM SYSTEM A, B, C, D, ETC, DRAWN TO SCALE, PLOT THE POSITION OF THE FIXED POINT F_1 IN LEFT HAND PORTION OF SPAN AB.

WHEN END A IS FREELY SUPPORTED: f_1 = ZERO.

" " A " FIXED: $f_1 = \cdot 333\, L_1$

SET OFF $BE = X_2 = \cdot 333(L_1 - L_2)$; $X_3 = \cdot 333(L_2 - L_3)$; $X_4 = \cdot 333(L_3 - L_4)$, ETC.

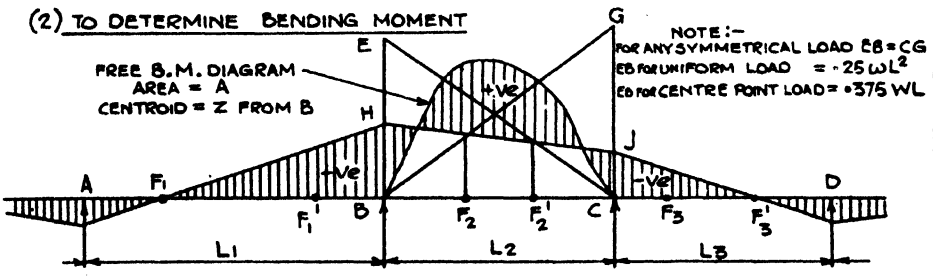
SET UP VERTICALS AT THE THIRD POINTS OF EACH SPAN. AT ANY CONVENIENT ANGLE DRAW GEJ. JOIN F_1G AND PRODUCE TO MEET SUPPORT LINE AT H. JOIN HJ TO INTERSECT BEAM LINE IN F_2 WHICH IS FIXED POINT FOR LEFT HAND PORTION OF SPAN L_2.

REPEAT SIMILAR CONSTRUCTION TO FIND F_3, F_4, ETC. TO THE END OF THE SYSTEM, AND, COMMENCING AT THE EXTREME RIGHT HAND SUPPORT, REPEAT THE WHOLE PROCESS WORKING TOWARDS THE LEFT, THUS ESTABLISHING THE FIXED POINTS IN THE RIGHT HAND PORTION OF EACH SPAN.

EQUAL SPANS. (i.e. $L_1 = L_2 = L_3$ ETC. = L)

DISTANCE OF FIXED POINT:	f_1	f_2	f_3	f_4 ETC.
FIXED AT A:	·3333 L	·2222 L	·2121 L	·2113 L
FREELY SUPPORTED AT A:	0	·2000 L	·2105 L	·2113 L

(2) TO DETERMINE BENDING MOMENT



NOTE:—
FOR ANY SYMMETRICAL LOAD EB = CG
EB FOR UNIFORM LOAD = $\cdot 25\, wL^2$
EB FOR CENTRE POINT LOAD = $\cdot 375\, WL$

DRAW OUT THE BEAM SYSTEM TO SCALE AND PLOT THE POSITIONS OF THE FIXED POINTS, F_1 F_2 F_3 ETC. IN THE LEFT HAND PORTION AND F_1' F_2' F_3' ETC. IN THE RIGHT HAND PORTION OF EACH SPAN. SET UP $BE = \frac{6Az}{L_2^2}$ AND $CG = \frac{6A(L_2 - z)}{L_2^2}$ (SEE NOTE ABOVE). JOIN BG & CE.

DRAW HJ THROUGH INTERSECTIONS OF VERTICALS FROM F_2 AND F_2' WITH BG AND CE.

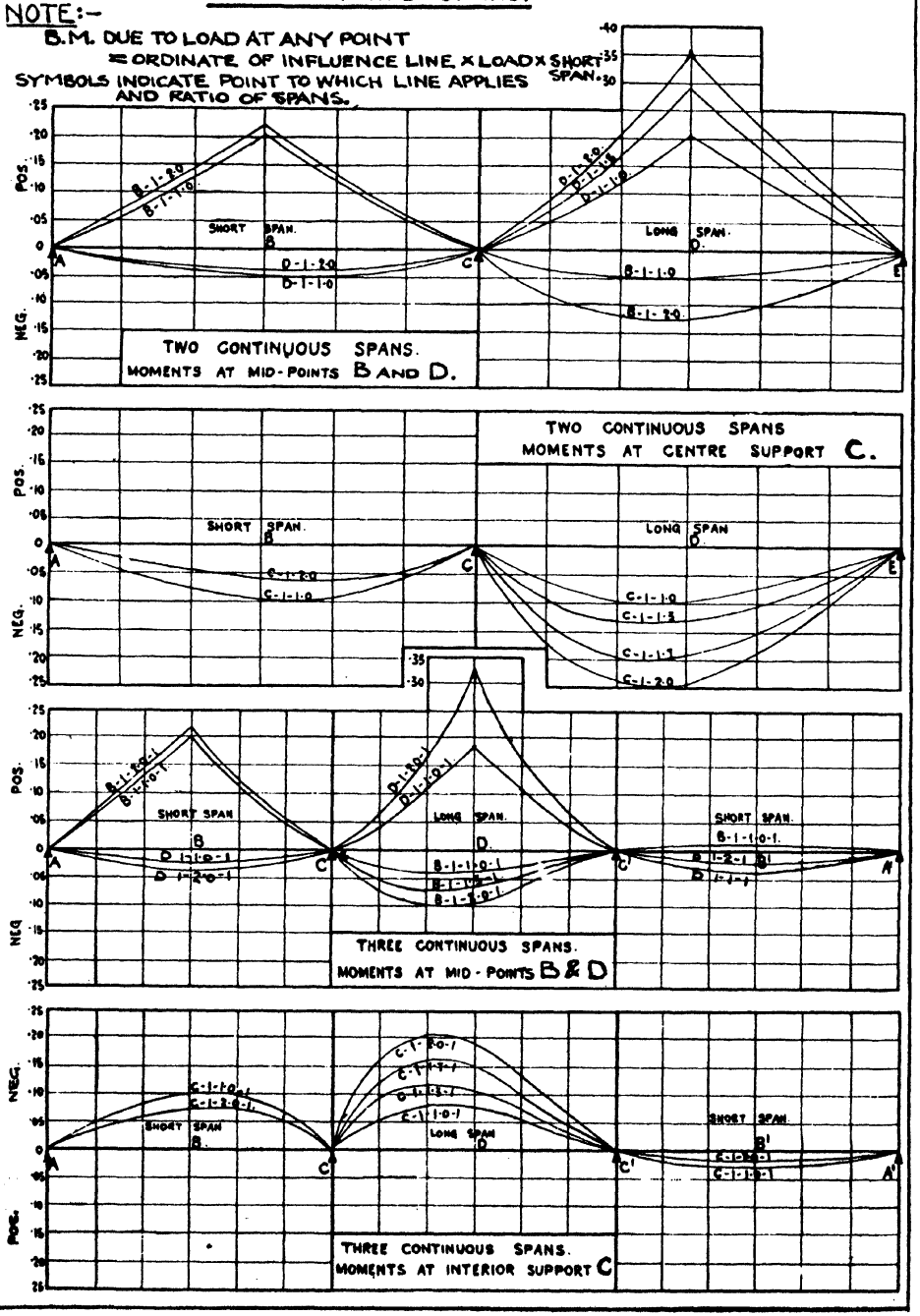
TO COMPLETE B.M. DIAGRAM FOR LOAD ON SPAN BC, CONNECT H TO F_1', J TO F_3', ETC.

REPEAT FOR OTHER LOADED SPANS AND COMBINE DIAGRAMS TO GIVE TOTAL MOMENTS.

INFLUENCE LINES FOR UNIT ROLLING LOAD ON TWO & THREE SPANS.

TABLE No. 12A

NOTE:—
B.M. DUE TO LOAD AT ANY POINT
= ORDINATE OF INFLUENCE LINE × LOAD × SHORT SPAN
SYMBOLS INDICATE POINT TO WHICH LINE APPLIES AND RATIO OF SPANS.



NOTES.

INFLUENCE LINES — TABLE Nº 12.

DATA FOR CONSTRUCTING INFLUENCE LINES FOR BENDING MOMENTS PRODUCED AT SUPPORTS AND MID-SPAN BY A SINGLE UNIT POINT LOAD MOVING ALONG A BEAM OF TWO OR THREE SPANS.

B.M. DUE TO ANY LOAD AT ANY POINT $= hWL$

h = ORDINATE OF APPROPRIATE INFLUENCE LINE AT POINT CONSIDERED.

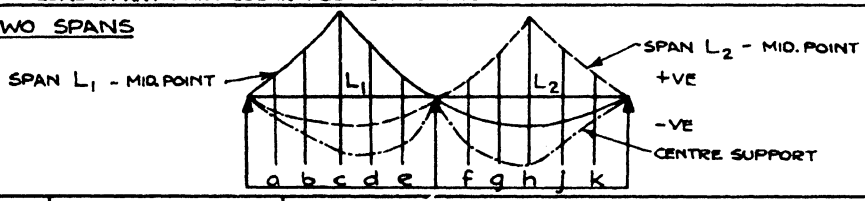
W = TOTAL LOAD.

L = SPAN (WHEN SPANS ARE UNEQUAL L = END SPAN = L_1)

ORDINATES FOR INTERMEDIATE SPAN RATIOS CAN BE INTERPOLATED.

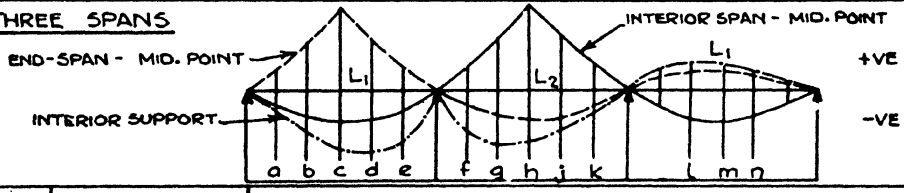
REFERENCE TO THE DIAGRAMS WILL INDICATE WHETHER THE MOMENT DUE TO THE LOAD IN ANY PARTICULAR POSITION IS POSITIVE OR NEGATIVE.

TWO SPANS



		SPAN RATIO $L_1 : L_2$	ORDINATES a	b	c	d	e	f	g	h	j	k
SHORT SPAN	MID. POINT	1 : 1	·063	·130	·203	·121	·052	·032	·046	·047	·037	·020
		1 : 1½	·067	·137	·213	·130	·058	·058	·083	·084	·067	·037
		1 : 2	·070	·142	·219	·136	·062	·085	·124	·125	·099	·054
LONG SPAN	CENTRE SUPPORT	1 : 1	·041	·074	·094	·093	·064	·064	·093	·094	·074	·041
		1 : 1½	·032	·059	·075	·074	·051	·115	·167	·169	·133	·073
		1 : 2	·027	·049	·063	·062	·042	·170	·247	·250	·198	·108
	MID. POINT	1 : 1	·020	·037	·047	·046	·032	·052	·121	·203	·130	·063
		1 : 1½	·016	·030	·038	·037	·025	·067	·167	·291	·183	·088
		1 : 2	·014	·025	·031	·031	·021	·082	·210	·375	·235	·113

THREE SPANS



		SPAN RATIO $L_1 : L_2 : L_1$	ORDINATES a	b	c	d	e	f	g	h	j	k	l	m	n
END SPAN	MID. POINT	1 : 1 : 1	·062	·127	·200	·117	·050	·029	·040	·038	·027	·013	·012	·013	·010
		1 : 1½ : 1	·066	·134	·209	·126	·056	·051	·070	·065	·046	·021	·012	·012	·010
		1 : 2 : 1	·068	·139	·215	·132	·060	·075	·102	·094	·065	·029	·012	·012	·009
INTERIOR	SUPPORT	1 : 1 : 1	·043	·079	·100	·099	·068	·057	·079	·075	·054	·026	·025	·025	·020
		1 : 1½ : 1	·036	·065	·082	·081	·056	·102	·139	·130	·092	·042	·024	·025	·020
		1 : 2 : 1	·030	·056	·070	·069	·048	·151	·204	·188	·129	·058	·023	·023	·019
INT.ᴿ SPAN	MID. POINT	1 : 1 : 1	·016	·030	·038	·037	·025	·042	·100	·175	·100	·042	·037	·038	·030
		1 : 1½ : 1	·013	·023	·029	·028	·020	·053	·135	·245	·135	·053	·028	·029	·023
		1 : 2 : 1	·010	·019	·023	·023	·016	·063	·167	·313	·167	·063	·023	·023	·019

EXAMPLES OF USE OF TABLE No. 13.

(*a*) To determine the bending moment at the penultimate (left-hand end) support of a system of four spans (constant moment of inertia, freely supported on end supports) subject to a central point load of 10 tons on the first and third spans (counting from left-hand end); end spans = 20 ft.; middle spans = 30 ft.; hence span-ratio is $1 : 1\frac{1}{2} : 1\frac{1}{2} : 1$.

		ft.-lb.	
With load on 1st span: (ordinate *c*)	B.M. = $-(0{\cdot}082 \times 10 \times 2{,}240 \times 20)$	= 36,800	(neg.)
With load on 3rd span: (ordinate *m*)	B.M. = $+(0{\cdot}035 \times 10 \times 2{,}240 \times 20)$	= 15,700	(pos.)
	Net B.M. at penultimate support	= 21,100	(neg.)

(*b*) For the determination of the moments at any of the critical sections in a beam system due to a train of point loads in any given position, the procedure is as follows:

Draw the beam system to some convenient linear scale.

With the ordinates given on *Tables* Nos. 12 and 13 construct the influence line (for a unit load) for the section to be considered, selecting a convenient scale for the bending moment.

Plot on this diagram the position of the train of loads.

Tabulate the value of (ordinate × load) for each point load.

Algebraically add the products (ordinate × load) to give the resultant bending moment at the section considered.

NOTES.

The influence line marked "*Penultimate Support*" on the four-span diagram on *Table* No. 13 refers to the second support from the left-hand side.

See succeeding pages for influence line curves for four and five spans.

INFLUENCE LINES — TABLE Nº 13.

DATA FOR CONSTRUCTING INFLUENCE LINES FOR BENDING MOMENTS PRODUCED AT SUPPORTS AND MID-SPAN BY A SINGLE UNIT POINT LOAD MOVING ALONG A CONTINUOUS BEAM OF FOUR OR MORE SPANS.

B.M. DUE TO ANY LOAD AT ANY POINT = hWL.

h = ORDINATE OF APPROPRIATE INFLUENCE LINE AT POINT CONSIDERED.

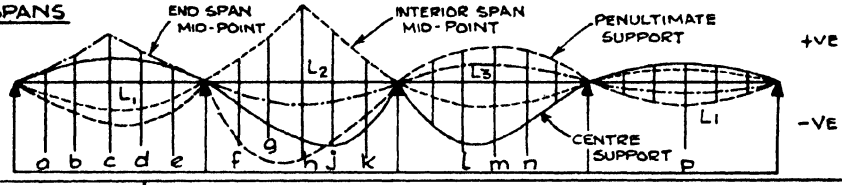
W = TOTAL LOAD.

L = SPAN. (WHEN SPANS ARE UNEQUAL L = END SPAN = L_1)

ORDINATES FOR INTERMEDIATE SPAN RATIOS CAN BE INTERPOLATED.

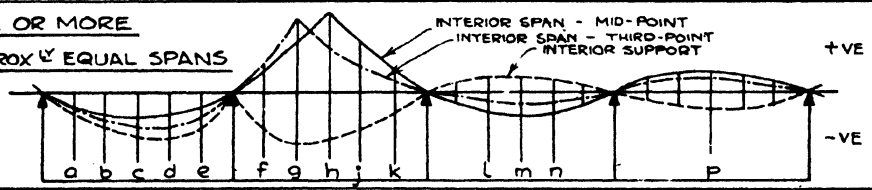
REFERENCE TO THE DIAGRAMS WILL INDICATE WHETHER THE MOMENT DUE TO THE LOAD IN ANY PARTICULAR POSITION IS POSITIVE OR NEGATIVE.

FOUR SPANS



		SPAN RATIO $L_1 \cdot L_2 \cdot L_2 \cdot L_1$	ORDINATES a	b	c	d	e	f	g	h	j	k	l	m	n	p
END SPAN	MID-POINT	1 · 1 · 1 · 1	·062	·127	·200	·117	·049	·028	·039	·037	·027	·013	·011	·010	·007	·003
		1 · 1½ · 1½ · 1	·066	·135	·209	·126	·056	·052	·071	·067	·048	·023	·019	·017	·012	·003
		1 · 2 · 2 · 1	·069	·140	·216	·133	·060	·077	·106	·100	·072	·034	·027	·025	·017	·003
PENULTIMATE	SUPPORT	1 · 1 · 1 · 1	·043	·079	·100	·099	·068	·057	·078	·074	·053	·025	·021	·020	·015	·007
		1 · 1½ · 1½ · 1	·035	·065	·082	·081	·055	·103	·142	·134	·096	·046	·037	·035	·025	·007
		1 · 2 · 2 · 1	·030	·054	·069	·068	·047	·155	·213	·200	·143	·068	·055	·050	·035	·006
INTR. SPAN	MID-POINT	1 · 1 · 1 · 1	·016	·029	·037	·036	·025	·041	·099	·173	·098	·040	·032	·030	·022	·010
		1 · 1½ · 1½ · 1	·013	·024	·030	·029	·020	·054	·138	·250	·140	·057	·043	·041	·029	·007
		1 · 2 · 2 · 1	·011	·020	·025	·025	·017	·066	·175	·325	·180	·073	·054	·050	·035	·006
CENTRE	SUPPORT	1 · 1 · 1 · 1	·012	·021	·027	·027	·018	·028	·058	·080	·085	·061	·085	·080	·058	·026
		1 · 1½ · 1½ · 1	·010	·017	·022	·022	·015	·038	·082	·116	·124	·091	·124	·116	·082	·022
		1 · 2 · 2 · 1	·008	·015	·019	·019	·013	·046	·104	·150	·163	·121	·163	·150	·104	·019

FIVE OR MORE APPROX. EQUAL SPANS

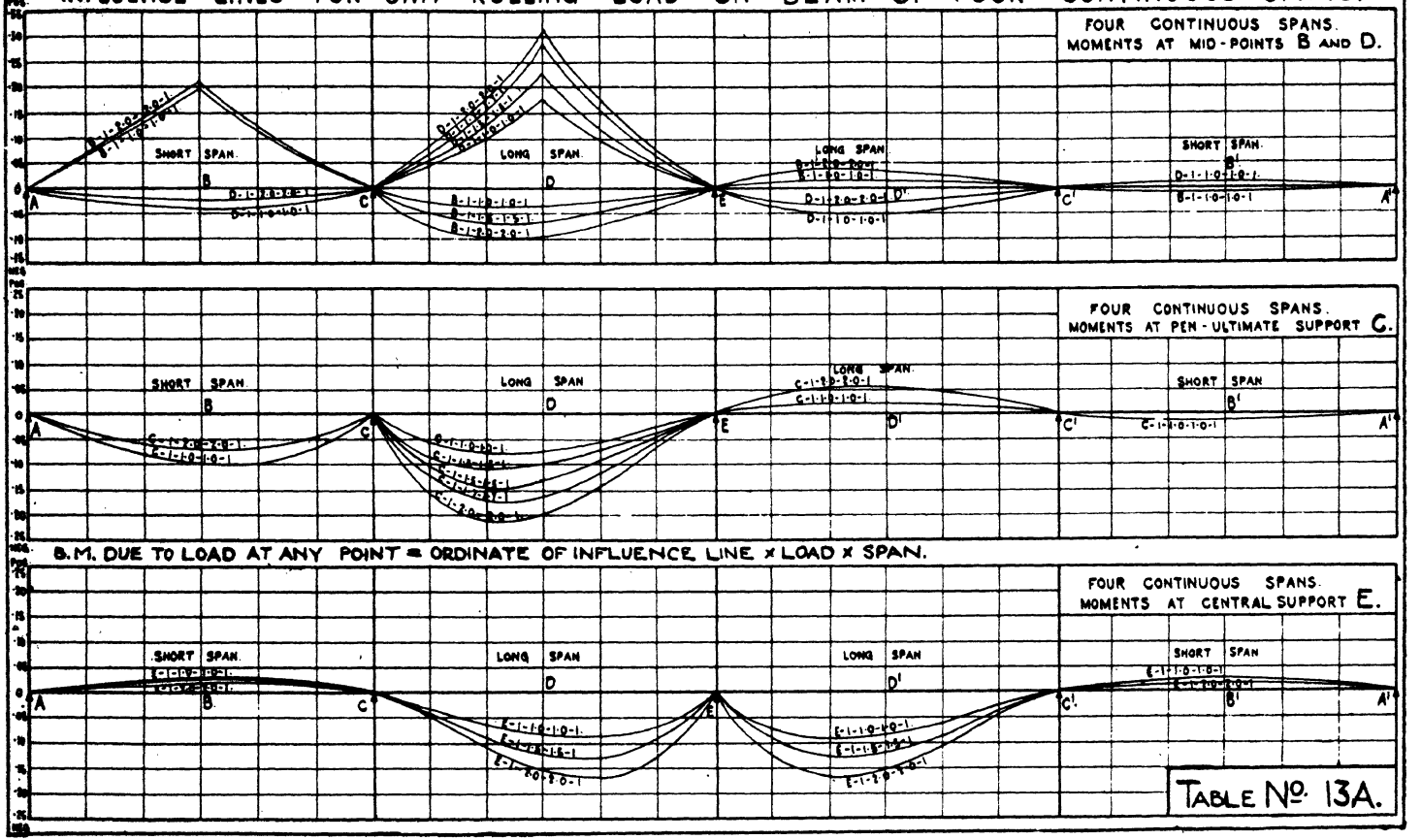


SECTION	ORDINATES a	b	c	d	e	f	g	h	j	k	l	m	n	p
INTR. SPAN: MID-POINT	·010	·021	·029	·031	·022	·039	·096	·171	·096	·039	·031	·029	·021	·008
INTR. SPAN: THIRD-POINT	·016	·033	·046	·049	·035	·062	·147	·087	·045	·017	·013	·012	·009	·003
INTERIOR SUPPORT	·027	·057	·079	·084	·061	·061	·084	·079	·057	·027	·023	·021	·015	·006

END SPAN: MID-POINT
PENULTIMATE SUPPORT

ORDINATES FOR END SPAN AND PENULTIMATE SUPPORT OF FOUR EQUAL SPANS SYSTEM CAN BE USED WITH REASONABLE ACCURACY.

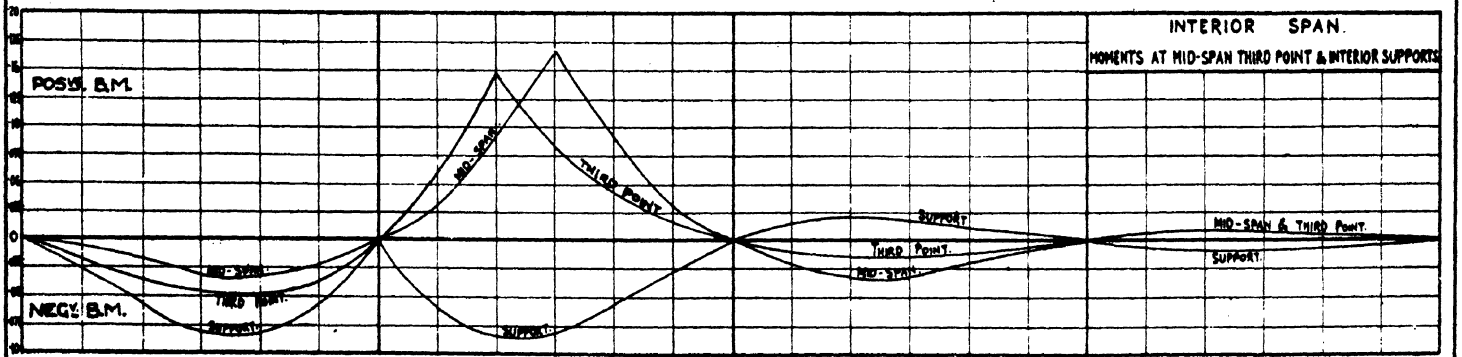
INFLUENCE LINES FOR UNIT ROLLING LOAD ON BEAM OF FOUR CONTINUOUS SPANS.



B.M. DUE TO LOAD AT ANY POINT = ORDINATE OF INFLUENCE LINE × LOAD × SPAN.

TABLE No. 13A.

INFLUENCE LINES FOR UNIT ROLLING LOAD ON FIVE OR MORE CONTINUOUS EQUAL SPANS



B.M. DUE TO LOAD AT ANY POINT = ORDINATE OF INFLUENCE LINE × LOAD × SPAN.

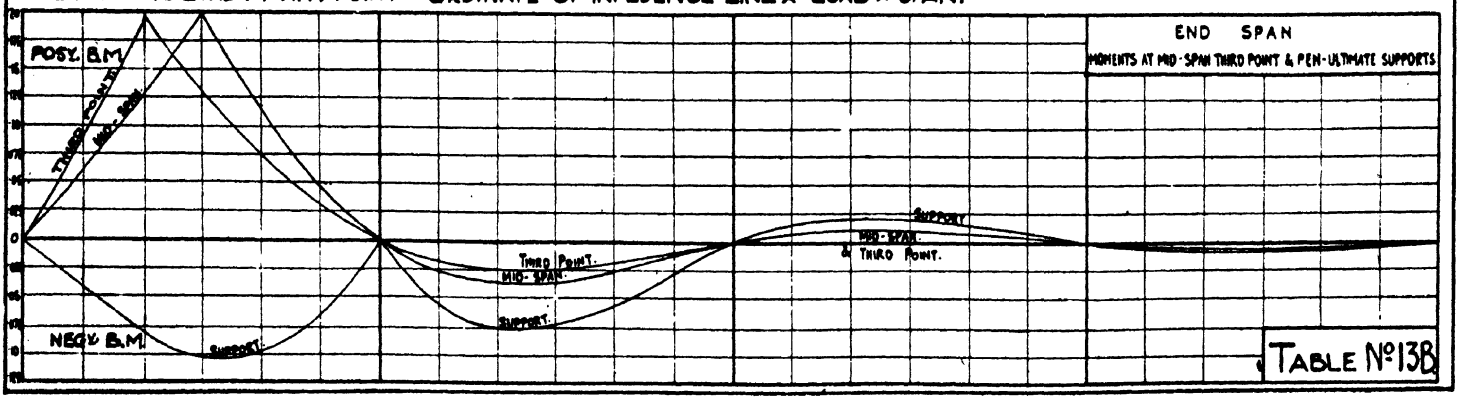


TABLE No. 13B

EXAMPLES OF USE OF TABLES Nos. 14, 14A, AND 14C.

(*a*) Determine the bending moments on a rectangular slab *ABCD* freely supported on all four sides (corners not held down) and subject to an inclusive load of 200 lb. per square foot, the lengths of sides being $L_B = AB = CD = 10$ ft. and $L_L = AD = BC = 12$ ft. 6 in.

Ratio of sides $= k = \frac{12{\cdot}5}{10{\cdot}0} = 1{\cdot}25$.

From *Table* No. 14, Grashof and Rankine coefficients : $K_B = 0{\cdot}71$; $K_L = 0{\cdot}29$.

Bending moments : Midspan, short span $= 0{\cdot}125 \times 0{\cdot}71 \times 200 \times 10^2 \times 12$
$= 21{,}300$ in. lb.

,, long span $= 0{\cdot}125 \times 0{\cdot}29 \times 200 \times 12{\cdot}5^2 \times 12$
$= 13{,}600$ in. lb.

Support bending moments = zero.

Since the long-span bending moment exceeds one-tenth (or one-fifth for Code) of short-span bending moment, the minimum requirements for distribution steel are satisfied.

(*b*) Find the bending moment in the panel in (*a*) if it is constructed monolithic with the supports, but discontinuous along all edges (i.e. nominal free support but corners held down).

From *Table* No. 14, using Pigeaud's coefficients : $K_B = 0{\cdot}45$; $K_L = 0{\cdot}17$.

Bending moments : Midspan, short span $= 0{\cdot}125 \times 0{\cdot}45 \times 200 \times 10^2 \times 12$
$= 13{,}500$ in. lb.

,, long span $= 0{\cdot}125 \times 0{\cdot}17 \times 200 \times 12{\cdot}5^2 \times 12$
$= 8{,}000$ in. lb.

(*c*) As (*b*) but using *Table* No. 14C : $\frac{L_B}{L_L} = \frac{10}{12{\cdot}5} = 0{\cdot}8$; $C_B = 0{\cdot}068$; $C_L = 0{\cdot}032$.

Bending moments :		in. lb.
short span—middle strip, at midspan	$= 0{\cdot}068 \times 200 \times 10^2 \times 12$	$= 16{,}300$
,, ,, at edge	$= -\frac{2}{3} \times 16{,}300$	$= -10{,}850$
column strip, at midspan	$= \frac{2}{3} \times 16{,}300$	$= 10{,}850$
,, ,, at edge	$= -\frac{2}{3} \times 10{,}850$	$= -7{,}230$
long span—middle strip, at midspan	$= 0{\cdot}032 \times 200 \times 12{\cdot}5^2 \times 12$	$= 12{,}000$
,, ,, at edge	$= -\frac{2}{3} \times 12{,}000$	$= -8{,}000$
column strip, at midspan	$= \frac{2}{3} \times 12{,}000$	$= 8{,}000$
,, ,, at edge	$= -\frac{2}{3} \times 8{,}000$	$= -5{,}333$

(*d*) Determine the bending moments if the panel in (*a*) is freely supported on side *CD* only and is continuous over the other three sides.

Table No. 14A applies (curves in bottom left-hand corner)

$AB = L_1 = 10$ ft. $AD = L_2 = 12$ ft. 6 in. $\frac{L_2}{L_1} = \frac{12{\cdot}5}{10} = 1{\cdot}25$.

Bending moments :		in. lb.
short span, midspan	$= 0{\cdot}057 \times 200 \times 10^2 \times 12$	$= 13{,}700$
,, ,, support	$= 0{\cdot}072 \times 200 \times 10^2 \times 12$	$= -17{,}300$
long span, midspan	$= 0{\cdot}012 \times 200 \times 12{\cdot}5^2 \times 12$	$= 4{,}510$
,, ,, support	$= 0{\cdot}013 \times 200 \times 12{\cdot}5^2 \times 12$	$= -4{,}890$

(*e*) Find the bending moments if the panel in (*a*) is continuous over all four sides and carries a load of 10,000 lb. concentrated on an area such that $A = 3$ ft. and $B = 1$ ft. (see lower part of *Table* No. 14). Thus

$$\frac{A}{L_L} = \frac{3{\cdot}75}{12{\cdot}5} = 0{\cdot}3\ ; \quad \frac{B}{L_B} = \frac{1{\cdot}0}{10{\cdot}0} = 0{\cdot}1\ ; \quad k = 1{\cdot}25.$$

From curves, $M_B = 2{\cdot}27$ and $M_L = 1{\cdot}68$.

Bending moments per foot width :		in. lb.
Short span, midspan	$= +0{\cdot}8 \times 2{\cdot}27 \times 10{,}000$	$= +18{,}200$
,, ,, support	$= -18{,}200$ in. lb.	
Long span, midspan	$= +0{\cdot}8 \times 1{\cdot}68 \times 10{,}000$	$= +13{,}400$
,, ,, support	$= -13{,}400$ in. lb.	

SLABS SPANNING IN TWO DIRECTIONS. RECTANGULAR PANELS. TABLE No 14

UNIFORMLY DISTRIBUTED LOAD OVER WHOLE AREA

SPAN RATIO	FREE SUPPORT ON FOUR SIDES (CORNERS NOT HELD DOWN) RANKINE & GRASHOF		CONTINUITY OR FREE SUPPORT ON FOUR SIDES (CORNERS HELD DOWN) PIGEAUD	
k	K_B	K_L	K_B	K_L
1·00	0·50	0·50	0·30	0·30
1·05	0·55	0·45	0·33	0·27
1·10	0·59	0·41	0·36	0·24
1·15	0·64	0·36	0·39	0·22
1·20	0·68	0·33	0·42	0·19
1·25	0·71	0·29	0·45	0·17
1·30	0·74	0·26	0·48	0·15
1·40	0·79	0·21	0·53	0·13
1·50	0·84	0·17	0·58	0·11
1·60	0·87	0·13	0·63	0·09
1·70	0·89	0·11	0·66	0·07
1·75	0·90	0·10	0·68	0·07
1·80	0·91	0·09	0·70	0·06
1·90	0·93	0·07	0·74	0·05
2·00	0·94	0·06	0·76	0·05
2·50	0·98	0·03	0·87	0·03
3·00	0·99	0·02	0·94	0·02

w = UNIFORMLY DISTRIBUTED LOAD (LB. PER SQ. FT.) $= \frac{W}{L_L L_B}$

L_B = ACTUAL SHORT SPAN (FT.) L_L = ACTUAL LONG SPAN (FT.)

$K = \frac{\text{LONGER EQUIVALENT SPAN}}{\text{SHORTER EQUIVALENT SPAN}}$ EQUIVALENT SPAN $= f \times$ ACTUAL SPAN

CONTINUOUS ONE END, UNSUPPORTED AT OTHER	f = 3·00
CONTINUOUS AT BOTH ENDS OF SPAN	= 0·67
FREE SUPPORT BOTH ENDS OF SPAN	= 1·00
FREE SUPPORT ONE END, CONTINUOUS AT OTHER	= 0·80

WHEN BOTH SPANS ARE IDENTICAL AS REGARDS END CONDITIONS:-

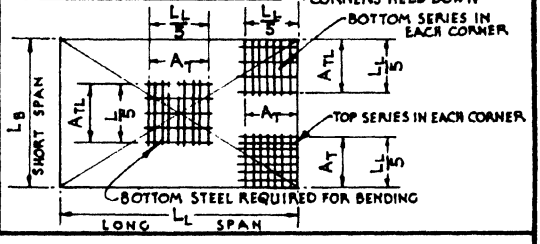
$K = \frac{\text{LONGER ACTUAL SPAN}}{\text{SHORTER ACTUAL SPAN}}$

B.Ms. FOR SAME CONDITIONS ON ALL FOUR SIDES.

	FREE	CONTINUOUS
SHORT SPAN: MIDSPAN B.M.	$+0{\cdot}125\,K_B w L_B^2$	$+0{\cdot}100\,K_B w L_B^2$
SUPPORT B.M	ZERO	$+0{\cdot}125\,K_B w L_B^2$
LONG SPAN: MIDSPAN B.M.	$+0{\cdot}125\,K_L w L_L^2$	$+0{\cdot}100\,K_L w L_L^2$
SUPPORT B.M.	ZERO	$-0{\cdot}125\,K_L w L_L^2$

CORNER REINFORCEMENT

GIVING CONDITIONS OF "CORNERS HELD DOWN"



PARTIALLY DISTRIBUTED AND POINT LOADS

(CURVES BASED ON PIGEAUD APPROX.)

FREELY SUPPORTED ON FOUR SIDES.

SHORT SPAN:

MIDSPAN B.M. = $+ M_B W$ IN.LB.

LONG SPAN:

MIDSPAN B.M. = $+ M_L W$ IN.LB.

CONTINUOUS OVER ONE OR MORE SUPPORTS

SHORT SPAN:

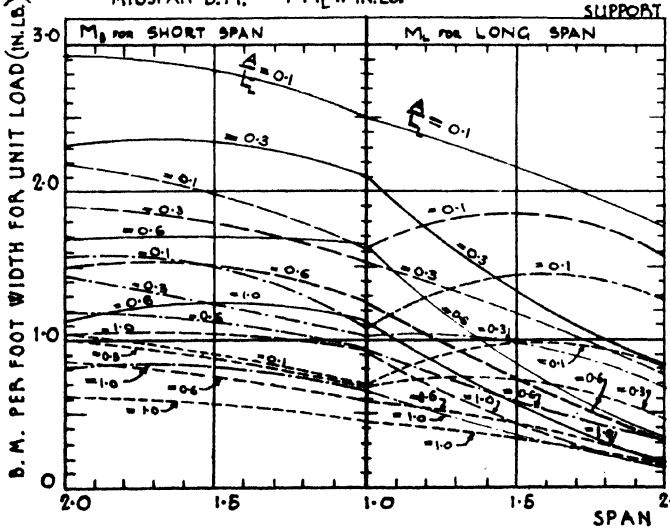
MIDSPAN B.M. = $+0{\cdot}8\,M_B W$ IN.LB.

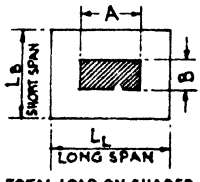
SUPPORT B.M. = $-0{\cdot}8\,M_B W$ " "

LONG SPAN:

MIDSPAN B.M = $+0{\cdot}8\,M_L W$ " "

SUPPORT B.M. = $-0{\cdot}8\,M_L W$ " "





TOTAL LOAD ON SHADED AREA A×B = W LB.

$\frac{B}{L_B}$ = 0·1 ———

= 0·3 — — —

= 0·6 —·—·—

= 1·0 - - - - -

INTERMEDIATE VALUES CAN BE INTERPOLATED.

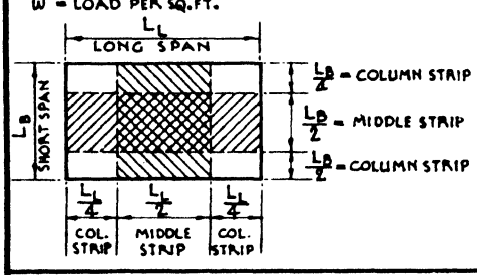
SLABS SPANNING IN TWO DIRECTIONS. TABLE Nº 14C

RECTANGULAR PANELS MONOLITHIC WITH SUPPORTS
BUT NOT CONTINUOUS OVER ALL SUPPORTS

RATIO OF SPANS $\frac{L_B}{L_L}$	BASIC BENDING MOMENT COEFFICIENTS							
	ONE EDGE DISCONTINUOUS		TWO EDGES DISCONTINUOUS		THREE EDGES DISCONTINUOUS		FOUR EDGES DISCONTINUOUS	
	SHORT SPAN C_B	LONG SPAN C_L	SHORT SPAN C_B	LONG SPAN C_L	SHORT SPAN C_B	LONG SPAN C_L	SHORT SPAN C_B	LONG SPAN C_L
1·0	0·031	0·031	0·037	0·037	0·044	0·044	0·050	0·050
0·9	0·036	0·025	0·043	0·030	0·050	0·036	0·057	0·041
0·8	0·041	0·020	0·048	0·024	0·056	0·028	0·068	0·032
0·7	0·047	0·015	0·054	0·018	0·062	0·022	0·072	0·025
0·6	0·052	0·011	0·059	0·013	0·068	0·016	0·080	0·018
0·5	0·064	0·008	0·068	0·009	0·074	0·011	0·083	0·013

B. M. PER FT. WIDTH	SHORT SPAN		LONG SPAN	
	MIDDLE STRIP	COLUMN STRIP	MIDDLE STRIP	COLUMN STRIP
MIDSPAN	$+C_B wL_B^2$	$+0{\cdot}67 C_B wL_B^2$	$+C_L wL_L^2$	$+0{\cdot}67 C_L wL_L^2$
AT CONTINUOUS EDGE	$-1{\cdot}33 C_B wL_B^2$	$-0{\cdot}89 C_B wL_B^2$	$-1{\cdot}33 C_L wL_L^2$	$-0{\cdot}89 C_L wL_L^2$
AT NON CONTINUOUS EDGE	$-0{\cdot}67 C_B wL_B^2$	$-0{\cdot}44 C_B wL_B^2$	$-0{\cdot}67 C_L wL_L^2$	$-0{\cdot}44 C_L wL_L^2$

W = LOAD PER SQ. FT.

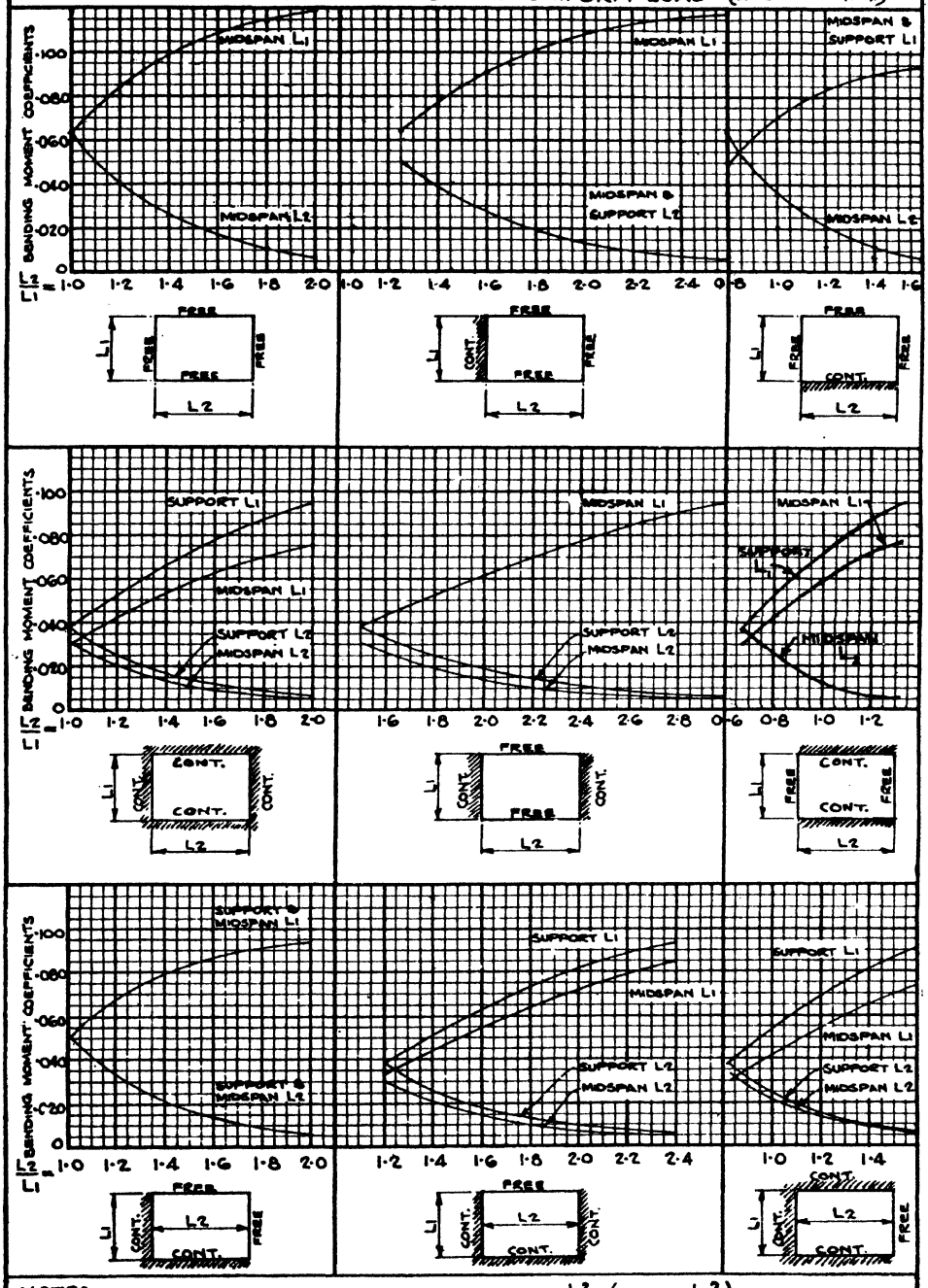


NOTE: THIS TABLE IS BASED ON U.S.A. JOINT COMMITTEE REPORT (1940)

SLABS SPANNING IN TWO DIRECTIONS

BENDING MOMENT COEFFICIENTS WITH UNIFORM LOAD (W LB. PER. SQ. FT.)

TABLE Nº 14A



NOTES. B.M. = BENDING MOMENT COEFFICIENT × wL_1^2 (OR × wL_2^2)

FOR EXAMPLE SEE PAGE 240.

FOR CASES OUTSIDE RANGE OF CURVES, CALCULATE BY RULES ON PAGE 45.

EXAMPLE OF USE OF TABLE No. 14B.

To find the maximum bending moments in a wall panel of a rectangular tank that can be considered as freely supported along the top edge and continuous along the bottom edge and along the two vertical sides. The height of the panel is 8 ft. (L_1) and the horizontal span is 12 ft. (L_2), the intensity of pressure being 500 lb. per sq. ft. along the bottom edge and decreasing uniformly to zero at the top edge.

Span ratio $= \frac{12}{8} = 1{\cdot}5$.

For case (*e*), $K = 2{\cdot}02$;

for mid-span of vertical span	$k_1 = 24$
for bottom edge	$k_1 = 11{\cdot}5$
for mid-span of horizontal span	$k_2 = 45$
for sides	$k_2 = 22{\cdot}5$.

The maximum bending moments will be as follows:

Mid-span of vertical span	$= + \frac{500 \times 8^2 \times 2{\cdot}02}{24 \times 3{\cdot}02} \times 12$	$= 10{,}700$ in.-lb.
Bottom edge	$= - \frac{500 \times 8^2 \times 2{\cdot}02}{11{\cdot}5 \times 3{\cdot}02} \times 12$	$= 22{,}400$,,
Mid-span of horizontal span	$= + \frac{500 \times 12^2}{45 \times 3{\cdot}02} \times 12$	$= 6{,}350$,,
Vertical sides (max. bending moment at about mid-height)	$= - \frac{500 \times 12^2}{22{\cdot}5 \times 3{\cdot}02} \times 12$	$= 12{,}700$,,

The negative bending moments are extreme theoretical values and for the purposes of designing the slab panel can be reduced by 15 per cent. A further reduction to allow for the width of the support (the critical section being at the edge of the support) can also usually be made, as long as the total reduction does not exceed 20 per cent. of the original value and the full value of the reduction is added to the corresponding mid-span positive bending moments; if the adjustment, within the 20 per cent. limit, makes the positive and negative bending moments equal, a more convenient detail for the slab reinforcement can be produced.

If a full 20 per cent. reduction is made in the negative bending moment calculated above, and if the positive moments are made equal to the negative bending moments, the results will be:

Max. neg. and pos. bending moments in vertical span	$= 0{\cdot}80 \times 22{,}400 = 17{,}900$ in.-lb.
Max. neg. and pos. bending moments in horizontal span	$= 0{\cdot}80 \times 12{,}700 = 10{,}200$,,

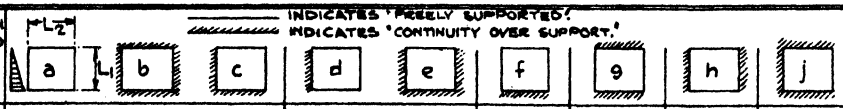
In designing the slab section it is usually necessary to combine a direct tension with the bending moment in the horizontal span.

NOTE.

The bending moment at "the edge of the support" does not necessarily mean the moment at "the edge of corner splays."

SLABS SPANNING IN TWO DIRECTIONS WITH TRIANGULARLY DISTRIBUTED LOADING.

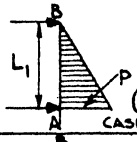
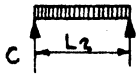
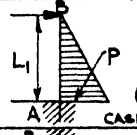
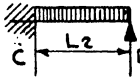
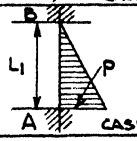
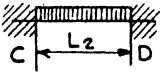
TABLE Nº14B



VALUES OF COEFFICIENT K

SPAN RATIO $\frac{L_2}{L_1}$	a	b	c	d	e	f	g	h	j
0·5		0·06		-		0·16	0·41	-	0·16
0·6		0·13		0·05		0·34	0·85	-	0·33
0·7		0·24		0·10		0·63	1·57	-	0·60
0·8		0·41		0·16		1·07	2·67	0·06	1·03
0·9		0·64		0·26		1·67	4·17	0·10	1·61
1·0		1·00		0·40		2·60	6·50	0·15	2·50
1·1		1·47		0·59		3·82	9·55	0·22	3·68
1·2		2·08		0·83		5·41	13·50	0·31	5·20
1·3		2·86		1·14		7·49	-	0·43	7·15
1·4		3·83		1·53		9·95	-	0·58	9·58
1·5		5·05		2·02		13·1	-	0·76	12·6
1·6		6·57		2·63		17·2	-	0·96	16·4
1·7		8·35		3·34		-	-	1·25	-
1·8		10·50		4·20		-	-	1·58	-
1·9		13·00		5·20		-	-	1·95	-
2·0		16·00		6·40		-	-	2·40	-
2·25		-		10·30		-	-	3·88	-
2·5		-		15·60		-	-	5·85	-

BENDING MOMENTS

VERTICAL SPAN = L_1	k_1	HORIZONTAL SPAN = L_2	k_2
 AT MIDSPAN (AT A AND B, B.M. = 0) CASES (a)(d) & (h)	+ 12	CASES (a)(f) & (g)  AT MIDSPAN (AT C AND D, B.M = 0)	+ 15
 AT MIDSPAN AT SUPPORT A (AT B, B.M. = 0) CASES (c)(e) & (f)	+ 24 −11·5	CASES (c)(d) & (j) AT SUPPORT C  AT MIDSPAN (AT D, B.M = 0)	− 15 + 27
 AT SUPPORT B AT MIDSPAN AT SUPPORT A CASES (b)(g) & (j)	− 22 + 30 − 15	CASES (b)(e) & (h)  AT SUPPORTS C & D AT MIDSPAN	− 22·5 + 45
$B.M. = \frac{p L_1^2 K}{k_1(1+K)}$		$B.M. = \frac{p L_2^2}{k_2(1+K)}$	

PROCEDURE:- p = MAX. INTENSITY OF LOADING.
FOR GIVEN PROPORTIONS OF PANEL AND SEQUENCE OF CONTINUITY, READ OFF VALUE OF K
SUBSTITUTE K AND APPROPRIATE VALUES OF k_1 AND k_2 IN B.M. FORMULÆ.

EXAMPLES OF USE OF TABLES Nos. 15 AND 16

(See also Chapter V.)

(*a*) The bases of the columns of a rectangular portal frame, 10 ft. wide and 15 ft. high, are equivalent to "hinged"; the beam and columns are of identical section. To find the moments at the head of the columns and at the centre of the beam, if a point load of 10,000 lb. is placed at the centre of the beam:

$$L = 10{\cdot}0\,;\ H = 15{\cdot}0\,;\ I_{AB} = I_{BC} = \mathrm{I}\,;\ R = \frac{I \times 15}{I \times 10} = 1{\cdot}5.$$

From tabulated values, $\frac{A}{L} = \frac{WL}{8}$ for central point load (*Table No. 7D*)

$$= \frac{10{,}000 \times 10}{8} = 12{,}500 \text{ ft.-lb.}$$

Moment at head of columns (= moment at ends of beam) $= M_B = M_C$

$$= \frac{3}{3 + 2R} \cdot \frac{A}{L} = \left(\frac{3}{3 + 2 \times 1{\cdot}5}\right) 12{,}500 = 6{,}250 \text{ ft.-lb. (negative) (}\textit{Table No. 16}\text{).}$$

From tabulated values, free B.M. for central point load (*Table No. 7*)

	$= \frac{WL}{4} = 25{,}000$ ft.-lb.	(positive)
Less end moment	$= 6{,}250$,,	(negative)
Mid-span	$= 18{,}750$,,	(positive)

(*b*) To find the approximate wind moments and forces in the columns of a gantry support; the maximum load on each pair of columns is 100 tons, and the horizontal wind load is 5 tons. The height of the columns is 48 ft. braced at top and bottom and with intermediate braces at 12-ft. centres; distance apart of columns at base = 20 ft.:

$$P = 5 \text{ tons}\,;\ W = \frac{100}{2} = 50 \text{ tons per column.}$$

Table No. 15:—

$H = 48{\cdot}0\,;\ h = 12{\cdot}0\,;\ D = 20{\cdot}0.$

B.M. in columns at junction with brace $= \frac{5 \times 12{\cdot}0}{4} = 15$ ft.-tons.

B.M. in brace at junction with columns $= 2 \times 15 = 30$,,

Shear on brace (constant throughout length) $= \frac{2 \times 30}{20} = 3$ tons.

The vertical load on the leeward column $= F_B = 50 + \frac{5 \times 48}{20} = 62$ tons plus dead load of columns and braces.

The vertical load on the windward column $= F_A = 50 - \frac{5 \times 48}{20} = 38$ tons plus dead load of columns and braces.

These loads and moments are combined as explained in Chapter XIV to design the sections or determine the stresses thereon.

FRAMED STRUCTURES — TABLE No. 15

NOTATION

THE SUFFIXES AB RELATE TO JOINT OR SUPPORT A OF ANY MEMBER AB
" BA " " " " B " " AB
SIMILARLY FOR OTHER MEMBERS BC, AC, ETC.

L_{AB} = LENGTH OF MEMBER AB

I_{AB} = MOMENT OF INERTIA (CONCRETE UNITS) OF AB

K_{AB} = STIFFNESS FACTOR OF AB $= \frac{I_{AB}}{L_{AB}}$

E = YOUNG'S MODULUS FOR CONCRETE

θ_A = SLOPE OF DEFORMED MEMBER AB AT A.

WITH NO LOAD ON AB, $P_{AB} = Q_{BA} = 0$

$$P_{AB} = \frac{2A_{AB}}{L_{AB}^2}(2L_{AB} - 3Z_{AB})$$

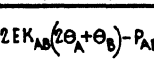
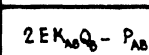
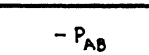
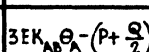
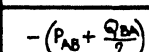
$$Q_{BA} = \frac{2A_{AB}}{L_{AB}^2}(3Z_{AB} - L_{AB})$$

FOR SYMMETRICAL LOAD ON AB $P_{AB} = Q_{BA} = \frac{A_{AB}}{L_{AB}}$ (SEE TABLE No 7D)

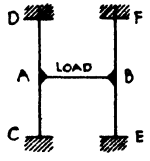
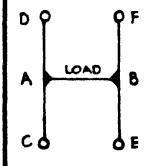
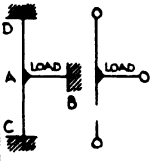
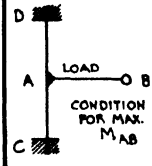
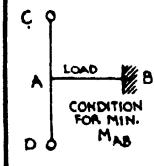
A_{AB} = AREA OF FREE B.M. DIAGRAM FOR LOAD ON AB

Z_{AB} = DISTANCE FROM A TO CENTROID OF FREE B.M. DIAGRAM FOR LOAD ON AB.

SLOPE-DEFLECTION GENERAL FORMULÆ NON-ELASTIC SUPPORTS

–					
M_{AB}	$2EK_{AB}(2\theta_A+\theta_B)-P_{AB}$	$2EK_{AB}Q_B - P_{AB}$	$-P_{AB}$	$3EK_{AB}\theta_A-(P+\frac{Q}{2})$	$-(P_{AB}+\frac{Q_{BA}}{2})$
M_{BA}	$2EK_{AB}(2\theta_B+\theta_A)+Q_{BA}$	$4EK_{AB}\theta_B+Q_{BA}$	$-Q_{BA}$	ZERO	ZERO

EXTERIOR COLUMNS

–				 CONDITION FOR MAX. M_{AB}	 CONDITION FOR MIN. M_{AB}
M_{AC}	$\frac{2K_{AC}}{2K_{AC}+2K_{AD}+K_{AB}}P_{AB}$	$\frac{3K_{AC}}{3K_{AC}+3K_{AD}+2K_{AB}}P_{AB}$	$\frac{K_{AC}}{K_{AC}+K_{AD}+K_{AB}}P_{AB}$	$\frac{4K_{AC}}{4K_{AC}+4K_{AD}+3K_{AB}}P_{AB}$	$\frac{3K_{AC}}{3K_{AC}+3K_{AD}+4K_{AB}}P_{AB}$

$M_{AD} = \frac{K_{AD}}{K_{AC}} \cdot M_{AC}$ $\quad M_{AB} = -(M_{AC} + M_{AD})$ $\quad$ FOR ⊓ AND ⊏ FRAMES, $K_{AD} = 0$

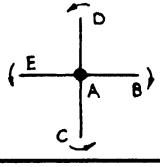
B.M. IN EXTERNAL COLUMNS OF BUILDING FRAMES (CODE OF PRACTICE AND LONDON BUILDING ACT.)

$$M_{AC} = \frac{K_{AC}}{K_{AC}+K_{AD}+nK_{AB}}P_{AB}$$

FOR FRAMES OF ONE BAY WIDTH ONLY $n = 0{\cdot}5$
" " MORE THAN ONE BAY WIDTH $n = 1{\cdot}0$

$$M_{AD} = \frac{K_{AD}}{K_{AC}} \cdot M_{AC} \qquad M_{AB} = -(M_{AC} + M_{AD})$$

INTERIOR COLUMNS



$$M_{AC} = \frac{K_{AC}}{K_{AC}+K_{AD}+K_{AB}} \cdot \frac{A_{AB}}{L_{AB}}$$

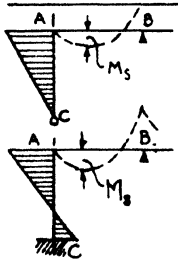
$$M_{AD} = \frac{K_{AD}}{K_{AC}} \cdot M_{AC}$$

FOR SYMMETRICAL LIVE LOAD ON AB AND FOR $K_{AE} = K_{AB}$ APPROX. AND $L_{AE} = L_{AB}$ APPROX.

M_{AB} AND M_{AE} ARE IN ALL CASES LESS THAN B.M. ASSUMING KNIFE EDGE SUPPORT FOR BEAM EB AT A.

APPROXIMATE TREATMENT

B.M. IN EXTERIOR COLUMNS



$M_{AC} = \frac{M_S}{3}$

$M_{AC} = \frac{M_S}{3}$

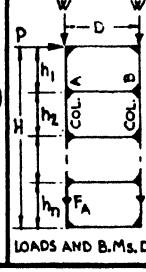
$M_{CA} = +\frac{M_S}{6}$

EQUIVALENT DIRECT LOAD ON COLUMN

TO ALLOW FOR BENDING ON COLUMNS DESIGN FOR EQUIVALENT DIRECT LOAD = F X ACTUAL LOAD (BASED ON STATIC REACTIONS)

F = 1·0 TO 1·1
F = 1·4
F = 1·3
F = 1·5

BRACED COLUMNS



$F_A = W - \frac{PH}{D}$

$F_B = W + \frac{PH}{D}$

B.M. IN COL. AT ANY JOINT $= \frac{Ph_x}{4}$

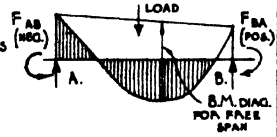
SHEAR ON COL. $= \frac{P}{2}$

B.M. IN BRACE $= \frac{P}{4}(h_x + h_{x-1})$

LOADS AND B.Ms. DUE TO DEAD WEIGHT TO ADD.

FRAME ANALYSIS BY MOMENT DISTRIBUTION TABLE Nº 16 A

MOMENT - DISTRIBUTION METHOD.

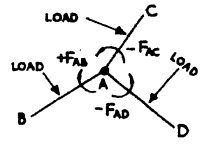


① CONSIDER EACH MEMBER AS FIXED ENDED; END MOMENTS ($F = CL$) DUE TO EXTERNAL LOADS ON GIVEN MEMBER CAN BE DERIVED FROM TABLE Nº 7D OR CALCULATED.

CONVENTION OF SIGNS FOR B.M. :- RESTRAINING MOMENT CLOCKWISE = POSITIVE.

IF SYMMETRICAL LOAD, $F_{AB} = -F_{BA} = \frac{\text{AREA OF FREE B.M. DIAG}}{\text{SPAN}} = \frac{A}{L_{AB}}$

IF NO EXTERNAL LOAD, $F_{AB} = F_{BA} =$ ZERO.



② WHERE TWO OR MORE MEMBERS MEET, SUM OF B.Ms. AT JOINT = ZERO (FOR EQUILIBRIUM)

i.e. $M_{AB} + M_{AC} + M_{AD} + \cdots = 0.$

SINCE ΣF ($= +F_{AB} - F_{AC} - F_{AD}$ etc.) IS UNLIKELY TO EQUAL ZERO, A BALANCING-MOMENT $= -\Sigma F$ MUST BE INTRODUCED AT A.

③ THE BALANCING MOMENT IS DISTRIBUTED BETWEEN THE MEMBERS MEETING AT A IN PROPORTION TO THEIR RELATIVE STIFFNESSES ($K = \frac{I}{L}$) BY MULTIPLYING $-\Sigma F$ BY THE DISTRIBUTION-FACTOR (D) FOR EACH MEMBER. $D_{AB} = \frac{K_{AB}}{\Sigma K}$; $D_{AC} = \frac{K_{AC}}{\Sigma K}$; ETC. $\Sigma D = 1 \cdot 0$

AT FREE END $D = 1 \cdot 0$

AT FIXED END $D = 0$

DISTRIBUTED MOMENT FOR AB $= D_{AB}(-\Sigma F)$. ETC.

④ EFFECT OF APPLYING DISTRIBUTED MOMENT AT ONE END OF MEMBER, IS TO PRODUCE A MOMENT OF HALF THE MAGNITUDE BUT THE SAME SIGN AT THE REMOTE END (TERMED CARRY-OVER). THUS DISTRIBUTED MOMENT $-\Sigma F \cdot D_{AB}$ APPLIED AT A ON AB, PRODUCES MOMENT OF $-0 \cdot 5 \Sigma F \cdot D_{AB}$ AT B.

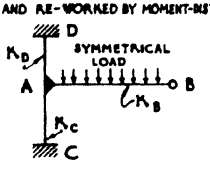
⑤ THE CARRY-OVER MOMENTS AT A FROM THE DISTRIBUTED MOMENTS AT B, C, D, ETC., WILL PRODUCE FURTHER UNBALANCED MOMENTS AT A, WHICH MUST BE DISTRIBUTED AND THE CARRY-OVER PROCESS REPEATED. THESE SUCCESSIVE OPERATIONS ARE REPEATED UNTIL THE UNBALANCED MOMENT IS NEGLIGIBLE. (SEE EXAMPLES BELOW).

EXAMPLE

THE EXAMPLE ON PAGE 56 (SLOPE-DEFLECTION) IS EXTENDED NUMERICALLY AND RE-WORKED BY MOMENT-DISTRIBUTION

ASSUME:- FOR SYMMETRICAL LOAD ON AB, VALUE OF $\frac{A}{L} = 24{,}000$ FT.LB.

RATIO OF STIFFNESS FACTORS:- $K_B = K$; $K_C = \frac{2}{3}K$; $K_D = \frac{1}{3}K$



FORMULA DERIVED FROM SLOPE-DEFLECTION (p.56)

$$M_{AB} = -\frac{6(K_D + K_C)}{3K_B + 4K_C + 4K_D} \cdot \frac{A}{L} = -\frac{6}{7} \times 24{,}000 = -20{,}571 \text{ FT.LB.}$$

$$M_{AD} = -\frac{K_D}{K_C + K_D} \cdot M_{AB} = -\tfrac{1}{8}(-20{,}571) = +6{,}857 \text{ FT.LB.} \quad M_{DA} = \tfrac{1}{2} M_{AD} = +3{,}428 \text{ FT.LB.}$$

$$M_{AC} = -M_{AB} - M_{AC} = 20{,}571 - 6{,}857 = +13{,}714 \text{ FT.LB.} \quad M_{CA} = \tfrac{1}{2} M_{AC} = +6{,}857 \text{ FT.LB.}$$

BY MOMENT-DISTRIBUTION.

$$D_{AB} = \frac{K}{K + \frac{2}{3}K + \frac{1}{3}K} = \tfrac{1}{2}; \quad D_{AC} = \tfrac{1}{3}; \quad D_{AD} = \tfrac{1}{6}\left[= 1 - (\tfrac{1}{2} + \tfrac{1}{3})\right]$$

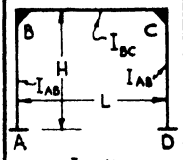
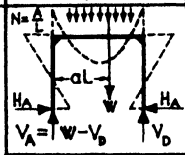
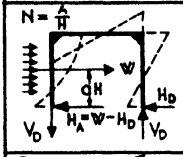
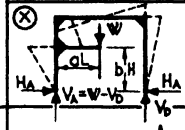
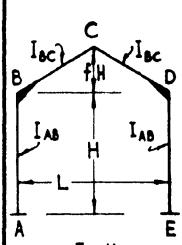
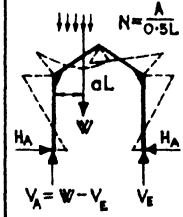
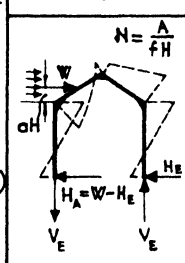
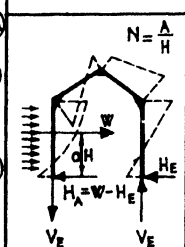
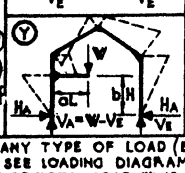
	M_{CA}	M_{AC}	M_{AB}	M_{AD}	M_{DA}	M_{BA}
DISTRIBUTION FACTORS (D)	0	⅓	½	⅛	0	1·0
FIXED END B.Ms. (F)	0	0	−24,000	0	0	+24,000
1ST. DISTRIBUTION	0	+8,000	+12,000	+4,000	0	−24,000
1ST. CARRY-OVER	+4,000	0	−12,000	0	+2,000	+ 6,000
2ND. DISTRIBUTION	0	+4,000	+ 6,000	+2,000	0	− 6,000
2ND. CARRY-OVER	+2,000	0	− 3,000	0	+ 1,000	+ 3,000
3RD. DISTRIBUTION	0	+1,000	+ 1,500	+ 500	0	− 3,000
3RD. CARRY-OVER	+ 500	0	− 1,500	0	+ 250	+ 750
4TH. DISTRIBUTION	0	+ 500	+ 750	+ 250	0	− 750
SUMMATIONS	+6,500	+13,500	−20,250	+ 6,750	+3,250	0 FT.LB.

NOTE: M_{AB} AFTER 1ST. DISTRIBUTION = −12,000 FT.LB.
" 2ND. " = −18,000 " "
" 3RD. " = −19,500 " "
" 4TH. " = −20,250 " "
" 5TH. " = −20,437 " "

ACCURACY INCREASES WITH INCREASE IN NUMBER OF DISTRIBUTION OPERATIONS, BUT RESULTS ARE NORMALLY SUFFICIENTLY ACCURATE AFTER TWO DISTRIBUTIONS FOR BEAMS (SEE TABLE Nº 11A) AND FOUR FOR OTHER CASES.

ETC. COMPARED WITH $M_{AB} = -20{,}571$ FT.LB. BY SLOPE DEFLECTION.

PORTAL FRAMES. TABLE Nº 16

FRAME FORM	LOADING	ENDS	REACTIONS AND BENDING MOMENTS.
 $R = \frac{I_{BC}H}{I_{AB}L}$		HINGED	$H_A = \frac{3N}{K_3H}$ $V_D = aW$ $M_A = M_D = 0$ $M_B = M_C = H_AH$
		FIXED	$H_A = \frac{3N}{K_1H}$ $V_D = aW - 2K_2N$ $M_B = H_AH - M_A$ $M_C = H_AH - M_D$ $\left.\begin{matrix}M_A\\M_D\end{matrix}\right\rangle = \left(\frac{1}{K_1} \mp K_2\right)N$
$K_1 = R + 2$ $K_2 = \frac{3(1-2z_1)}{6R+1}$ $K_3 = 2R+3$ $K_4 = 3R+1$ $K_5 = 2R+1$ $K_6 = R+1$		HINGED	$H_D = 0{\cdot}5aW + \frac{3RZ_1}{K_3H}N$ $V_D = \frac{WaH}{L}$ $M_C = H_DH$ $M_A = M_D = 0$ $M_B = H[H_A - W(1-a)]$
		FIXED	$H_D = 0{\cdot}5aW + \frac{3N}{K_1H}(K_5z_1 - K_6)$ $V_D = \frac{1}{L}(M_B + M_C)$ $M_C = H_DH - M_D$ $C_1 = \frac{1}{K_1}[K_3(1-z_1) - Rz_1]$ $C_2 = \frac{1}{2K_2}[WaHK_4 + 6RN]$ $\left.\begin{matrix}M_A\\M_D\end{matrix}\right\rangle = C_1 \pm C_2$ $M_B = H(H_D + aW) - M_A$
	(X) 	HINGED	$H_D = 1{\cdot}5\frac{WaL}{HK_3}\left[(1-b^2)R + 1\right]$ $M_B = WaL - H_AH$ $M_C = H_AH$ $V_D = aW$ $M_A = M_D = 0$
 $R = \frac{I_{BC}H}{I_{AB}\sqrt{(\frac{1}{2})^2 + (fH)^2}}$		HINGED WHEN ALSO HINGED AT C	$H_A = \frac{aLW}{2H(1+f)}$ $M_C = 0$ $V_E = aW$ $M_A = M_E = 0$ $M_B = M_D = H_AH$
		HINGED	$H_A = \frac{WaL(3+2f) + 6(1+z_1f)N}{4HK_7}$ $M_C = 0{\cdot}5WaL - H_AH(1+f)$ $V_E = Wa - K_{16}$ $M_B = H_AH - M_A$
		FIXED	$H_A = \frac{0{\cdot}5WaLK_{10} + 3K_{11}N}{HK_{12}}$ $M_C = 0{\cdot}5(WaL + M_A + M_E) - H_AH(1+f)$ $\left.\begin{matrix}M_A\\M_E\end{matrix}\right\rangle = \left(\frac{0{\cdot}5WaLK_{13} + K_{14}N}{K_{12}}\right) \mp 0{\cdot}5K_{16}$ $M_D = H_AH - M_E$
$K_7 = R + 3 + f(3-f)$ $K_8 = 2R + 6 + 3f + af(3+2f)$ $K_9 = 2R + 6 + 3f$ $K_{10} = 3R + f(4R+1)$ $K_{11} = R - f + 2z_1f(1+R)$ $K_{12} = R(R+4) + f^2 + 2Rf(3+2f)$ $K_{13} = R + f(2R+1)$ $K_{14} = R - f(3+2f) + 3z_1f(2+R+f)$ $K_{15} = (R+4+3f)R$ $K_{16} = \frac{3(1-z_1)N}{3R+1}$ $K_{17} = R + 2 + f + 2z_1(R+f)$ $K_{18} = 2(R+3) + 2f(3+f) - 3z_1(R+2) - 3z_1f$		HINGED WHEN ALSO HINGED AT C	$H_E = \frac{W(1+a)}{2(1+f)}$ $M_C = 0$ $V_E = \frac{WH}{L}(1+a)$ $M_A = M_E = 0$ $M_B = H_AH$ $M_D = H_EH$
		HINGED	$H_E = \frac{WHK_8 + 6(1+z_1f)N}{4HK_7}$ $M_C = 0{\cdot}5WH(1+af) + M_D(1-f)$ $V_E = \frac{1}{L}[WH(1+a) - M_E - M_A]$ $M_D = H_EH - M_E$ $M_B = H_AH - M_A$
		FIXED	$H_E = \frac{WH(K_{15} + aK_{10}) + 6K_{11}N}{2HK_{12}}$ $M_C = 0{\cdot}5[WH(1+a) - M_A + M_E] - H_EH(1+f)$ $\left.\begin{matrix}M_A\\M_E\end{matrix}\right\rangle = \frac{1}{K_{12}}\left[\frac{WH}{2}(f-a)K_{13} - NK_{14}\right] \mp \frac{WH(3f+2) + 6(1-z_1)N}{4K_4}$
		HINGED WHEN ALSO HINGED AT C	$H_E = \frac{WaW}{2(1+f)}$ $M_C = 0$ $V_E = \frac{WaH}{L}$ $M_D = H_EH$ $M_B = H(Wa - H_E)$ $M_A = M_E = 0$
		HINGED	$H_E = \frac{WaHK_9 + 6Rz_1N}{4HK_7}$ $M_C = 0{\cdot}5WaH - H_EH(1+f)$ $V_E = \frac{1}{L}(WaH - M_E - M_A)$ $M_B = H(Wa - H_E) - M_A$ $M_D = H_EH - M_E$
		FIXED	$H_E = \frac{1}{2HK_{12}}(WaHK_{15} - 6RK_{17}N)$ $M_C = 0{\cdot}5[WaH - M_A + M_E] - H_EH(1+f)$ $\left.\begin{matrix}M_A\\M_E\end{matrix}\right\rangle = \frac{1}{K_{12}}(0{\cdot}5WaHfK_{13} + RK_{18}N) \mp \frac{1}{4K_4}[WaH(3R+2) + 6RN]$
	(Y) 	HINGED WHEN ALSO HINGED AT C	$H_A = \frac{aLW}{2H(1+f)}$ $M_C = 0$ $V_E = aW$ $M_B = WaL - H_AH$ $M_D = H_AH$ $M_A = M_E = 0$
		HINGED	$H_A = \frac{3WaL}{4H} \cdot \frac{R(1-b^2) + 2 + f}{f(3+f) + 3 + R}$ $M_C = 0{\cdot}5WaL - H_AH(1+f)$

FORMULAE APPLY TO ANY TYPE OF LOAD (EXCEPT CASES (X) AND (Y)) AND GIVE NUMERICAL VALUES OF REACTIONS AND MOMENTS; SEE LOADING DIAGRAMS FOR DIRECTION OF ACTION.
CENTRE OF GRAVITY OF TOTAL LOAD W IS AT aL OR aH.
CENTROID OF FREE B.M. DIAGRAM IS AT z_1L, OR z_1H, OR z_1fH FROM L.H. SUPPORT (VERTICAL LOADS) OR LOWER SUPPORT (HORIZ. LOADS)

$\frac{A}{L}, \frac{A}{0{\cdot}5L}, \frac{A}{H}$ OR $\frac{A}{fH}$ = $\frac{\text{AREA OF FREE B.M. DIAG.}}{\text{LOADED LENGTH}}$ = N (SEE TABLE Nº 7D)

EXAMPLES OF USE OF TABLE No. 17.

(*a*) To determine the variation in pressure under a plain concrete foundation 10 ft. long by 2 ft. thick by 8 ft. wide carrying a load of 100 tons placed 1 ft. eccentrically.

Weight of base $= \frac{8 \times 10 \times 2 \times 140}{2{,}240} = 10$ tons. $\therefore$ total load $= 110$ tons.

Taking moments about the short side of base $= (10 \times 5) + (100 \times 4) = 450$ ft.-tons.

Eccentricity $= \frac{10}{2} - \frac{450}{110} = 0{\cdot}91$ ft.

Since $\frac{L}{6} = \frac{10}{6} = 1{\cdot}17$ ft. and $e = 0{\cdot}91$ ft., $e < \frac{L}{6}$.

$$\therefore k = \left(1 \pm \frac{6 \times 0{\cdot}91}{10}\right) = 1{\cdot}545 \text{ or } 0{\cdot}455.$$

Max. ground pressure $= p_1 = \frac{1{\cdot}545 \times 110}{10 \times 8} = 2{\cdot}13$ tons per sq. ft.

Min. ground pressure $= p_2 = \frac{0{\cdot}455 \times 110}{10 \times 8} = 0{\cdot}636$,, ,, ,,

(*b*) To find the maximum ground pressure under the base in (*a*) if the applied load is 2 ft. off centre.

Moments about short side of base $= (10 \times 5) + (100 \times 3) = 350$ ft.-tons.

Eccentricity $= 5 - \frac{350}{110} = 1{\cdot}82$ ft.; $e > \frac{L}{6}$.

$$\therefore k = \frac{4}{3\left(1 - \frac{2 \times 1{\cdot}82}{10}\right)} = 2{\cdot}09.$$

Max. ground pressure $= \frac{2{\cdot}09 \times 110}{10 \times 8} = 2{\cdot}89$ tons per sq. ft.

(*c*) To find the minimum depth of the foundation in Example (*b*) to prevent spewing if the foundation is in dry sandy soil.

From *Table* No. 5, k_2 can be taken as 0·27 and the weight of soil can be assumed as 100 lb. per cu. ft.

Substituting in formula on *Table* No. 17,

$$\text{Minimum } h = \frac{2{\cdot}89 \times 2{,}240}{100} \times 0{\cdot}27^2 = 4{\cdot}73 \text{ ft., say, 5 ft.}$$

(*d*) To find the bending moment at the position of load W_2 on a base 50 ft. long and 5 ft. wide, carrying five unequal point loads as shown in the diagram on *Table* No. 17. The loads (in tons) are $W_1 = 50$, $W_2 = 45$, $W_3 = 40$, $W_4 = 35$, and $W_5 = 30$; the distances are $z_1 = 45$, $z_2 = 37$, $z_3 = 28$, $z_4 = 18$, and $z_5 = 5$ ft. Thus $\Sigma W = 200$ and $\Sigma Wz = (50 \times 45) + (45 \times 37) + (40 \times 28) + (35 \times 18) + (30 \times 5) = 5{,}815$.

$$\therefore e = \frac{5{,}815}{200} - \frac{50}{2} = 4{\cdot}075 \text{ ft. and is } < \frac{L}{6} (= 8\tfrac{1}{3} \text{ ft.})$$

$$\therefore k = 1 \pm \frac{6 \times 4{\cdot}075}{50} = 1{\cdot}489 \text{ and } 0{\cdot}511.$$

Maximum pressure: $p_1 = \frac{1{\cdot}489 \times 200}{50 \times 5} = 1{\cdot}19$ tons per sq. ft.

Minimum pressure: $p_2 = \frac{0{\cdot}511 \times 200}{50 \times 5} = 0{\cdot}41$,, ,, ,,

At the load W_2, $a = 50 - 37 = 13$ ft.

$$\therefore p_x = \left(\frac{50 - 13}{50}\right)(1{\cdot}19 - 0{\cdot}41) + 0{\cdot}41 = 0{\cdot}987 \text{ ton per sq. ft.}$$

Also $x_1 = 8$, $x_2 = 0$.

$\therefore \Sigma Wx = W_1x_1 + W_2x_2 = 400 + 0 = 400$ ft.-tons.

B.M. at x.x.

$$= 400 - \frac{13^2}{6}(2 \times 1{\cdot}19 + 0{\cdot}987)\,5 = 400 - 470 = -70 \text{ ft.-tons.}$$

(The B.M. at other sections could be found in a similar manner.)

FOUNDATIONS. PRESSURES. TABLE Nº 17.

SAFE PRESSURES ON GROUND. (L.B.A. DENOTES LONDON BUILDING A)

NATURE OF GROUND	SAFE PRESSURE GENERAL	SAFE PRESSURE L.B.A.	MINIMUM DEPTH
MUD, MUDDY CLAYS, ETC.	0 TO 1/4	-	3'-0"
ALLUVIAL EARTH	1/2 TO 3/4	1/2	2'-0"
ARTIFICIAL FILLING	1/4 TO 3/4	1/2	2'-0"
CLAY - SOFT	1	1	2'-0"
" - ORDINARY	2	2	2'-6"
" - DRY	3	3	3'-0"
" - SOUND YELLOW	3 TO 5	-	3'-0"
" - BLUE	4 TO 6	4	3'-0"
SAND - WET OR LOOSE	1	1/2 TO 1	4'-0"
" - DRY WITH CLAY	2	2	3'-6"
" - DRY OR CONFINED	3	2 TO 4	3'-6"
" - IN RIVER BED	3 1/2 TO 4	-	25'-0"
GRAVEL - COMPACT	4 TO 7	4	3'-0"
" - BOULDER	5 TO 7	-	3'-0"
" - WITH SAND	6	-	20'-0"
MARL, FIRM SHALE	5 TO 7	-	-
CHALK - SOFT	1 1/2	-	-
" - HARD WHITE	2 1/2 TO 6	-	-
ROCK - VERY SOFT	2	-	-
" - MODERATELY HARD	5 TO 10	-	-
" - HARD	12 (MIN)	-	-

SAFE PRESSURES ON VARIOUS MATERIALS. (TONS PER SQ. FT.)

MATERIAL (SEE ALSO B.S. Nº 1145)	SAFE PRESSURE GENERAL	SAFE PRESSURE L.B.A.
CONCRETE - MIX A	10	-
CONCRETE - MIX C	15	12
BLUE BRICK	12	12
HARD BRICK (LONDON STOCKS)	8	8
ORDINARY BRICK	6	5
GRANITE	15	-
PORTLAND STONE	12	-
LIMESTONE	9	-

ALL BRICKWORK IN P.C. MORTAR.

MAX. GROUND PRESSURE AT DEPTH h FT. TO PREVENT SPEWING:—

SAND AND GRANULAR SOILS:

$$p_{max.} = \frac{wh}{k_2^2}$$

CLAY:

$$p_{max.} = \frac{wh}{k_2^2} + \frac{2k}{\sqrt{k_2}}\left(\frac{1}{k_2} + 1\right)$$

w = WT. PER CU. FT. OF SOIL.

k_2 = PER TABLE Nº 5 (WITH $\theta = a$ for clay)

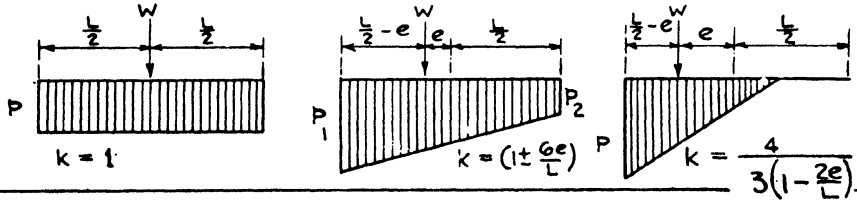
SEE PAGE 26 FOR VALUES OF a & k.

WHEN MINIMUM TABULATED DEPTHS ARE EXCEEDED, SAFE PRESSURE CAN BE INCREASED BY "EXTRA DEPTH × w" PROVIDED FORMULÆ ABOVE ARE SATISFIED.

PRESSURES UNDER FOUNDATIONS.

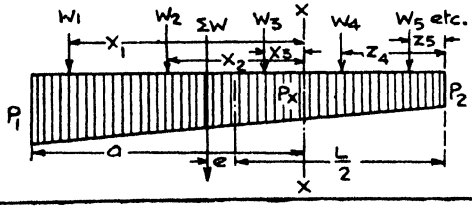
p = INTENSITY OF PRESSURE $= \dfrac{kW}{L \times \text{BREADTH}}$

$e = 0$ $\qquad$ $e < \frac{L}{6}$ $\qquad$ $e > \frac{L}{6}$



COMBINED FOUNDATIONS.

(1) METHOD APPLICABLE TO ANY NUMBER OF LOADS. (ALSO APPLICABLE WHEN $e = 0$ i.e. WHEN $P_1 = P_2 = P_x$)



$$e = \frac{\Sigma Wz}{\Sigma W} - \frac{L}{2}$$

CALCULATE P_1 AND P_2 AS ABOVE.

$$P_x = \left(\frac{L-a}{L}\right)(P_1 - P_2) + P_2$$

$$\text{B.M. AT XX} = \Sigma Wx - \frac{a^2}{6}(2P_1 + P_x)B$$

EXAMPLES OF USE OF TABLE No. 18.

(*a*) To design a base for a 15-in. square column carrying 60 tons, (i) with ground pressure not exceeding 4 tons per sq. ft., and (ii) with ground pressure not exceeding $1\frac{1}{2}$ tons per sq. ft. Assume 1 : 2 : 4 mix (stresses as in *Table* No. 23) generally.

(i) Adopt type (*a*). (For plain concrete base use 1 : 6 mix ; S_p = 80 lb. per sq. in.)

$$B \nless 12\sqrt{\frac{60}{20}} = 20{\cdot}8 \text{ in.}$$; say, reinforced concrete footing 21 in. square.

$$d \nless \frac{560 \times 60}{150}\left(\frac{1}{15} - \frac{15}{21^2}\right) = 7{\cdot}4 \text{ in.}$$; reinforced concrete footing, say 12 in. deep.

Check : $B \ngtr 15 + (12 \times 37) = 45$ in. Satisfactory.

$$A \nless 12\sqrt{\frac{60}{4}} = 46{\cdot}5,$$ say, plain concrete base 48 in. square.

$$D \nless \frac{560 \times 60}{80}\left(\frac{1}{21} - \frac{21}{48^2}\right) = 16{\cdot}2 \text{ in.}$$; plain concrete base 18 in. thick.

Check : $A \ngtr 21 + (2 \times 18) = 57$ in. Satisfactory.

(ii) Adopt type (*b*) :

$$A \nless 12\sqrt{\frac{\text{say } 62}{1{\cdot}5}} = 77 \text{ in.,}$$ say, 6 ft. 6 in. square.

$$R = \frac{1{\cdot}5}{78} = 0{\cdot}192.$$

For punching shear : $$D \nless \frac{560 \times 60}{78 \times 150}\left(\frac{1}{0{\cdot}192} - 0{\cdot}192\right) = 14{\cdot}5,$$ say, 18 in.

$$\text{B.M. on x.x.} = \frac{60 \times 78}{24}(2 + 0{\cdot}192)(1 - 0{\cdot}192)^2 = 281 \text{ in.-tons.}$$

NOTES.

Independent Bases, Types (b) and (c).

The minimum resistance moment of bases of these types is given by

$$\text{R.M.} = QC_2d^2 \text{ in.-lb.}$$

where Q = resistance moment factor (*see Table* No. 27)
d = effective depth (in.) under column.
C_2 = dimension given on *Table* No. 18.
(If the base is of uniform thickness $C_2 = A$ (or a).)

The area of reinforcement required (in each direction mutually at right-angles) is given by

$$A_r = \frac{\text{B.M. (in.-lb.) on } XX.}{a_1dt} \text{ sq. in.}$$

where a_1 = lever-arm factor (see *Table* No. 27).
t = allowable tensile stress (lb. per sq. in.).

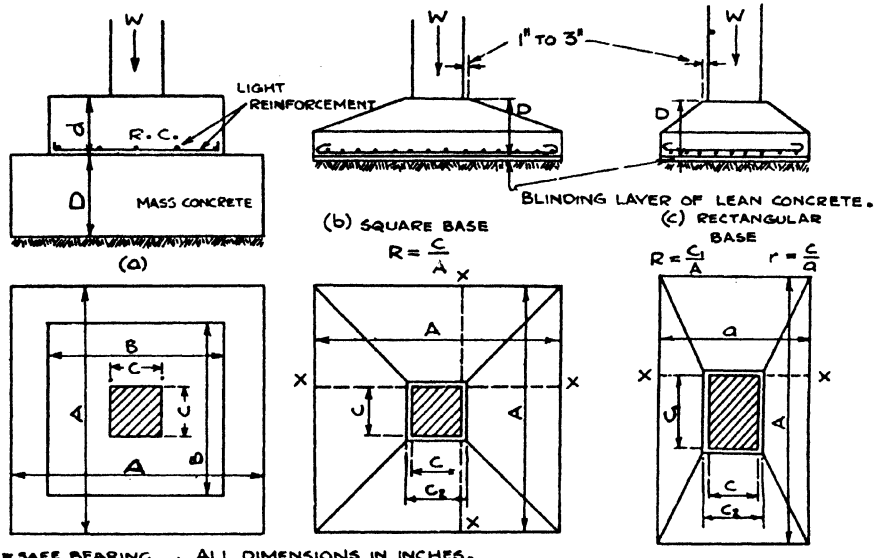
This reinforcement should be distributed symmetrically under the column in a band of width $C + 2d$.

If $C + 2d < A$ (or a) additional reinforcement should be provided outside the limits of this band.

When $A \ngtr C + 2d$ bending moments can be neglected and only nominal reinforcement provided.

FOUNDATIONS . INDEPENDENT BASES. TABLE No. 18

INDEPENDENT BASES WITH CONCENTRIC LOADS.



C_2 = SAFE BEARING STRESS (tons per ft.²)

S_p = SAFE PUNCHING SHEAR STRESS (lb. per in.²)

ALL DIMENSIONS IN INCHES.

W = LOAD FROM COLUMN IN TONS.

W_B = WEIGHT OF BASE IN TONS.

P = SAFE BEARING PRESSURE ON GROUND IN TONS PER SQ. FT.

(a)

$B \not< 12\sqrt{\frac{W}{C_2}} \quad \not> C + 2d$

$A \not< 12\sqrt{\frac{W}{P}} \quad \not> B + 2D$

$d \not< \frac{560W}{S_p}\left(\frac{1}{C} - \frac{C}{B^2}\right)$

$D \not< \frac{560W}{S_p}\left(\frac{1}{B} - \frac{B}{A^2}\right)$

(b)

$A \not< 12\sqrt{\frac{W + W_B}{P}}$

B.M. ON X.X. IN INCH-TNS:-

$= \frac{WA}{24}(2+R)(1-R)^2$

$D \not< \frac{560W}{A S_p}\left(\frac{1}{R} - R\right)$

(c)

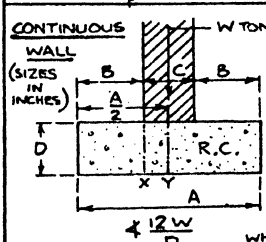
$Aa \not< \frac{144(W+W_B)}{P}$

B.M. ON X.X. IN INCH-TNS:-

$= \frac{WA}{24}(2+r)(1-R)^2$

$D \not< \frac{1120W}{S_p}\left(\frac{1-Rr}{C_1+C}\right)$

CONTINUOUS WALL



$D \not< \frac{187\,WB}{A_s}$

(OR AS REQUIRED FOR BENDING MOMENT)

$\bar{B.M.}$ AT X $= \frac{WB^2}{2A}$

B.M. PER FOOT AT Y:-

$= \frac{WB^2}{2A} + \frac{WBC}{4A}$

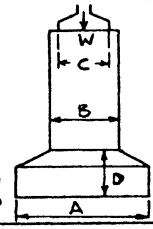
$\not< \frac{12W}{P}$

WHEN $B < D$, B.M. NEGLECTED

S = SAFE SHEAR STRESS ON CONCRETE. (lb. per in.²)

MASS CONCRETE PIERS.

SIZES IN INCHES.



(SQUARE)

$C \not< 12\sqrt{\frac{W}{C_2}}$

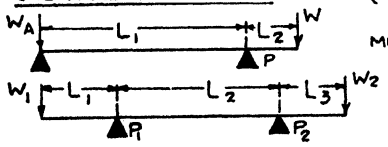
$B \not< C$

$A \not< 12\sqrt{\frac{W_T}{P}} \quad \not> B + 2D$

WHERE W_T = W + WT. OF PIER (TONS)

$D \not< \frac{560W_T}{S_p}\left(\frac{1}{B} - \frac{B}{A^2}\right)$

BALANCED FOUNDATIONS.



(MOMENTS AND REACTIONS DUE TO DEAD WT. OF BEAM TO BE ALLOWED FOR IN ADDITION.)

MIN. ANCHORAGE WT. $= W_A = \frac{WL_2}{L_1}$; $P = W\left(1 + \frac{L_2}{L_1}\right)$

$P_1 = \frac{W_1(L_2+L_1) - W_2 L_3}{L_2}$

$P_2 = \frac{W_2(L_2+L_3) - W_1 L_1}{L_2}$

NOTE:- BASES TO TAKE P_1 ETC. DESIGNED FOR CONCENTRIC LOAD.

EXAMPLES OF USE OF TABLE No. 19.

(*a*) To find the safe load on a pile 40 ft. long, 14 in. square, driven down to a gravel bed. Driving record as follows: 2 tons single-acting steam hammer dropping 42 in.; 12 blows per final inch of penetration; weight of helmet, dolly, and stationary part of hammer = $\frac{1}{2}$ ton; dolly in good condition at end of driving; weight of pile $= \frac{40 \times 196}{2{,}240} = 3\frac{1}{2}$ tons; $P = 3\frac{1}{2} + \frac{1}{2} = 4$ tons; $\frac{P}{w} = \frac{4}{2} = 2$:

(i) *By Dutch Formula:*

From *Table* No. 19 with $\frac{P}{w} = 2$ and $w = 2$, safe load may be from 40 to 60 tons for $Hn = 360$. Adopting lower value with dolly in good condition, $Hn = 42 \times 12 = 504$, and allowing for 10 per cent. increase due to type of hammer, safe load $= 40 \times \frac{504}{360} \times 1{\cdot}1 = 49$ tons.

(ii) *By Hiley Formula:*

Effective drop $= H_1 = 0{\cdot}90 \times 42 = 38$ in.

For $\frac{P}{w} = 2$, $e = 0{\cdot}37$.

For medium driving, $c = 0{\cdot}295$ for $L = 40$ ft.

Settlement load

$$= \frac{2 \times 38 \times 0{\cdot}37 \times 12}{1 + (0{\cdot}295 \times 12)} + 2 + 4 = 73{\cdot}5 + 6 = 79{\cdot}5 \text{ tons.}$$

(Check: driving pressure $= \frac{79{\cdot}5 \times 2{,}240}{14^2} = 910$ lb. per sq. in.

Easy	driving =	500	lb. per sq. in.
Medium	,, =	1,000	,, ,, ,,
Hard	,, =	1,500	,, ,, ,,
Very hard	,, =	2,000	,, ,, ,,)

$$\text{Safe load} = \text{say } \frac{79{\cdot}5}{2} = 40 \text{ tons.}$$

Hence by two formulæ the safe load seems to be between 40 and 50 tons.

(*b*) To find the safe load on a 15-in. square pile driven 40 ft. into firm mud without finding harder strata:

By Rhode formula: $k_1 = 0{\cdot}0001$; $k_2 = 0{\cdot}021$;

$B = D = 15$ in.

Safe load $= 40[(0{\cdot}0001 \times 40 \times 30) + 0{\cdot}021] = 5{\cdot}64$ tons, say, 6 tons exclusive of weight of pile.

(*c*) To find safe load on same pile as in (*b*) if bearing on wet clay:

k_3 (applicable to material driven through) $= 0{\cdot}015$

k_4 (applicable to material founded upon) $= 0{\cdot}023$

$W = (40 \times 0{\cdot}015 \times 30) + (0{\cdot}023 \times 225) = 23$ tons including weight of pile.

NOTES.

In all pile loading calculations it must be remembered that a formula can only pretend to give comparative values that must be combined with the results of tests and experience when assessing the safe load to which a given pile, driven under certain conditions, can be subjected.

For short piles the toggle holes provided near the head of the pile for guiding can generally be used for lifting purposes without producing excessive bending moments.

PILES.

TABLE Nº 19.

DUTCH FORMULA.

SAFE LOAD $= W = \dfrac{\omega^2 H n}{c(P+\omega)}$ $\quad c = 4$ TO 6

ω = WEIGHT OF HAMMER IN TONS. $\quad$ H = DROP OF HAMMER IN INCHES.

P = WEIGHT OF PILE, HELMET, ETC. $\quad$ n = NUMBER OF BLOWS PER INCH PENETRATION.

VALUES OF W FOR n = 10 AND H = 30" (TONS) $\quad$ MAXᴹ W: C = 4. $\quad$ MINᴹ W: C = 6.

ω	½ TON.		¾ TON.		1 TON.		1½ TON.		2 TON.		3 TON.		4 TON.	
P/ω	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.
½	-	-	-	-	40	60	60	90	80	120	120	180	160	240
1	-	-	23	34	30	45	45	68	60	90	90	135	120.	180
1½	12	18	18	27	24	36	36	54	48	72	72	108	96	144
2	10	15	15	23	20	30	30	45	40	60	60	90	80	120
2½	9	13	13	20	17	26	26	39	34	51	51	77	68	102
3	8	12	12	17	15	23	23	34	30	45	45	68	60	90
3½	7	10	10	15	13	20	20	30	26	40	40	60	52	80
4	6	9	9	14	12	18	18	27	24	36	36	54	48	72

USE MAXIMUM VALUE IF NO HELMET IS USED OR IF DOLLY IS DESTROYED AT END OF DRIVING.

USE MINIMUM VALUE IF DOLLY AND PACKING ARE IN GOOD CONDITION AT END OF DRIVING.

FOR OTHER VALUES OF n AND H:— $\quad W = \dfrac{\text{TABULATED VALUES} \times Hn}{360}$

VALUES OF W CAN BE INCREASED BY 10% IF SINGLE ACTING STEAM HAMMER IS EMPLOYED.

HILEY FORMULA. (MODIFIED)

SETTLEMENT LOAD $= W_M = \dfrac{\omega H_1 e n}{1 + cn} + \omega + P$

SAFE LOAD $= \dfrac{W_M}{3}$ TO $\dfrac{W_M}{1.5}$ $\quad$ (FOR SIGNIFICATION OF ω, n, AND P SEE ABOVE)

(LOWER FACTOR OF SAFETY ASSOCIATED WITH HARDER DRIVING.)

VALUES OF $\dfrac{\text{EFFECTIVE DROP } (H_1)}{\text{ACTUAL DROP}}$:—

FREELY FALLING DROP HAMMER = 1·00

SINGLE ACTING STEAM HAMMER = ·90

WINCH OPERATED DROP HAMMER = ·80

VALUES OF P/ω	½	1	1½	2	2½	3	4	5	6	7	8
EFFICIENCY OF BLOW = e	·69	·53	·44	·37	·33	·30	·25	·21	·19	·17	·15

VALUES OF C = TEMPORARY ELASTIC COMPRESSION FACTOR.	LENGTH OF PILE	10'-0"	20'-0"	30'-0"	40'-0"	50'-0"	60'-0"
	500 LBS/IN² EASY DRIVING	·125	·128	·155	·170	·185	·200
	1000 LBS/IN² MEDIUM DRIVING	·205	·235	·265	·295	·325	·355
	1500 LBS/IN² HARD DRIVING	·285	·325	·370	·415	·460	·505
	2000 LBS/IN² VERY HARD DRIVING	·335	·395	·455	·515	·575	·635

TABULATED VALUES ARE MINIMUM AND ASSUME DOLLY AND PACKING IN GOOD CONDITION, AND GRAVEL SOIL.

FOR MEDIUM DRIVING ADD :-

FOR CLAY SOIL ·05"

FOR SOFT AND PEATY SOIL ·20"

FRICTION PILES. (RECTANGULAR SECTION : B" × D")

(i) CONSISTENTLY SOFT MATERIAL :—

$$W = L_e\left[k_1 L_e (B+D) + k_2\right]$$ (RHODE MODIFIED)

(ii) THROUGH SOFT MATERIAL AND BEARING ON SOMEWHAT FIRMER STRATA :—

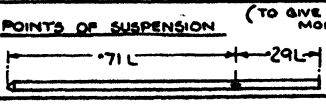
$$W = L_e k_3 (B+D) + k_4 BD$$ (ADAPTED FROM A. HUNTER)

L_e = EMBEDDED LENGTH OF PILE IN FEET.

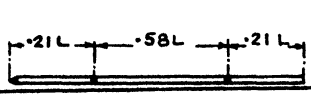
VALUES OF FACTORS k.

TYPE OF GROUND	k_1	k_2	k_3	k_4
SOFT SILT	-	-	·012	-
FIRMER SILT, STIFF MUD, OR LOAM.	·0001	·021	·015	·007
WET CLAY	·0006	·043	·016	·023
DRY CLAY OR SAND	(·0014)	(·091)	·033	·042
COARSE SAND OR GRAVEL	(·0022)	(·143)	·043	·114

POINTS OF SUSPENSION (TO GIVE MINIMUM MOMENTS)



SINGLE POINT LIFTING.



DOUBLE POINT LIFTING

EXAMPLES OF USE OF TABLE No. 20.

(*a*) To design a retaining wall (Type *a*) for the following conditions:

$h = 10$ ft.; $H = 12$ ft.
$P = 2{,}000$ lb. per ft. run of wall. $z = 3$ ft. 6 in.
$P_0 = 2{,}150$,, ,, ,, $y = 5$ ft. 3 in.
Weight of earth = 100 lb. per cb. ft.; surcharge = 224 lb. per sq. ft.
B.M. at A.A. = 2,000 × 3 ft. 6 in. = 7,000 ft.-lb. = 84,000 in.-lb. (See Note below.)

From *Table* No. 30, stresses 16,000 and 600, a suitable section for the wall stem would be 10 in. Therefore stem of wall at *AA* and base of wall at edge of haunches would be 10 in. thick tapering to 6 in.

Assume $L = 6$ ft. and toe projects 6 in. in front of wall face.

To determine W and x (by moments about toe of wall):

Wall stem: 11 ft. 2 in. × 96 av.	= 1,070 lb.	× 1 ft.	=	1,070	ft.-lb.
Wall base: 6 ft. × 96 av.	= 576 ,,	× 2 ft. 6 in.	=	1,440	,,
Haunches: say	= 70 ,,	× 1 ft.	=	70	,,
Earth: 4 ft. 8 in. by 11 ft. 4 in. × 100	= 5,300 ,,	× 3 ft. 8 in.	=	19,400	,,
Surcharge: 4 ft. 8 in. × 224	= 1,045 ,,	× 3 ft. 8 in.	=	3,840	,,
	W = 8,061 lb.		Wx =	25,820	ft.-lb.

$$x = \frac{25{,}820}{8{,}061} = 3{\cdot}2 \text{ ft.}$$

Stability: $x \nless 1{\cdot}5\dfrac{2{,}150 \times 5{\cdot}25}{8{,}061} = 2{\cdot}1$ ft. Satisfactory.

Sliding: $W \nless 3{\cdot}75 \times 2{,}150 = 8{,}060$ lb. Satisfactory.

Ground pressure: $e = \dfrac{2{,}150 \times 5 \text{ ft. } 3 \text{ in.}}{8{,}061} + \dfrac{6}{2} - 3{\cdot}2 = 1{\cdot}20$

$6e = 1{\cdot}2 \times 6 = 7{\cdot}2$ which is greater than $L = 6$ ft.

By the appropriate formula given on *Table* No. 17

$$p = \frac{4 \times 8{,}061}{3 \times 1(6 - 2 \times 1{\cdot}2)}$$

Max press. = 1·32 ton per sq. ft.

(*b*) To design a sheet pile wall 12 ft. high, freely cantilevering, with angle of repose of material = 35 deg.:

Here $H = 12$ ft. $\theta = 35$ deg.
Hence $k'_1 = 1{\cdot}4$. $k'_2 = 1{\cdot}0$.
Driven length of pile = 1·0 × 12 = 12 ft.
Total length of pile = 12 + 12 = 24 ft.
Span of sheeting for purpose of calculating bending moment = L = 1·4 × 12 = 16·8 ft.

The pressures in front of and behind the wall, and the point of application of these pressures can be computed as indicated on pp. 89 to 92. Having calculated the moments, suitable sections can be determined from Chapter XI.

(*c*) To determine the moments and forces in the walls of a cylindrical water tank 20 ft. in diameter, 30 ft. deep, with walls 10 in. thick at bottom:

Here $D = 20$ ft.; $H = 30$ ft.; $d = 0{\cdot}83$ ft.; $w = 62{\cdot}4$ lb. per cb. ft.

With $\dfrac{H}{D} = \dfrac{30}{20} = 1{\cdot}5$; $\dfrac{H}{d} = \dfrac{30}{0{\cdot}83} = 36$;

$F = 0{\cdot}003$; $K = 0{\cdot}20$.
B.M. (vertical) at A: B.M. = $0{\cdot}003 \times 62{\cdot}4 \times 30^3 = 5{,}050$ ft.-lb.
Position of point of maximum ring tension: $L = 0{\cdot}20 \times 30 = 6$ ft. from bottom.
Max. ring tension = 0·5 × 62·4 × 30 × 20(1 − 0·20) = 15,000 lb. per ft. height of wall.

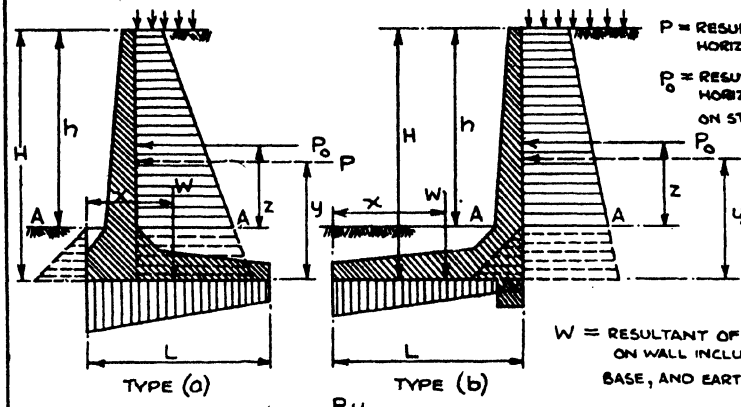
NOTE.

When a single splay only is provided at the base of a retaining wall, the critical bending moment in the stem of the wall may be at the bottom of the splay (instead of at section *AA*). The resistance for this section should be based on a thickness less than the thickness of the concrete at the bottom of 45-deg. splays.

RETAINING WALLS & TANK WALLS — TABLE

CANTILEVER RETAINING WALLS.

(FOR CALCULATION OF EAR PRESSURES SEE TABLE



P = RESULTANT OF ALL ACTIVE HORIZONTAL PRESSURES.

P_o = RESULTANT OF ALL ACTIVE HORIZONTAL PRESSURES ACTING ON STEM OF WALL ABOVE A.A.

UNITS:-
FORCES ..... LBS.
WEIGHTS .... LBS.
DIMENSIONS .. FEET
B.M'S.... FT.LBS.

(FORMULÆ ARE PER UNIT LENGTH OF WALL)

W = RESULTANT OF ALL VERTICAL LOADS ON WALL INCLUDING WT. OF STEM AND BASE, AND EARTH ON BASE, SURCHARGE, ETC.

STABILITY : $x \not< 1{\cdot}5 \dfrac{Py}{W}$

SLIDING : APPLICABLE TO DESIGNS OMITTING RIB SHOWN IN (b). $W \not< 3{\cdot}75P$

GROUND PRESSURE : e = ECCENTRICITY OF COMBINED P AND W = $\dfrac{Py}{W} + \dfrac{L}{2} - x$

SUBSTITUTE e IN APPROPRIATE FORMULA GIVEN ON TABLE Nº 17.

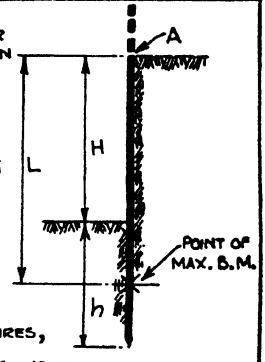
BENDING MOMENT : AT A.A. = $P_o z$

SHEET PILE WALLS

θ	CONDITION AT A					
	FREE		HINGED		FIXED	
	k'_1	k'_2	k'_1	k'_2	k'_1	k'_2
20°	2·0	2·0	1·23	1·23	1·11	1·11
30°	1·5	1·3	1·08	·88	1·07	·87
35°	1·4	1·0	1·07	·67	1·06	·66
45°	1·3	·8	1·06	·56	1·05	·55

SPAN OF SHEETING FOR MOMENT CALCULATION $= L = k'_1 H$.

MINIMUM EMBEDDED LENGTH OF SHEETING $= h = k'_2 H$

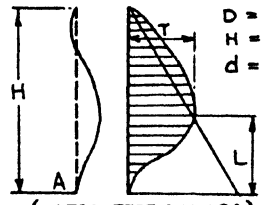


θ = ANGLE OF REPOSE OF EARTH IN FRONT OF SHEETING.

TABULATED VALUES ASSUME APPROX. TRIANGULARLY DISTRIBUTED PRESSURES, AND ARE SUFFICIENTLY ACCURATE FOR PRELIMINARY SCHEMES; FOR MAJOR STRUCTURES, CONSIDERATION SHOULD BE GIVEN TO ACTUAL CONDITIONS.

RESTRAINT AT BASE OF WALLS OF CYLINDRICAL TANKS.

FACTORS		F				K			
H/d		10	20	30	40	10	20	30	40
VALUES OF H/D	·1	·075	·047	·036	·028	-	-	-	-
	·2	·046	·028	·022	·015	-	·50	·45	·40
	·3	·032	·019	·014	·010	·55	·43	·38	·33
	·4	·024	·014	·010	·007	·50	·39	·35	·30
	·5	·020	·012	·009	·006	·45	·37	·32	·27
	1·0	·012	·006	·005	·003	·37	·28	·24	·21
	2·0	·006	·005	·002	·002	·30	·22	·19	·16
	4·0	·004	·002	·002	·001	·27	·20	·17	·14



D = DIA. OF TANK.
H = DEPTH OF TANK.
d = WALL THICKNESS AT A.
ω = WT. PER CU. FT. OF CONTAINED LIQUID

(UNITS: FEET AND LBS.)

B.M. AT A: $M = F \omega H^3$

MAX. RING TENSION : $T = {\cdot}5 \omega H \cdot D (1-K)$

POSITION OF MAX. RING TENSION: $L = KH$.

EXAMPLE OF USE OF TABLE No. 21.

(*a*) A single rectangular cell measuring in the clear 12 ft. 9 in. by 10 ft. 6 in. without cross walls or ties, and subject to a uniform horizontal pressure of 200 lb. per sq. ft. at a given depth; to find the maximum moments and horizontal direct tensions at the given depth, assuming that the walls span horizontally.

Assuming the walls are 6 in. thick, the effective spans of the walls will be 13 ft. 3 in. and 11 ft.; hence $\frac{B}{D} = \frac{13{\cdot}25}{11{\cdot}0} = 1{\cdot}2$.

$k = 9{\cdot}7$ (approx.) from *Table* No. 21.

M_1 = B.M. at corners $= \frac{PD^2}{k} = \frac{200 \times 11^2}{9{\cdot}7} = 2{,}500$ ft.-lb.

Free moment in $D = \frac{200 \times 11^2}{8}$	= 3,025	ft.-lb.
Less corner moment	= 2,500	,,
Positive B.M. at mid-span of D	= 525	,,
Free moment in $B = \frac{200 \times 13{\cdot}25^2}{8}$	= 4,390	ft.-lb.
Less corner moment	= 2,500	,,
Positive B.M. at mid-span of B	= 1,890	,,

Direct tension in short side

$$= \frac{PB}{2} = 0{\cdot}5 \times 200 \times 12{\cdot}75 = 1{,}275 \text{ lb. per ft.}$$

Direct tension in long side

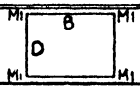
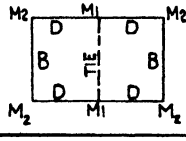
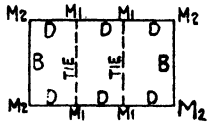
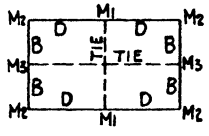
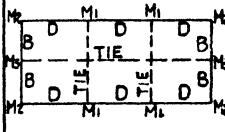
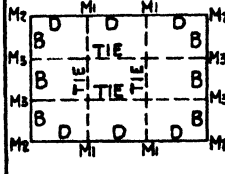
$$= \frac{PD}{2} = 0{\cdot}5 \times 200 \times 10{\cdot}5 = 1{,}050 \text{ lb. per ft.}$$

The appropriate moments and tensions would be combined as described in Chapter XIV.

NOTES:

MOMENTS IN BIN WALLS. TABLE Nº21

(WITH UNIFORM HORIZONTAL PRESSURE (P) AT ANY GIVEN DEPTH).

FORM OF BIN.	BENDING MOMENTS. $M = \frac{PD^2}{k}$ — FORMULÆ	BENDING MOMENT COEFFICIENTS k — VALUES OF $\frac{B}{D}$ 0·5	0·8	1·0	1·2	1·5	1·8	2·0	REACTION FORMULÆ.
	$M_1 = \frac{P(D^3 + B^3)}{12(D+B)}$	16	14·3	12	9·68	6·86	4·92	4	$R_B = \frac{PB}{2}$ $R_D = \frac{PD}{2}$
	$M_1 = \frac{P(D^3 + 3D^2B - B^3)}{12(D+2B)}$	10·1	10·8	12	14·2	22·6	97	MINUS 60	$R_B = \frac{PB}{2}$
	$M_2 = \frac{P(D^3 + 2B^3)}{12(D+2B)}$	19·2	15·4	12	9·15	6·2	4·36	3·53	$R_D = \frac{PD}{2} + \frac{M_2 - M_1}{D}$
	$M_1 = \frac{P(3D^3 + 6D^2B - B^3)}{12(3D+5B)}$	11·2	11·5	12	12·75	14·7	18	22·3	$R_B = \frac{PB}{2}$ R_D (END SPAN) $= \frac{PD}{2} + \frac{M_2 - M_1}{D}$
	$M_2 = \frac{P(3D^3 + 5B^3)}{12(3D+5B)}$	18·2	15·1	12	9·3	6·35	4·5	3·6	R_D (CENTRE SPAN) $= \frac{PD}{2}$
	$M_1 = \frac{P(D+B)(2D-B)}{24}$	10·68	11·1	12	13·6	19·2	42·9	—	$R_B = \frac{PB}{2} + \frac{M_2 - M_3}{B}$
	$M_2 = \frac{P(D^3 + B^3)}{12(D+B)}$	16	14·3	12	9·68	6·86	4·92	4	$R_D = \frac{PD}{2} + \frac{M_2 - M_1}{D}$
	$M_3 = \frac{P(D+B)(2B-D)}{24}$	—	22·2	12	7·8	4·8	3·3	2·67	
	$M_1 = \frac{P(6D^3 + 6D^2B - B^3)}{12(6D+5B)}$	11·5	11·66	12	12·55	13·95	16·4	19·2	$R_B = \frac{PB}{2} + \frac{M_2 - M_3}{B}$
	$M_2 = \frac{P(6D^3 + 5B^3)}{12(6D+5B)}$	15·4	14·10	12	9·85	7·09	5·12	4·17	R_D (END SPAN) $= \frac{PD}{2} + \frac{M_2 - M_1}{D}$
	$M_3 = \frac{P(5B^3 + 9DB^2 - 3D^3)}{12(6D+5B)}$	—	22·6	12	7·75	4·75	3·25	2·65	R_D (CENTRE SPAN) $= \frac{PD}{2}$
	$M_1 = \frac{P(5D^3 + 6D^2B - B^3)}{60(D+B)}$	11·4	11·61	12	12·6	14·11	16·71	20	R_B (END SPAN) $= \frac{PB}{2} + \frac{M_2 - M_3}{B}$ R_B (CENTRE SPAN) $= \frac{PB}{2}$
	$M_2 = \frac{P(D^3 + B^3)}{12(D+B)}$	16	14·3	12	9·68	6·86	4·92	4	R_D (END SPAN) $= \frac{PD}{2} + \frac{M_2 - M_1}{D}$ R_D (CENTRE SPAN) $= \frac{PD}{2}$
	$M_3 = \frac{P(5B^3 + 6DB^2 - D^3)}{60(D+B)}$	80	20	12	8·1	5·12	3·53	2·86	

NOTE :-

IN ADDITION TO BINS, THESE VALUES APPLY TO CONTINUOUS SPANS, WHERE INDEFINITE CONTINUITY, IN SIMILAR SEQUENCE, OCCURS.

EXAMPLES OF USE OF TABLE No. 22.

(*a*) For an example of the design of hopper bottoms see the typical examples given in the pages following *Table* No. 40.

(*b*) To determine the principal forces on the bottom of a "balanced" cylindrical tank (i.e. Intze type), given the following:

$D = 40$ ft.; $d = 25$ ft.; $\phi = 48$ deg.; $\theta = 40$ deg.
$W_1 = 582{,}000$ lb.; $W_2 = 639{,}000$ lb.; $W_3 = 296{,}000$ lb.

From *Table* No. 40; $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = 1{\cdot}55$; $\cot\theta = 1{\cdot}192$;
$\cot\phi = 0{\cdot}900$.

Vertical shear along periphery of domed portion $= F_1 = \frac{582{,}000}{3{\cdot}14 \times 25} = 7{,}400$ lb. per ft. run.

Thrust at periphery of domed portion $= T_1 = 7{,}400 \times 1{\cdot}55 = 11{,}450$ lb. per ft. run.

The values of F_1 and T_1 determine the thickness of the dome at the springing (see Chapter VII).

Outward horizontal thrust on ring beam at B from dome $= P_1 = 7{,}400 \times 1{\cdot}192 = 8{,}820$ lb. per ft.

Shear per foot along inner periphery of conical portion

$$= F_2 = \frac{639{,}000 + 296{,}000}{3{\cdot}14 \times 25} = 11{,}900 \text{ lb. per ft.}$$

Inward thrust on ring beam at B from conical section $= P_2 = 11{,}900 \times 0{\cdot}900 = 10{,}700$ lb. per ft.

Resultant circumferential force in ring beam at $B = \frac{d}{2}(P_1 - P_2) = 0{\cdot}5 \times 25$ $(8{,}820 - 10{,}700)$ = thrust of 23,500 lb.

(If P_1 exceed P_2, this circumferential force would be tensile; the ideal case is for P_1 and P_2 to be equal, thereby producing zero force in B—see note below.)

Shear per foot along outer periphery of conical portion $= F_3 = \frac{296{,}000}{3{\cdot}14 \times 40} = 2{,}350$ lb. per ft.

Outward thrust on ring beam at top of conical portion $= P_3 = 2{,}350 \times 0{\cdot}900 = 2{,}120$ lb. per ft.

Ring tension in beam at top of conical section $= 0{\cdot}5 \times 40 \times 2{,}120 = 42{,}400$ lb.

Vertical walls must be reinforced for ring tension due to the horizontal pressure of water; ring tension $= \frac{62{\cdot}4Dh}{2}$ lb. where h = head of water at section considered.

Conical portion must be reinforced to take a similar ring tension, and this steel can either be distributed throughout the height of the conical section or be concentrated in the ring beams at the top and bottom of the section.

NOTES.

In Intze tanks of large diameter, the width of ring beam B may be considerable, in which case the weight of water immediately above the beam should not be considered as contributing to the forces on the dome and conical portion. With a wide beam, for the dome calculations

W_1 = weight of contents over the net area of the dome
d = internal diameter of ring beam,

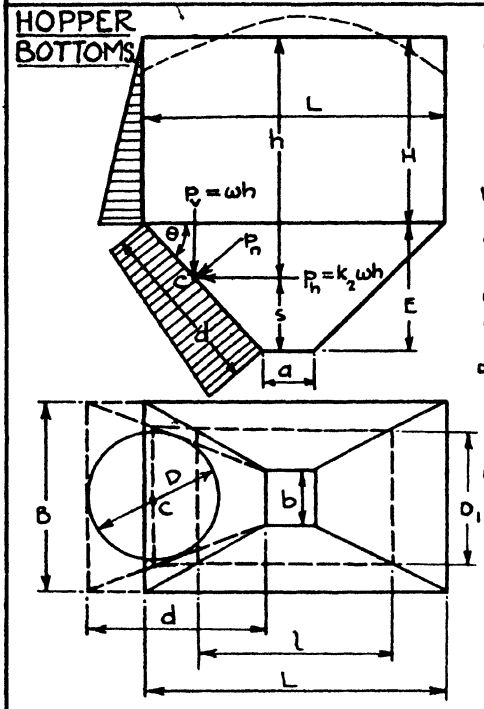
and for the conical portion

W_2 = weight of contents over the net area of the cone
d = external diameter of ring beam.

If this adjustment were made for a ring beam of reasonable width in Example (*b*) above, P_1 would balance P_2.

BOTTOMS OF BUNKERS & TANKS. TABLE Nº22.

HOPPER BOTTOMS



ω = WT. PER CU. FT. OF FILLING.

ω_s = WT. OF SLAB PER SQ. FT.

INTENSITY OF PRESSURE NORMAL TO SLAB:-

$$p_n = \omega h (k_2 \sin^2\theta + \cos^2\theta) + \omega_s \cos\theta$$

(SEE PAGE 312 FOR VALUES OF p_n)
(SEE TABLE Nº 5 FOR VALUES OF k_2)

$$W_1 = \omega\left[\frac{S}{3}\left(ab + lD_1 + \sqrt{ab.lD_1}\right) + lD_1h\right] + \omega_1$$

$$W = \omega\left[\frac{E}{3}\left(ab + LB + \sqrt{ab.LB}\right) + LBH\right] + \omega_2$$

ω_1 = WT. OF BOTTOM BELOW LEVEL C.

ω_2 = WT. OF COMPLETE BOTTOM.

DETERMINATION OF HORIZONTAL STEEL AT MIDSPAN AND CORNERS:-

B.M. = $\cdot 375\, p_n D^2$ IN.LBS/FT.

DIRECT TENSION = $\cdot 5\, p_h \sin\theta\, l$ LBS/FT.

DETERMINATION OF LONGITUDINAL STEEL:-

AT CENTRE OF SLOPE:

B.M. = $\cdot 375\, p_n D^2$ IN. LBS/FT.

DIRECT TENSION = $\dfrac{W_1}{2 \sin\theta_1 (l + D_1)}$

AT TOP OF SLOPE:

B.M. = $\cdot 375\, p_n D^2$ IN. LBS/FT.

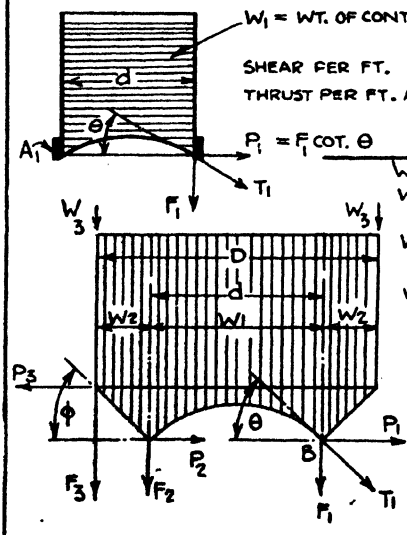
DIRECT TENSION = $\dfrac{W}{2 \sin\theta_1 (L + B)}$

VERTICAL HANGING-UP FORCE AT BASE OF WALLS

$= \dfrac{W}{2(L + B)}$ LBS./FT.

DOMICAL BOTTOMS OF CYLINDRICAL TANKS.

(UNITS:- FEET & LBS.)



W_1 = WT. OF CONTENTS ABOVE DOME INCLUDING WT. OF DOME.

SHEAR PER FT. $= F_1 = \dfrac{W_1}{3 \cdot 14\, d}$

THRUST PER FT. AT PERIPHERY OF DOME $= T_1 = F_1 \operatorname{cosec.} \theta$.

$P_1 = F_1 \cot.\theta$

RING TENSION IN BEAM AT $A_1 = \cdot 16\, W_1 \cot.\theta$.

W_1 AS ABOVE.

W_2 = WT. OF CONTENTS IN PORTION ABOVE CONE INCLUDING WT. OF CONE.

W_3 = WT. OF WALLS ETC. AND ALL LOADS FROM ROOF, ET CARRIED BY WALLS.

VALUES OF F_1, T_1 AND P_1 AS ABOVE.

$$F_2 = \frac{W_2 + W_3}{3 \cdot 14\, d} \qquad F_3 = \frac{W_3}{3 \cdot 14\, D}$$

$$P_2 = F_2 \cot.\phi. \qquad P_3 = F_3 \cot.\phi$$

RING TENSION IN BEAM AT B: $= \cdot 5\, d (P_1 - P_2)$

(F_2, F_3, P_2 & P_3 EXPRESSED IN LBS. PER FOOT.)

(IDEAL CASE: $P_1 = P_2$).

NOTES.

Summary of Working Stresses.

Concrete : c = one-third of ultimate strength of 6-in. cubes at 28 days.

	Maximum Working Stresses.
Compression due to pure bending (or combined bending and direct force) :	
Buildings, bridges, bunkers, etc.	c
Impermeable construction : in contact with water	$0{\cdot}8c$
ditto beam ribs remote from water face	c
At support section of continuous beams (only permissible if theoretical negative bending moment is used without reduction)	$1{\cdot}125c$
Direct compression (applied to whole area of columns with independent binders and to core section of columns with helical binding)	$0{\cdot}8c$
Shear stress : When whole shear resisted by concrete	$0{\cdot}1c$ ($\not>$ 150 lb. per sq. in.)
When whole shear resisted by reinforcement	$0{\cdot}4c$
Punching shear	$0{\cdot}2c$
Bond stress (see Chapter X)	$0{\cdot}1c + 25$ ($\not>$ 150 lb. per sq. in.)

(See *Table* No. 23 for limitations of shear and bond stress for impermeable construction, and Chapter IX for values of direct tension on concrete.)

Reinforcement : Tension due to bending (or combined bending and direct force) with ordinary mild steel :

	lb. per sq. in.
Buildings, bunkers, etc.	18,000
Bridges	16,000
With high yield-point steel = 50 per cent. of yield stress and	$\not>$ 27,000
Impermeable construction : Beams and slabs in contact with water	12,000
Beam ribs remote from water-face	16,000
Compression due to bending or direct compression	= stress in surrounding concrete $\times (m - 1)$
"Steel-beam" theory	18,000
Direct tension : Normal construction	18,000
Impermeable construction	12,000
Shear reinforcement : When whole shear is resisted by reinforcement	18,000
Ditto in impermeable construction	12,000

CONCRETE MIXES AND STRESSES — TABLE No. 23.

			PLAIN CONCRETE		REINFORCED CONCRETE								
	NOMINAL VOLUMETRIC PROPORTIONS		1:8	1:6	1:2:4		1:1 2/3:3 1/3 (APPROX)		1:1½:3		1:1:2		UNITS
PROPORTIONS	MIX	CEMENT	112	112	112		112		112		112		LBS.
		FINE AGG.	10	7½	2½		2 1/12 (2)		1⅞		1¼		CU. FT.
		COARSE AGG.			5		4 1/6 (4)		3¾		2½		CU. FT.
	QUANTITIES PER CU. YD. MIXED CONC.	CEMENT	(DEPENDS ON GRADING OF COMBINED AGGREGATE)		485		550		600		815		LBS
		FINE AGGREGATE			11 (14½)		10½ (14)		10¼ (13¼)		9⅓ (12)		CU. FT.
		COARSE AGGREGATE			22		21		20½		18⅔		CU. FT.
	REFERENCES	QUALITY.	ORD.	ORD.	ORD	A	ORD.	A	ORD.	A	ORD.	A	-
		LONDON BY-LAWS	V	IV	III	III A	-	-	II	II A	I	I A	-
		CODE	-	-	IV	IV (H.G)	-	-	III	III (H.G)	I	I (H.G)	-
ULTIMATE STRENGTH	COMP^N (C_u) AT 28 DAYS (SITE CUBES)	NORMAL USE (LONDON BY-LAWS, CODE, ETC.)	1110	1480	2250	2850 (2850)	2450	3155 (3300)	2550	3300	2925	3750	LB/IN.²
		M. OF T.	-	-	2250	-	2580	-	2700	-	3600	-	"
WORKING STRESSES	COMPRESSION	BENDING $c = \frac{C_u}{3}$	-	-	750	950 (760)	817	1051 (880)	850	1100	975	1250	"
		DIRECT (c_o) $0 \cdot 8c$	15 TON PER FT.²	20 TON PER FT.²	600	760 (760)	653	841 (880)	680	880	780	1000	"
		M. O. T. (c)	-	-	750	-	860	-	900	-	1200	-	"
	SHEAR	WITHOUT REINFORCEMENT $0 \cdot 1c$ (⌀150)	-	-	75	95 (75)	82	105 (85)	85	110	98	125	"
		WITH REINFORCEMENT $0 \cdot 4c$	-	-	300	380 (220)	327	420 (250)	340	440	390	600	"
		PUNCHING $0 \cdot 2c$	-	-	150	170	164	210	170	220	195	300	"
	ADHESION $0 \cdot 1c + 25$ (⌀150)		-	-	100	120 (100)	107	130 (100)	110	135	123	150	"
MODULAR RATIO	NORMAL CALC^N. (IMPERMEABLE)		-	-	15	15 (14)	15	15 (12)	15	15	15	15	E_s/E_c
	SPECIAL CALC^NS.		-	-	17·8	14·0	16·3	12·6	15·7	12·1	13·7	10·7	
	NORMAL USES FOR CONCRETES OF VARIOUS MIXES.		FOUNDATIONS, PIERS & WALLS, BLINDING, SITE CONCRETE, FILLING.		BEAMS, SLABS, COLUMNS, WALLS, STAIRS, ETC. RETAINING WALLS FOUNDATIONS IMPERMEABLE (THICK) CONST^N. BRIDGES ROADS (BOTTOM COURSE)		PRE-CAST PILES, CONCRETE, DEPOSITED UNDER WATER. IMPERMEABLE CONSTRUCTION:- TANKS, CONCRETE IN CONTAMINATED GROUND, NON-SURFACED ROOF SLABS, ETC.) ROADS (SINGLE COURSE).		RIBBED FLOORS, ARCHES. (MEDIUM SPAN) COLUMNS. (MEDIUM LOAD) LONG PRE-CAST PILES ROADS (TOP COURSE)		ARCHES (LONG SPAN.) COLUMNS (HIGH LOAD.)		

NOTES

QUANTITIES OF FINE AGGREGATE ASSUME DRY MATERIALS; VALUES IN BRACKETS ASSUME 30% BULKING DUE TO DAMPNESS

QUALITY "A" (LONDON BY-LAWS) CORRESPONDS TO "HIGH GRADE" (CODE OF PRACTICE).

STRENGTHS AND STRESSES IN BRACKETS REFER TO "IMPERMEABLE CONSTRUCTION." SEE CHAPTER IX FOR TENSILE STRESSES.

FOR RULE FOR PROPERTIES OF MIXES NOT TABULATED SEE CHAPTER IX.

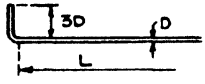
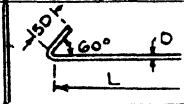
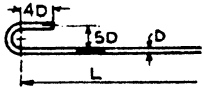
NOTES.

REINFORCEMENT — ADHESION

TABLE Nº 24.

MINIMUM LAP LENGTHS. (TENSION)

(SEE ALSO CHAP. X)

STRESS IN BAR. LB. PER IN.²	LAP LENGTH = N × DIA. OF BAR. VALUES OF N FOR 1:2:4 MIX.							
	ORD.	A.	ORD.	QUAL. A.	ORD.	A.	ORD.	A
	100 LB/IN.²	120 LB/IN.²	100 LB/IN.²	120 LB/IN.²	100 LB/IN.²	120 LB/IN.²	100 LB/IN.²	120 LB/IN.²
UP TO 10,000	40	40	35	31	30	26	25	21
11,000	41½	40	37½	33	32½	28	27½	23
12,000	44	40	40	35	35	30	30	25
13,000	46½	41	42½	37	37½	32	32½	27
14,000	49	44	45	40	40	35	35	30
15,000	51½	46	47½	42	42½	37	37½	32
16,000	54	48	50	44	45	39	40	34
17,000	56½	50	52½	46	47½	41	42½	36
18,000	59	52	55	48	50	43	45	38
20,000	64	56	60	52	55	47	50	42
22,500	67¾	61	63¾	57	58¾	52	53¾	47
25,000	71¾	67	67½	53	62½	58	57½	53
27,000	76½	72	72½	68	67½	63	62½	58
TYPE OF ANCHORAGE	STRAIGHT							

FOR BOND STRESSES FOR OTHER MIXES SEE TABLE Nº 23.

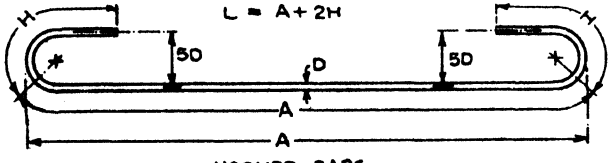
MIN.M COMPRESSION LAP LENGTH = 24 DIAMS. & FOR IMPERMEABLE CONSTRUCTION = 30 DIA.

STANDARD BENDING DIMENSIONS.

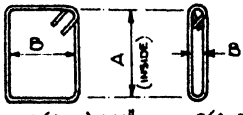
ONLY DIMENSIONS A, B, H & L NEED BE ENTERED ON BENDING SCHEDULE.

DIA. OF BAR	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1⅛"	1¼"	1⅜"	1½"
H	3"	3"	3"	4½"	4½"	4½"	6"	7½"	7½"	9"	10½"	12"	12"	13½"
H 60°	1½"	1½"	3"	3"	3"	3"	3"	4½"	4½"	6"	6"	6"	7½"	7½"
H	1½"	1½"	1½"	1½"	1½"	1½"	2"	3"	3"	3"	4½"	4½"	4½"	4½"

L = TOTAL LENGTH OF BAR.

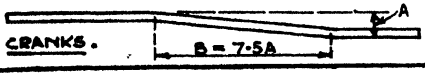


HOOKED BARS.



L = 2(A+B) + 10" L = 2(A+B) + 8"

BINDINGS.



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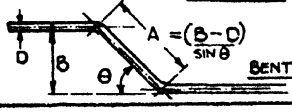


BENT-UP BARS.

EXAMPLES OF USE OF TABLE No. 25.

(*a*) To select suitable reinforcement for a beam requiring 7·42 sq. in. of steel. Width of beam = 10 in.:

$$\text{Maximum number of bars in a single layer} = \frac{10}{2\frac{1}{2}} = 4.$$

From *Table* No. 25, four $1\frac{1}{2}$-in. bars = 7·07 sq. in. which is insufficient; hence most suitable number of bars may be six; that is, 4 in bottom layer and 2 in second layer:

Six $1\frac{1}{4}$-in. bars = 7·36 sq. in. which should be satisfactory.

If the area must equal or exceed 7·42 sq. in., use

Three $1\frac{1}{2}$-in. bars in bottom layer = 5·30 sq. in.
Three 1-in. bars in top layer = 2·36 ,,
7·66 ,,

(*b*) To select suitable reinforcement for a square column requiring 3·06 sq. in.

Use four 1-in. bars = 3·14 sq. in.
or eight $\frac{3}{4}$-in. bars = 3·53 sq. in.

The 1-in. bars would be more economical, but size of column may necessitate eight bars.

(*c*) To select suitable reinforcement for a 6-in. slab requiring 0·4 sq. in. of steel per foot width.

With maximum spacing:—$\frac{5}{8}$-in. bars at 9-in. centres = 0·409 sq. in.
With minimum size bar:—$\frac{3}{8}$-in. bars at 3-in. centres = 0·442 ,,

Most economical and practicable would be an intermediate diameter and spacing, say, $\frac{1}{2}$-in. bars at 6-in. centres = 0·393 sq. in.

NOTES.

REINFORCEMENT — AREAS & WEIGHTS TABLE No. 25A

PLAIN ROUND BARS OR WIRE	DIAM. D.	3/16"	1/4"	5/16"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
	AREA	0·028	0·049	0·077	0·110	0·196	0·307	0·442	0·601	0·785	0·994	1·227	1·484	1·767
	WEIGHT PER FT.	0·094	0·167	0·261	0·375	0·667	1·043	1·502	2·044	2·670	3·379	4·173	5·049	6·008
	GAUGE No. L.W.G.	20	18	16	14	12	10	8	6	5	4	3	2	1
	DIAM. D.	·036	·048	·064	·080	·104	·128	·160	·192	·212	·232	·252	·276	·300
	AREA	·0010	·0018	·0032	·0050	·0085	·0123	·0201	·0289	·0353	·0423	·0499	·0598	·0707
	WEIGHT PER FT.	·0034	·0061	·0109	·017	·029	·044	·068	·098	·120	·143	·169	·203	·240
TWIN TWISTED	SIZE D.	3/16"	7/32"	1/4"	9/32"	5/16"	11/32"	3/8"	13/32"	7/16"	1/2"	9/16"	5/8"	11/16"
	AREA OF TWIN BAR	0·055	0·751	0·098	0·124	0·153	0·186	0·221	0·259	0·301	0·393	0·497	0·614	0·742
	WEIGHT PER FT. OF TWIN BAR	0·188	0·256	0·334	0·423	0·522	0·631	0·751	0·884	1·022	1·335	1·69	2·086	2·53
TWISTED SQUARE	SIZE D.	No. 6g	No. 5g	1/4"	5/16"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	–
	AREA (MIN)	0·037	0·045	0·063	0·098	0·140	0·250	0·391	0·563	0·766	1·000	1·266	1·565	INS.
	WEIGHT PER FT.	0·126	0·153	0·213	0·332	0·478	0·85	1·33	1·91	2·60	3·40	4·304	5·32	LB.

NOTES. AREA OF ROUND BARS = 0·7854 D^2 — WT. OF STEEL = 0·28 LB./IN.^{3} = 3·4 LB./IN.^{2} PER FT. LENGTH
" " TWIN TWISTED = 1·5708 D^2 — WT. OF ROUND BARS = 2·67 D^2 LB./FT.
" " TWISTED SQUARE = D^2 (MIN.) — TWIN-TWISTED ALSO OBTAINABLE IN 3/4", 13/16" AND 7/8" SIZES

REINFORCEMENT — AREAS (INS²) PLAIN ROUND BARS TABLE Nº25

AREAS FOR GIVEN NUMBER OF BARS.

DIA.	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
1	0·028	0·049	0·077	0·110	0·150	0·196	0·307	0·442	0·601	0·785	0·994	1·227	1·484	1·767
2	0·055	0·098	0·153	0·221	0·301	0·393	0·614	0·884	1·203	1·571	1·988	2·45	2·97	3·53
3	0·082	0·147	0·230	0·331	0·451	0·589	0·920	1·325	1·804	2·36	2·98	3·68	4·45	5·30
4	0·110	0·196	0·307	0·442	0·601	0·785	1·227	1·767	2·41	3·14	3·98	4·91	5·94	7·07
5	0·138	0·245	0·384	0·552	0·752	0·982	1·534	2·21	3·01	3·93	4·97	6·14	7·42	8·84
6	0·165	0·295	0·460	0·663	0·902	1·178	1·841	2·65	3·61	4·71	5·96	7·36	8·91	10·60
7	0·193	0·344	0·537	0·773	1·052	1·374	2·15	3·09	4·21	5·50	6·96	8·59	10·39	12·37
8	0·221	0·393	0·614	0·884	1·202	1·571	2·45	3·53	4·81	6·28	7·95	9·82	11·88	14·14
9	0·248	0·442	0·690	0·994	1·353	1·767	2·76	3·98	5·41	7·07	8·95	11·04	13·36	15·90
10	0·276	0·491	0·767	1·104	1·503	1·963	3·07	4·42	6·01	7·85	9·94	12·27	14·85	17·67
11	0·304	0·540	0·844	1·215	1·654	2·16	3·37	4·86	6·61	8·64	10·93	13·50	16·33	19·44
12	0·331	0·589	0·920	1·325	1·804	2·36	3·68	5·30	7·22	9·42	11·93	14·73	17·82	21·21
13	0·359	0·638	0·997	1·436	1·954	2·55	3·99	5·74	7·82	10·21	12·92	15·95	19·30	22·97
14	0·387	0·687	1·074	1·547	2·10	2·75	4·30	6·19	8·42	11·00	13·92	17·18	20·79	24·74
15	0·414	0·736	1·151	1·657	2·25	2·95	4·60	6·63	9·02	11·78	14·91	18·41	22·27	26·51
16	0·442	0·785	1·227	1·768	2·41	3·14	4·91	7·07	9·62	12·57	15·90	19·64	23·76	28·27
17	0·469	0·836	1·304	1·878	2·56	3·34	5·22	7·51	10·22	13·35	16·90	20·86	26·24	30·04
18	0·497	0·884	1·381	1·989	2·71	3·53	5·52	7·95	10·82	14·14	17·89	22·09	26·73	31·81
19	0·525	0·933	1·457	2·10	2·86	3·73	5·83	8·39	11·43	14·92	18·89	23·32	28·21	33·57
20	0·552	0·982	1·534	2·21	3·01	3·93	6·14	8·84	12·03	15·71	19·88	24·54	29·70	35·34

AREAS PER FT. WIDTH FOR VARIOUS SPACINGS.

3"	0·110	0·196	0·307	0·442	0·601	0·785	1·227	1·767	2·405	3·142				
3½"	0·095	0·168	0·263	0·379	0·515	0·673	1·052	1·515	2·06	2·69				
4"	0·083	0·147	0·230	0·331	0·451	0·589	0·920	1·325	1·804	2·356				
4½"	0·074	0·131	0·205	0·295	0·401	0·524	0·818	1·178	1·604	2·09				
5"	0·066	0·118	0·184	0·265	0·361	0·471	0·736	1·060	1·443	1·885				
5½"	0·060	0·107	0·167	0·241	0·328	0·428	0·669	0·964	1·312	1·714				
6"	0·055	0·098	0·153	0·221	0·301	0·393	0·614	0·884	1·203	1·571				
6½"	0·051	0·091	0·142	0·204	0·278	0·363	0·566	0·816	1·110	1·450				
7"	0·047	0·084	0·131	0·189	0·258	0·337	0·526	0·757	1·031	1·346				
7½"	0·044	0·079	0·123	0·177	0·241	0·314	0·491	0·707	0·962	1·257				
8"	0·041	0·074	0·115	0·166	0·225	0·295	0·461	0·663	0·902	1·178				
8½"	0·039	0·069	0·108	0·156	0·212	0·277	0·433	0·624	0·849	1·109				
9"	0·037	0·065	0·102	0·147	0·200	0·262	0·409	0·589	0·802	1·047				
9½"	0·035	0·062	0·097	0·140	0·190	0·248	0·388	0·558	0·760	0·992				
10"	0·033	0·059	0·092	0·133	0·180	0·236	0·368	0·530	0·722	0·942				
10½"	0·032	0·056	0·088	0·126	0·172	0·224	0·351	0·505	0·687	0·898				
11"	0·030	0·054	0·084	0·120	0·164	0·214	0·335	0·482	0·656	0·857				
12"	0·028	0·049	0·077	0·110	0·150	0·196	0·307	0·442	0·601	0·785				
15"	0·022	0·039	0·061	0·088	0·120	0·157	0·245	0·353	0·481	0·628				
18"	0·018	0·033	0·051	0·074	0·100	0·131	0·205	0·295	0·401	0·524				
24"	0·014	0·025	0·038	0·055	0·075	0·098	0·153	0·221	0·301	0·393				

GENERAL DATA

DIA.	PERI-METER	WT. PER FOOT	METRIC SIZE
INS.	INS.	LBS.	M.M.
3/16"	·589	·094	4·76
1/4"	·786	·167	6·35
5/16"	·98	·261	7·94
3/8"	1·18	·375	9·52
7/16"	1·37	·511	11·1
1/2"	1·57	·667	12·7
5/8"	1·96	1·043	15·9
3/4"	2·36	1·502	19·1
7/8"	2·75	2·044	22·2
1"	3·14	2·670	25·4
1 1/8"	3·54	3·379	28·6
1 1/4"	3·92	4·173	31·8
1 3/8"	4·31	5·049	34·9
1 1/2"	4·71	6·008	38·1

SEE ALSO
TABLE Nº 24 - ADHESION.
TABLE Nº 26 - WEIGHTS.

EXAMPLES OF USE OF TABLE No. 26.

(*a*) To find the weight of 13 ft. of 1-in bar: By direct reading from the table, weight = 34·71 lb.

(*b*) To find the weight of 5,673 ft. of $\frac{1}{2}$-in. bar: = 5,673 × 0·667 = 3,800 lb.

(*c*) To find the number of tons of steel in 10,000 ft. of $\frac{3}{4}$-in. bar: $= \frac{10,000}{1,491}$ = 6·7 tons.

(*d*) To estimate the weight of reinforcement in a slab having $\frac{1}{2}$-in. bars at 6-in. centres transversely and $\frac{5}{16}$-in. bars at 15-in. centres longitudinally (bottom steel only).

$\frac{1}{2}$-in. at 6 in.	= 12·02	lb. per sq. yd.
$\frac{5}{16}$-in. at 15 in.	= 1·88	,, ,, ,,
	13·90	
Add 10 per cent. for laps, hooks, etc.	= 13·90 + 1·39	= 15·29 lb. per sq. yd.
Add $2\frac{1}{2}$ per cent. for rolling margin		= 0·38 ,, ,, ,,
	Total	= 15·67 ,, ,, ,,

NOTES.

The amount of No. 14 or 16 gauge soft-iron binding wire required in the assembly of reinforcement is about 5 lb. per ton of bars.

REINFORCEMENT - WEIGHTS (LBS) PLAIN ROUND BARS TABLE Nº 26

DIA.		3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
LBS. PER FT.		0·094	0·167	0·261	0·375	0·511	0·667	1·043	1·502	2·044	2·670	3·379	4·173	5·049	6·008
FT. PER TON		23830	13415	8582	5975	4384	3358	2148	1491	1096	839	663	537	444	373
WT. PER GIVEN NUMBER & SIZES.	1	·094	·167	·261	·376	0·51	0·67	1·04	1·50	2·04	2·67	3·38	4·17	5·05	6·01
	2	·188	·334	·522	·752	1·02	1·34	2·09	3·00	4·09	5·34	6·76	8·34	10·10	12·02
	3	·282	·501	·783	1·128	1·53	2·00	3·13	4·51	6·13	8·01	10·14	12·52	15·15	18·02
	4	·376	·668	1·044	1·504	2·04	2·67	4·17	6·01	8·18	10·68	13·52	16·69	20·20	24·03
	5	·470	·835	1·305	1·880	2·55	3·34	5·22	7·51	10·22	13·35	16·90	20·86	25·25	30·04
	6	·564	1·002	1·566	2·256	3·06	4·01	6·26	9·01	12·26	16·02	20·28	25·03	30·29	36·05
	7	·658	1·169	1·827	2·632	3·57	4·68	7·30	10·51	14·31	18·69	23·66	29·20	35·34	42·06
	8	·752	1·336	2·088	3·008	4·08	5·34	8·34	12·02	16·35	21·36	27·04	33·38	40·39	48·06
	9	·846	1·503	2·349	3·384	4·59	6·01	9·39	13·52	18·40	24·03	30·42	37·55	45·44	54·07
	10	·940	1·670	2·610	3·760	5·11	6·68	10·43	15·02	20·44	26·70	33·80	41·72	50·49	60·08
	11	1·034	1·837	2·871	4·136	5·62	7·35	11·47	16·52	22·48	29·37	37·18	45·89	55·54	66·09
	12	1·128	2·004	3·132	4·512	6·13	8·02	12·52	18·02	24·53	32·04	40·56	50·06	60·59	72·10
	13	1·222	2·171	3·393	4·888	6·64	8·68	13·56	19·53	26·57	34·71	43·94	54·24	65·64	78·10
	14	1·316	2·338	3·654	5·264	7·15	9·35	14·60	21·03	28·62	37·38	47·32	58·41	70·68	84·11
	15	1·410	2·505	3·915	5·640	7·67	10·02	15·65	22·53	30·66	40·05	50·70	62·58	75·74	90·12
	16	1·504	2·672	4·176	6·016	8·18	10·69	16·69	24·03	32·70	42·72	54·08	66·75	80·78	96·13
	17	1·598	2·839	4·437	6·392	8·69	11·36	17·73	25·53	34·75	45·39	57·46	70·92	85·83	102·14
	18	1·692	3·006	4·698	6·768	9·20	12·02	18·77	27·04	36·79	48·06	60·84	75·10	90·88	108·14
WT. PER SQ. YARD - BARS AT VARIOUS CENTRES.	2 1/2"	4·06	7·23	11·27	16·24	22·08	28·86	45·06	64·88	88·50	115·34	146·02	180·23	218·12	260·00
	3"	3·39	6·02	9·41	13·53	18·40	24·04	37·60	54·07	73·70	96·12	121·68	150·19	181·60	216·28
	3 1/2"	2·90	5·15	8·07	11·60	15·72	20·61	32·20	46·40	63·60	82·39	104·30	128·73	155·80	185·39
	4"	2·54	4·51	7·07	10·15	13·80	18·03	28·16	40·55	55·40	72·09	91·26	112·64	136·32	162·22
	4 1/2"	2·26	4·01	6·27	9·02	12·26	16·03	25·03	36·10	49·20	64·08	81·20	100·13	121·21	144·20
	5"	2·03	3·61	5·64	8·12	11·01	14·42	22·52	32·50	44·20	57·67	73·20	90·09	109·05	129·77
	5 1/2"	1·85	3·28	5·13	7·38	10·01	13·12	20·50	29·51	40·26	52·59	66·50	81·99	99·25	117·20
	6"	1·69	3·01	4·71	6·76	9·200	12·02	18·77	27·02	36·84	48·10	60·84	75·09	90·87	108·14
	6 1/2"	1·56	2·78	4·34	6·25	8·49	11·09	17·32	24·95	34·67	44·40	56·25	69·48	83·89	99·82
	7"	1·45	2·58	4·03	5·80	7·85	10·31	16·09	23·17	31·68	41·20	52·20	64·37	77·89	92·89
	7 1/2"	1·35	2·41	3·77	5·41	7·36	9·62	15·02	21·63	29·50	38·50	48·80	60·08	72·71	86·80
	8"	1·27	2·26	3·52	5·07	6·90	9·02	14·08	20·31	27·70	36·10	45·63	56·30	68·16	81·25
	8 1/2"	1·19	2·12	3·32	4·78	6·50	8·49	13·25	19·06	26·07	33·97	43·05	53·10	64·15	76·60
	9"	1·13	2·00	3·14	4·51	6·14	8·02	12·51	18·02	24·53	32·08	40·60	50·06	60·56	72·30
	9 1/2"	1·07	1·90	2·97	4·27	5·79	7·59	11·86	17·05	23·28	30·38	38·60	47·42	57·39	68·50
	10"	1·02	1·80	2·82	4·06	5·51	7·22	11·26	16·22	22·08	28·80	36·58	45·06	54·52	64·88
	11"	0·92	1·64	2·57	3·69	5·02	6·55	10·24	14·74	20·07	26·22	33·30	40·96	49·57	58·90
	12"	0·85	1·50	2·36	3·38	4·60	6·02	9·39	13·52	18·41	24·00	30·42	37·54	45·44	54·07
	15"	0·68	1·20	1·88	2·71	3·68	4·81	7·54	10·82	14·72	19·20	24·38	30·10	36·35	43·26
	18"	0·56	1·00	1·57	2·26	3·06	4·01	6·26	9·01	12·26	16·02	20·28	25·04	30·29	36·05

EXAMPLES OF USE OF TABLES Nos. 27, 28, AND 29.

(See also page facing *Table* No. 28.)

(*a*) To design a rectangular beam section to take a bending moment of 500,000 in.-lb. with maximum stresses of 17,000 and 700 lb. per sq. in.:

(i) From *Table* No. 27: $Q = 116{\cdot}7$; hence $bd^2 = \frac{500{,}000}{116{\cdot}7} = 4{,}280$.

For a section with a total depth of 21 in. and 12 in. wide,
$bd^2 = 12 \times 19{\cdot}5^2 = 4{,}560$.

$$r = \frac{17{,}000}{700} = 24{\cdot}3\text{; from the table, } a_1 = 0{\cdot}87.$$

$$\text{Hence } A_T = \frac{500{,}000}{0{\cdot}87 \times 19{\cdot}5 \times 17{,}000} = 1{\cdot}73 \text{ sq. in.}$$
$$= \text{three } \tfrac{7}{8}\text{-in. bars (}\textit{Table}\text{ No. 25).}$$

(ii) Alternative method if stresses are 16,000 and 600 (or any other pairs of stresses tabulated on *Tables* Nos. 28 and 29): Assume effective depth = 20 in., for which, from *Table* No. 28, M. of R. of beam 1 in. wide without compression steel = 38,100 in.-lb.

$$b = \frac{500{,}000}{38{,}100} = 13{\cdot}1 \text{ in., say, 14 in. wide.}$$

$$A_T = 13{\cdot}1 \times 0{\cdot}135 = 1{\cdot}78 \text{ sq. in.} = \text{three } \tfrac{7}{8}\text{-in. bars.}$$
Total depth of beam = 20 + 1·5 = 21·5 in., say, 22 in.

Therefore a section 22 in. by 14 in. with three $\frac{7}{8}$-in. bars is satisfactory; this would have to be checked for shear.

(*b*) To find the maximum stresses in a rectangular section 15 in. overall depth (effective depth = $13\frac{1}{2}$ in.) and 9 in. wide, reinforced in tension with two 1-in. bars, and subject to a B.M. of 200,000 in.-lb. $A_T = 1{\cdot}57$ (*Table* No. 25).

$$p = \frac{1{\cdot}57 \times 100}{9 \times 13{\cdot}5} = 1{\cdot}29.$$

From *Table* No. 27: $a_1 = 0{\cdot}85$ and $r = 18$,

$$t = \frac{200{,}000}{0{\cdot}85 \times 13{\cdot}5 \times 1{\cdot}57} = 11{,}100 \text{ lb. per sq. in.}$$

$$c = \frac{11{,}100}{18} = 617 \text{ lb. per sq. in.}$$

(*c*) To find the resistance moment of the section specified in (*b*) if stresses are not to exceed either 18,000 lb. or 700 lb. per sq. in.

p (as above) = 1·29, $a_1 = 0{\cdot}85$, $n_1 = 0{\cdot}46$.

R.M. (steel) = $1{\cdot}57 \times 18{,}000 \times 0{\cdot}85 \times 13{\cdot}5 = 324{,}000$ in.-lb.

$$\text{R.M. (concrete)} = 9 \times 0{\cdot}46 \times \frac{700}{2} \times 0{\cdot}85 \times 13{\cdot}5^2 = 224{,}000 \text{ in.-lb.}$$

Hence the compressive stress controls, and maximum R.M. = 224,000 in.-lb.

(*d*) To find the maximum stresses in a rectangular section 15 in. overall depth ($d = 13{\cdot}5$ in.) and 9 in. wide which is reinforced in tension with two 1-in. bars ($A_T = 1{\cdot}57$ sq. in.), and subject to a B.M. of 200,000 in.-lb.; $m = 17{\cdot}8$.

$$p = \frac{1{\cdot}57 \times 100}{9 \times 13{\cdot}5} = 1{\cdot}29.$$

$$\text{From } \textit{Table} \text{ No. 27: } n_1 = \sqrt{\left(\frac{17{\cdot}8 \times 1{\cdot}29}{100}\right)^2 + \frac{2 \times 17{\cdot}8 \times 1{\cdot}29}{100}} - \frac{17{\cdot}8 \times 1{\cdot}29}{100}$$
$$= 0{\cdot}488$$

$$a_1 = 1 - \frac{0{\cdot}488}{3} = 0{\cdot}837$$

$$r = m\left(\frac{1}{n_1} - 1\right) = 17{\cdot}8\left(\frac{1}{0{\cdot}488} - 1\right) = 25{\cdot}8.$$

$$t = \frac{200{,}000}{0{\cdot}837 \times 13{\cdot}5 \times 1{\cdot}57} = 11{,}300 \text{ lb. per sq. in.}$$

$$c = \frac{11{,}300}{25{\cdot}8} = 440 \text{ lb. per sq. in.}$$

RECTANGULAR BEAM FACTORS — TABLE No. 27

RATIO OF STRESSES $\frac{t}{c} = r$	NEUTRAL AXIS FACTOR n_1	LEVER ARM FACTOR a_1	PERCENTAGE OF STEEL p	VALUES OF $Q = \cdot 5 n_1 a_1 c = \frac{R.M.}{bd^2}$ — MAX. CONC. STRESS c.	MAXIMUM STEEL STRESS = t. 12,000	14,000	15,000	16,000	17,000	18,000	20,000	22,400
5	·75	·75	7·50	500	83·9	77·1	74·1	71·3	68·7	66·3	62·0	57·5
10	·60	·80	3·00	550	96·7	89·4	86·4	82·9	80·1	77·3	72·5	67·3
12	·56	·82	2·30	600	110·3	102·1	98·4	95·0	91·8	88	83·5	78
14	·52	·83	1·86	650	123·9	115·2	111·3	107·7	104·2	100·8	94·9	87·8
15	·50	·83	1·66	700	137·9	128·6	124·4	120·4	116·7	113	106·6	100
16	·49	·84	1·52	750	152·3	142·2	137·6	133·5	129·2	126	118·8	111·5
17	·47	·84	1·38	800	166·6	156·3	151·2	146·9	142·7	138·5	131·2	125·5
18	·46	·85	1·26	850	180·5	170·5	168	160·5	157	152	143·5	138
19	·44	·85	1·16	900	196	186	179·5	174·5	168	165·5	157	150·5
20	·43	·86	1·07	950	211·5	200	194	188	183	179	170	163
21	·42	·86	·99	1000	226	214	208·5	203	198	193	184	173
22	41	·87	·93									
23	·40	·87	·86									
24	·39	·87	·80									
25	·38	·88	·75									
26·7	·36	·88	·675									
28	·35	·88	·63									
29	·34	·89	·59									
30	·33	·89	·56									
32	·32	·89	·50									
35	·30	·90	·43									
40	·27	·91	·34									
45	·25	·92	·28									
50	·23	·92	·23									

FORMULÆ (FOR SIMPLE BENDING).

(APPLICABLE TO SINGLY REINFORCED RECTANGULAR SECTIONS)

DEPTH OF NEUTRAL AXIS :– $n_1 d = \dfrac{d}{1 + \frac{r}{m}} = n$

OR $n_1 = \sqrt{(\cdot 01mp)^2 + \cdot 02mp} - \cdot 01mp$

LEVER ARM :– $a_1 d = 1 - \dfrac{n}{3} = a$

RESISTANCE MOMENT OF TENSION STEEL :– $R.M. = A_T t a$

RESISTANCE MOMENT OF CONCRETE :– $R.M. = Q b d^2$

WHERE $Q = \cdot 5 n_1 a_1 c$

t = MAX. ALLOWABLE STRESS IN TENSION STEEL
c = " " " " CONCRETE
m = MODULAR RATIO (TABULATED VALUES ARE FOR $m = 15$)
d = EFFECTIVE DEPTH OF SECTION; b = BREADTH.
p = PERCENTAGE OF TENSILE REINFORCEMENT = $\dfrac{100 A_T}{bd}$
A_T = AREA OF TENSILE REINFORCEMENT.

$r = \dfrac{t}{c}$

NOTES ON THE USE OF TABLES Nos. 28, 29, AND 31 FOR VARIOUS STRESSES.

If given stresses are not tabulated but have any of the ratios $\left(\frac{t}{c}\right)$ given below, modify the appropriate resistance moments or steel areas thus:

Stress Ratio	$13\frac{2}{3}$	(Slabs)	$- k_3 \times$	values on	*Table*	31	for	12,000/880
,, ,,	19	(Beams)	$- k_2 \times$	,, ,,	,,	29	,,	18,000/950
,, ,,	20	(Beams)	$- k_1 \times$	,, ,,	,,	28	,,	16,000/800
,, ,,	24	(Beams)	$- k_2 \times$	,, ,,	,,	29	,,	18,000/750
,, ,,	24	(Slabs)	$- k_3 \times$	,, ,,	,,	31	,,	18,000/750
,, ,,	$26\frac{2}{3}$	(Beams)	$- k_1 \times$	,, ,,	,,	28	,,	16,000/600

Where $k_1 = \dfrac{\text{Given steel stress}}{16{,}000}$ $\quad k_2 = \dfrac{\text{Given steel stress}}{18{,}000}$

$k_3 = \dfrac{\text{Given steel stress}}{12{,}000}$

EXAMPLES OF USE OF TABLES Nos. 27, 28, AND 29.

(See also page facing *Table* No. 27 and *Fig.* 37.)

(*a*) To find the amount of compressive and tensile reinforcement that should be provided in a 12-in. square section subject to a bending moment of 150,000 in.-lb.; maximum stresses not to exceed 16,000 lb. and 600 lb. per sq. in.:

From *Table* No. 27: $Q = 95$; $d = 10{\cdot}5$ in.

R.M. of concrete $= 95 \times 12 \times 10{\cdot}5^2 = 126{,}500$ in.-lb.

$R_s = 150{,}000 - 126{,}500 = 23{,}500$ in.-lb.

$$r = \frac{16{,}000}{600} = 26{\cdot}7;\ \text{from } \textit{Table} \text{ No. 27: } a_1 = 0{\cdot}88;\ n_1 = 0{\cdot}36.$$

Assume $f = 1\frac{1}{2}$ in. and $m = 15$.

$a_s = 10{\cdot}5 - 1{\cdot}5 = 9$ in. $n = 0{\cdot}36 \times 10{\cdot}5 = 3{\cdot}78$ in.

$$f_c = \frac{3{\cdot}78 - 1{\cdot}5}{3{\cdot}78} \times 14 \times 600 = 5{,}060 \text{ lb. per sq. in.}$$

$$A_c = \frac{23{,}500}{5{,}060 \times 9} = 0{\cdot}515 \text{ sq in., say, two } \tfrac{5}{8}\text{-in. bars.}$$

$$a_r = \frac{150{,}000}{(0{\cdot}515 \times 5{,}060) + (0{\cdot}5 \times 12 \times 3{\cdot}78 \times 600)} = 9{\cdot}2 \text{ in.}$$

$$A_T = \frac{150{,}000}{9{\cdot}2 \times 16{,}000} = 1{\cdot}02 \text{ sq. in., say, two } \tfrac{7}{8}\text{-in. bars (}\textit{Table}\text{ No. 25).}$$

(*b*) Section in example (*a*) subject to a moment of 200,000 in.-lb.:

$R_s = 200{,}000 - 126{,}500 = 73{,}500$ in.-lb.

$$A_c = \frac{73{,}500}{5{,}060 \times 9} = 1{\cdot}61 \text{ sq. in.}$$

Approximate $a_r = 9{\cdot}1$ in.

$$A_T = \frac{200{,}000}{16{,}000 \times 9{\cdot}1} = 1{\cdot}37 \text{ sq. in.}$$

Since A_c is greater than A_T, the "steel-beam theory" is applicable, and

$$A_T = A_c = \frac{200{,}000}{16{,}000 \times 9} = 1{\cdot}39 \text{ sq. in.,}$$

say, two 1-in. bars top and bottom.

(*c*) To select the shallowest rectangular section 10 in. wide to resist a B.M. of 500,000 in.-lb. with normal stresses of 16,000 and 600, but if $A_c = A_T$ maximum concrete stress may be 800 lb. per sq. in. (that is, "steel-beam theory" not allowed).

$$\text{R.M. per inch width} = \frac{500{,}000}{10} = 50{,}000 \text{ in.-lb.}$$

With $A_c = A_T$ and stresses 16,000 and 800, from *Table* No. 28, effective depth $= 13$ in. (R.M. $= 52{,}000$); total depth, say, 14 in. $A_c = A_T = 10 \times 0{\cdot}290 = 2{\cdot}90$ sq. in., say, three $1\frac{1}{8}$-in. bars.

(*d*) To select a rectangular section to take 400,000 in.-lb. without compression steel, with maximum stresses 18,000 and 675 lb. per sq. in.

Stress ratio $= 26{\cdot}7$; therefore use the tabulated values given on *Table* No. 28 for 16,000 and 600;

$$k_1 = \frac{18{,}000}{16{,}000} = 1{\cdot}125.$$

Therefore select a section from *Table* No. 28 to give a R.M. $= \dfrac{400{,}000}{1{\cdot}125}$ $= 355{,}000$ in.-lb.

With effective depth $= 18$ in., R.M. per inch $= 30{,}800$ in.-lb.

$$\text{Hence } b = \frac{355{,}000}{30{,}800} = 11{\cdot}5 \text{ in.}$$

$$A_T = 11{\cdot}5 \times 0{\cdot}121 = 1{\cdot}4 \text{ sq. in.}$$

Therefore a suitable section would be 12 in. wide, 20 in. overall depth with three $\frac{3}{4}$-in. bars in bottom.

RESISTANCE MOMENTS -- RECT^R BEAMS. TABLE Nº28.

MAXIMUM STRESS IN TENSION STEEL = 16,000 LBS. PER SQ. IN.

EFFECTIVE DEPTH INS.	MAX. C = 600 LBS. PER IN.²				MAX. C = 800 LBS. PER IN.²			
	$A_c = 0$		$A_c = A_T$		$A_c = 0$		$A_c = A_T$	
	R.M.	A_T.	R.M.	A_T.	R.M.	A_T.	R.M.	A_T.
10	9,540	·068	15,000	·100	14,700	·107	28,000	·200
11	11,500	·074	18,500	·115	17,800	·117	36,000	·230
12	13,700	·081	21,500	·128	21,100	·128	43,000	·260
13	16,100	·088	25,500	·140	24,800	·139	52,000	·290
14	18,700	·095	30,000	·150	28,700	·150	60,000	·320
15	21,400	·101	35,000	·165	33,000	·160	70,000	·335
16	24,400	·108	40,000	·175	37,600	·171	80,000	·350
17	27,500	·115	45,000	·185	42,400	·182	90,000	·370
18	30,800	·121	50,000	·198	47,500	·192	100,000	·390
19	34,500	·128	55,000	·210	53,000	·203	110,000	·410
20	38,100	·135	61,000	·223	58,800	·214	122,000	·430
21	42,000	·142	67,000	·235	65,000	·225	134,000	·450
22	46,000	·148	75,000	·245	71,000	·235	148,000	·475
23	50,000	·155	84,000	·258	77,500	·246	162,000	·500
24	56,000	·162	92,500	·270	86,000	·257	178,000	·525
25	59,500	·169	102,000	·280	92,000	·267	196,000	·550
26	64,000	·175	110,000	·290	99,000	·278	215,000	·580
27	69,000	·182	120,000	·300	107,000	·288	235,000	·610
28	74,500	·189	130,000	·310	115,000	·300	255,000	·640
29	80,000	·196	138,000	·325	124,000	·310	275,000	·665
30	85,500	·203	148,000	·340	132,000	·320	293,000	·700
31	92,000	·209	158,000	·350	141,000	·331	320,000	·730
32	98,000	·216	168,000	·365	150,000	·342	343,000	·760
33	104,000	·222	179,000	·375	168,000	·352	364,000	·790
34	110,000	·229	189,000	·390	170,000	·363	383,000	·815
35	117,000	·236	200,000	·400	180,000	·374	415,000	·840
36	123,000	·243	215,000	·417	190,000	·385	440,000	·875

R.M. = RESISTANCE MOMENT IN INCH LBS. PER ONE INCH WIDTH OF SECTION.

A_T = AREA OF TENSILE REINFORCEMENT IN INS.² PER ONE INCH WIDTH.

A_c = AREA OF COMPRESSIVE REINFORCEMENT IN INS.² (VALUES OF R.Ms. WITH $A_c = A_T$ ARE APPROXIMATE ONLY, AS EXACT VALUES DEPEND UPON SIZE AND ARRANGEMENT OF BARS.)

$m = 15$

FOR NOTES ON THE USE OF THIS TABLE FOR OTHER STRESSES SEE TABLE Nº 27.

NOTES.

(For examples of use of this Table, see pages facing *Tables* Nos. 27 and 28.)

RESISTANCE MOMENTS – RECT^R BEAMS. TABLE Nº29.

MAXIMUM STRESS IN TENSION STEEL = 18,000 LB. PER SQ. IN.

EFFECTIVE DEPTH INS.	C = 750 LB/IN²				C = 950 LB/IN²			
	$A_c = 0$		$A_c = A_T$		$A_c = 0$		$A_c = A_T$	
	R.M.	A_T.	R.M.	A_T.	R.M.	A_T.	R.M.	A_T.
10	12,600	0·080	23,000	0·142	17,900	0·116	43,000	0·275
11	15,300	0·088	27,000	0·156	21,700	0·128	53,000	0·303
12	18,200	0·096	33,000	0·170	25,800	0·139	63,000	0·330
13	21,300	0·104	38,000	0·185	30,300	0·151	73,000	0·358
14	24,700	0·112	44,000	0·200	35,100	0·162	85,000	0·385
15	28,450	0·120	51,000	0·213	40,300	0·174	98,000	0·413
16	32,300	0·128	58,000	0·227	45,800	0·186	111,000	0·440
17	36,400	0·136	65,000	0·241	51,700	0·197	126,000	0·468
18	40,800	0·144	73,000	0·256	58,000	0·209	141,000	0·495
19	45,500	0·152	82,000	0·270	64,600	0·220	157,000	0·523
20	50,400	0·160	90,000	0·284	71,600	0·232	174,000	0·550
21	55,600	0·168	100,000	0·298	79,000	0·244	192,000	0·578
22	61,000	0·176	109,000	0·312	86,600	0·255	210,000	0·605
23	65,400	0·184	120,000	0·327	94,600	0·267	230,000	0·633
24	72,500	0·192	130,000	0·341	103,100	0·278	250,000	0·660
25	78,800	0·200	142,000	0·355	111,900	0·290	271,000	0·688
26	85,200	0·208	153,000	0·369	121,000	0·302	293,000	0·715
27	91,800	0·216	165,000	0·383	130,500	0·313	316,000	0·743
28	98,800	0·224	177,000	0·398	140,300	0·325	340,000	0·770
29	106,000	0·232	190,000	0·412	150,700	0·336	364,000	0·798
30	113,400	0·240	203,000	0·426	161,100	0·348	391,000	0·828
31	121,100	0·248	217,000	0·440	172,000	0·360	417,000	0·853
32	129,000	0·256	232,000	0·454	183,300	0·371	445,000	0·880
33	137,200	0·264	246,000	0·469	195,000	0·383	473,000	0·908
34	145,700	0·272	261,000	0·483	206,900	0·394	502,000	0·935
35	154,500	0·280	277,000	0·497	219,200	0·406	533,000	0·963
36	163,000	0·288	293,000	0·511	232,000	0·418	564,000	0·990

R.M. = RESISTANCE MOMENT IN INCH-LB. PER ONE INCH WIDTH OF SECTION.

A_T = AREA OF TENSILE REINFORCEMENT IN IN.² PER ONE INCH WIDTH.

A_c = AREA OF COMPRESSIVE REINFORCEMENT IN IN.² (VALUES OF R.MS WITH $A_c = A_T$ ARE APPROXIMATE ONLY, AS EXACT VALUES DEPEND UPON SIZE AND ARRANGEMENT OF BARS.)

$m = 15$

* FOR NOTES ON USE OF THIS TABLE FOR OTHER STRESSES SEE TABLE Nº 27

EXAMPLES OF USE OF TABLE No. 30.

To design a slab to take a bending moment of 2,000 ft.-lb. with stresses not exceeding 16,000 lb. per sq. in. on steel and 700 lb. per sq. in. on concrete.

(*a*) Maximum concrete and steel stresses:

Effective depth required $= 0{\cdot}0912\sqrt{2{,}000} = 4{\cdot}07$ in.

Total slab thickness $= 5$ in.

Area of steel required $= 0{\cdot}0095\sqrt{2{,}000} = 0{\cdot}420$ sq. in.

(*b*) Specified total slab thickness $= 6$ in.; effective depth $= 5{\cdot}25$ in.

$$k_1 = \frac{5{\cdot}25}{\sqrt{2{,}000}} = 0{\cdot}118.$$

With $t = 16{,}000$, corresponding value of $k_2 = 0{\cdot}007$.

Area of steel required $= 0{\cdot}007\sqrt{2{,}000} = 0{\cdot}312$ sq. in.

(*c*) Specified total slab thickness $= 4\frac{1}{2}$ in.; effective depth $= 3{\cdot}75$ in.

$$k_1 = \frac{3{\cdot}75}{\sqrt{2{,}000}} = 0{\cdot}084.$$

With $c = 700$, corresponding value of $k_2 = 0{\cdot}014$ approx.

Area of steel required $= 0{\cdot}014\sqrt{2{,}000} = 0{\cdot}625$ sq. in.

NOTES.

Table No. 30 is based on $m = 15$.

EXAMPLE OF USE OF TABLE No. 31.

To design a slab to take a bending moment of 2,000 ft.-lb. (= 24,000 in.-lb.) per foot width. Maximum stresses: $t = 18{,}000$, and $c = 750$ lb. per sq. in.

Resistance moment of a 5-in. slab with these stresses = 27,360 in.-lb.

Area of steel required for this resistance moment = 0·408 sq. in. per foot width.

For B.M. = 24,000 in.-lb., area of steel $= \dfrac{0{\cdot}408 \times 24{,}000}{27{,}360} = 0{\cdot}36$ sq. in. per ft.

This is given by $\frac{1}{2}$-in. bars at $6\frac{1}{2}$-in. centres in a 5-in. slab.

SLAB DESIGN FACTORS

TABLE No 30

t	12,000		14,000		16,000		17,000		18,000		20,000		22,400	
c	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1
100	·0024	·432	·0019	·462	·0016	·490	·0014	·505	·0013	·517	·0011	·542	·0010	·570
150	·0035	·299	·0028	·318	·0023	·336	·0021	·344	·0020	·353	·0017	·369	·0014	·388
200	·0046	·231	·0037	·245	·0031	·259	·0028	·265	·0026	·271	·0022	·283	·0019	·297
250	·0057	·191	·0046	·202	·0038	·212	·0035	·217	·0032	·222	·0027	·231	·0023	·242
300	·0067	·164	·0054	·173	·0045	·181	·0041	·185	·0038	·189	·0033	·196	·0028	·205
350	·0077	·145	·0062	·151	·0052	·159	·0047	·162	·0044	·165	·0038	·172	·0032	·179
400	·0087	·130	·0070	·136	·0058	·142	·0053	·145	·0049	·148	·0042	·153	·0036	·160
450	·0096	·118	·0078	·124	·0065	·129	·0059	·131	·0055	·134	·0047	·139	·0040	·144
500	·0105	·109	·0085	·114	·0071	·118	·0065	·121	·0060	·123	·0052	·127	·0044	·132
525	·0110	·105	·0089	·1097	·0074	·114	·0068	·116	·0063	·118	·0054	·122	·0046	·127
550	·0114	·102	·0093	·1058	·0077	·1098	·0071	·112	·0066	·114	·0057	·118	·0048	·122
575	·0118	·098	·0096	·1022	·0080	·1061	·0074	·1079	·0068	·1097	·0040	·113	·0050	·117
600	·0123	·095	·0100	·0990	·0083	·1026	·0077	·1043	·0071	·1061	·0061	·110	·0052	·113
625	·0127	·092	·0103	·0960	·0086	·0994	·0079	·1011	·0073	·1027	·0063	·106	·0054	·110
650	·0131	·090	·0107	·0932	·0089	·0965	·0082	·0980	·0076	·0996	·0066	·103	·0056	·106
675	·0135	·087	·0110	·0906	·0092	·0937	·0085	·0952	·0078	·0967	·0068	·0996	·0058	·103
700	·0139	·085	·0113	·0882	·0095	·0912	·0087	·0926	·0081	·0941	·0070	·0968	·0060	·1001
725	·0143	·083	·0117	·0859	·0098	·0888	·0090	·0901	·0083	·0915	·0072	·0942	·0062	·0975
750	·0147	·081	·0120	·0838	·0101	·0866	·0093	·0879	·0086	·0892	·0074	·0917	·0064	·0947
775	·0151	·079	·0123	·0819	·0103	·0845	·0095	·0857	·0088	·0870	·0077	·0894	·0066	·0923
800	·0155	·078	·0127	·0800	·0106	·0825	·0098	·0837	·0091	·0849	·0079	·0873	·0067	·0900
825	·0159	·076	·0130	·0783	·0109	·0807	·0100	·0818	·0093	·0830	·0081	·0852	·0069	·0879
850	·0163	·074	·0133	·0766	·0112	·0789	·0103	·0800	·0095	·0811	·0083	·0833	·0071	·0859
875	·0166	·073	·0136	·0750	·0114	·0773	·0105	·0783	·0098	·0794	·0085	·0815	·0073	·0840
900	·0170	·071	·0139	·0736	·0117	·0757	·0108	·0767	·0100	·0778	·0087	·0798	·0075	·0822
925	·0174	·070	·0142	·0722	·0120	·0742	·0110	·0752	·0102	·0762	·0089	·0782	·0076	·0805
950	·0177	·069	·0146	·0708	·0122	·0728	·0113	·0738	·0105	·0748	·0091	·0766	·0078	·0789
975	·0181	·068	·0149	·0696	·0125	·0715	·0115	·0724	·0107	·0734	·0093	·0752	·0080	·0773
1000	·0185	·067	·0152	·068	·0127	·070	·0118	·071	·0109	·072	·0095	·074	·0082	·076

NOTES:- M = BENDING MOMENT IN FT. LBS. PER FOOT WIDTH.
t = MAX. ALLOWABLE TENSILE STRESS IN LBS. PER IN.^{2}
c = MAX. ALLOWABLE COMPRESSIVE STRESS IN LBS. PER IN.^{2}

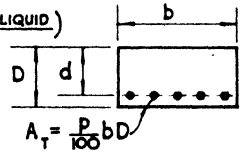
EFFECTIVE DEPTH = $k_1\sqrt{M}$ INCHES.

AREA OF TENSILE STEEL = $k_2\sqrt{M}$ SQ. IN. PER FT. WIDTH. m = 15

SLABS FOR LIQUID-CONTAINING STRUCTURES TABLE Nº 31A

NOTES

CONCRETE: 1-1⅔-3½ NOMINAL MIX AS ON TABLE Nº 23.
MINIMUM SLAB THICKNESS: $D \not< 4''$ OR $\not< \left(1'' + \frac{\text{DEPTH BELOW TOP OF LIQUID}}{40}\right)$
MODULAR RATIO $m = 12$.
MINIMUM COVER = 1 INCH.
MINIMUM PERCENTAGE OF REINFORCEMENT $p = 0{\cdot}3\%$.
c = MAX. CONCRETE COMPRESSIVE STRESS IN BENDING.
c_t = MAX. CONCRETE TENSILE STRESS.
t = MAX. TENSILE STRESS IN REINFORCEMENT.



CASE I. BENDING ONLY. TENSION ON FACE IN CONTACT WITH LIQUID

RESISTANCE TO CRACKING DETERMINES THICKNESS (D)

$c_t \not> 250$ LB/IN.² R.M. = $Q_T bD^2$

WHERE $Q_T = \frac{250}{1-n_1}\left[\frac{1}{3} - (1-n_1)n_1 + 0{\cdot}11\left(\frac{d}{D} - n_1\right)^2\right]$

$$n_1 = \frac{0{\cdot}5 + 0{\cdot}11p\frac{d}{D}}{1 + 0{\cdot}11p}$$

TABLE A - VALUES OF Q_T

p (PER CENT)		0·30	0·60	1·00
$\frac{d}{D}$ =	0·75	Q_T = 43·2	44·8	46·6
	0·80	43·8	45·9	48·9
	0·85	44·4	47·1	51·2
	0·90	45·2	48·5	53·5

DESIGN FOR STRENGTH (NEGLECT TENSION IN CONCRETE)

DETERMINES REINFORCEMENT (A_T)

$t \not> 12{,}000$ LB/IN.² [$c \not> 880$ LB/IN.² R.M. = $174\,bd^2$]

$$A_T = \frac{B.M.}{10{,}200d}$$

TABLE B - ECONOMICAL DESIGN
c_t = 250 LB/IN.² ON UNCRACKED SECTION.
t = 12,000 LB/IN.² ON CRACKED SECTION.

$\frac{d}{D}$	R.M. $= Q_T bD^2$	$p = \frac{100A_T}{bD}$	PRACTICAL RANGE OF APPLICABILITY
0·75	$44\frac{2}{3}bD^2$	0·585	UP TO 20,000 IN.LB. PER FT. WIDE
0·80	$45\frac{1}{3}$ "	0·560	18,000 TO 40,000 DITTO
0·85	$46\frac{2}{3}$ "	0·538	25,000 TO 72,000 DITTO
0·90	$47\frac{1}{3}$ "	0·519	OVER 50,000 DITTO

EXAMPLE.

TO DESIGN SLAB FOR B.M. = 24,000 IN.LB./FT.

FROM TABLE B, $\frac{d}{D} = 0{\cdot}80$ APPROX; $Q_T = 45\frac{2}{3}$

THEREFORE $D = \sqrt{\frac{24{,}000}{12 \times 45{\cdot}67}} = 6{\cdot}62$ I.E. 7" SLAB. $p = 0{\cdot}560\%$

WITH ½" BARS & 1" COVER, $d = 5{\cdot}75''$

A_T (APPROX) $= \frac{0{\cdot}560 \times 12 \times 6{\cdot}62}{100} = 0{\cdot}445$ IN.²

OR MORE ACCURATELY $A_T = \frac{24{,}000}{10{,}200 \times 5{\cdot}75} = 0{\cdot}408$ IN.² SAY ½" BARS AT 6" CENTRES.

CASE II. BENDING ONLY. TENSION ON FACE REMOTE FROM LIQUID

IF D LESS THAN 9 INS. DESIGN AS FOR CASE I.

IF D = OR > 9 INS. DESIGN FOR STRENGTH ONLY; $c \not> 880$ LB/IN.² $t \not> 12{,}000$ LB/IN.²

$d = \sqrt{\frac{B.M.}{174b}}$ $D = d + 1''$ (MIN.) + HALF BAR DIA. $A_T = \frac{B.M.}{10{,}200d}$

SEE TABLE Nº 31 FOR R.M. AND A_T FOR THIS CONDITION.

CASE III. DIRECT TENSION ONLY

DESIGN FOR STRENGTH (NEGLECT CONCRETE) DETERMINES REINFORCEMENT. $t \not> 12000$ LB/IN.²

DIRECT TENSION = T (LB/FT.) $A_T \not< \frac{T}{12{,}000}$ IN.² PER FT.

RESISTANCE TO CRACKING DETERMINES THICKNESS. $c_t \not> 175$ LB/IN.² $D \not< \frac{T}{2{,}100} - 0{\cdot}92A_T$

CASE V

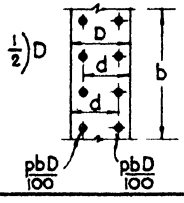
AS CASE IV BUT WITH TENSION ON FACE REMOTE FROM LIQUID.

WITH $D < 9''$ ADOPT CASE IV
WITH $D \not< 9''$ ADOPT METHOD ON TABLE Nº 36 WITH $m = 12$, $t = 12{,}000$, $c = 880$.

CASE IV. DIRECT TENSION COMBINED WITH BENDING MOMENT. TENSION ON LIQUID FACE. $e = \frac{B.M.}{T}$

CASE (a)

$e \not> \left(\frac{d}{D} - \frac{1}{2}\right)D$



$c_t \not> 175$ LB/IN.² $t = 12{,}000$ LB/IN.² ASSUME D AND $\frac{d}{D}$

FIND $p = \frac{T}{240bD}\left[1 + \frac{e}{D\left(\frac{d}{D} - \frac{1}{2}\right)}\right]$

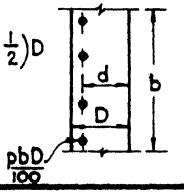
SUBSTITUTE IN

$$\frac{T}{bD}\left[\frac{1}{1+0{\cdot}22p} + \frac{e}{D\left[0{\cdot}0417 + 0{\cdot}11p\left(\frac{d}{D} - \frac{1}{2}\right)^2\right]}\right] \not> 175$$

ADJUST D UNTIL FORMULA SATISFIED.

CASE (b)

$e > \left(\frac{d}{D} - \frac{1}{2}\right)D$



$c_t \not> 250$ LB/IN.² $t \not> 12000$ LB/IN.² ASSUME D AND $\frac{d}{D}$ (FROM TABLE B)

FIND $p = \frac{T}{120bD}\left(1 + \frac{\frac{e}{D} + \frac{1}{2} - \frac{d}{D}}{0{\cdot}85\frac{d}{D}}\right)$ AND $n_1 = \frac{0{\cdot}5 + 0{\cdot}11p\frac{d}{D}}{1 + 0{\cdot}11p}$

SUBSTITUTE IN

$$\frac{T}{bD}\left[\frac{1}{1+0{\cdot}11p} + \frac{e(1-n_1)}{\left[\frac{1}{3} - (1-n_1)n_1 + 0{\cdot}11p\left(\frac{d}{D} - n_1\right)^2\right]D}\right] \not> 250$$

ADJUST D UNTIL FORMULA SATISFIED

SLABS. RESISTANCE MOMENTS AND REINFORCEMENT. TABLE Nº 31.

STRESSES ON STEEL AND CONCRETE		t = 18,000 c = 750		t = 18,000 c = 750		t = 12,000 c = 880		
MODULAR RATIO		m = 15		m = 17·8		m = 12		
TOTAL THICKNESS OF SLAB	EFFECTIVE DEPTH d.	1:2:4 MIX (ORD.) LONDON BY-LAWS.		1:2:4 MIX (ORD.) CODE OF PRACTICE.		1 : 1⅔ : 3⅓ (QUAL. A.) IMPERMEABLE CONSTⁿ.		
		R.M. = 1512 d^2	A_T = ·096d	R.M. = 1644 d^2	A_T = ·107d	d	R.M.	A_T
3"	2·25	7,650	0·22	8,300	0·25		NOT LESS THAN 4" THICK.	
3½"	2·75	11,480	0·26	12,460	0·29			
4"	3·25	16,030	0·31	17,420	0·35	2·81	16,500	0·58
4½"	3·75	21,320	0·36	23,120	0·40	3·31	22,900	0·67
5"	4·25	27,360	0·41	29,750	0·45	3·81	30,300	0·73
5½"	4·75	34,200	0·46	37,150	0·51	4·31	38,800	0·89
6"	5·25	41,750	0·50	45,400	0·56	4·81	48,400	0·99
6½"	5·75	50,100	0·55	54,400	0·62	5·31	58,800	1·09
7"	6·25	59,100	0·60	64,300	0·67	5·81	70,700	1·19
7½"	6·75	69,000	0·65	75,000	0·72	6·25	81,500	1·29
8"	7·1	76,400	0·68	83,000	0·76	6·63	91,200	1·37
8½"	7·6	87,400	0·73	95,000	0·81	7·06	104,000	1·46
9"	8·1	99,250	0·78	108,000	0·87	7·50	117,500	1·55
9½"	8·6	111,900	0·83	121,500	0·92	8·00	133,800	1·65
10"	9·0	122,500	0·86	133,200	0·96	8·50	151,000	1·75
10½"	9·5	136,500	0·91	148,300	1·02	9·0	169,000	1·85
11"	10·0	151,200	0·96	164,400	1·07	9·5	188,500	1·96
11½"	10·5	166,700	1·01	181,200	1·12	10·0	208,800	2·06
12"	11·0	183,000	1·06	199,000	1·18	10·31	222,000	2·13
15"	14·0	296,500	1·34	322,250	1·50	13·31	371,000	2·75
18"	16·5	412,000	1·58	447,500	1·76	16·13	544,000	3·32
24"	21·5	699,000	2·06	760,000	2·30	22·75	1,075,000	4·68
STRESS RATIO t/c		24		24		13⅔		
		R.Ms. FOR COMBINATIONS OF STRESSES HAVING THESE RATIOS ARE PRO RATA TO t.						

NOTES. R.M. = Qbd^2. TABULATED VALUES OF RESISTANCE MOMENTS ARE IN INCH-LB. PER FOOT WIDTH OF SLAB.

A_T = AREA OF TENSILE REINFORCEMENT IN SQ. IN. PER FOOT WIDTH OF SLAB.

THIS TABLE APPLIES FOR SLABS REINFORCED IN TENSION ONLY.

IF ACTUAL COMBINATION OF COVER AND BAR DIAMETER GIVES EFFECTIVE DEPTH CONTRARY TO ABOVE, R.M AND A_T OR TOTAL THICKNESS SHOULD BE ADJUSTED.

EXAMPLES OF USE OF TABLE No. 32.

(*a*) To find the moment of resistance in compression and the corresponding amount of tensile reinforcement for a tee beam, the rib of which is 10 in. wide and extends 16 in. below the soffit of a 6-in slab. Span of beam = 18 ft. and distance between adjacent beams = 8 ft. Maximum stresses not to exceed 17,000 and 700 lb. per sq. in.

Effective breadth:

(1) Available slab = 96 in.

(2) $\frac{\text{Span}}{3} = 72$ in.

(3) $12d_s + b = (12 \times 6) + 10 = 82$ in.

Hence maximum $b_e = 72$ in.; effective depth, say, 20·5 in.

From *Table* No. 27, $n_1 \left(\text{for } r = \frac{17{,}000}{700}\right) = 0{\cdot}39 \times 20{\cdot}5 = 8$ in.

and neutral axis falls below slab.

$a_1 = 20{\cdot}5 - 3 = 17{\cdot}5$ in.

R.M. in compression

$$= \frac{700 \times 72 \times 17{\cdot}5 \times 6}{2 \times 8}(2 \times 8 - 6) = 3{,}300{,}000 \text{ in.-lb.}$$

$$A_T = \frac{3{,}300{,}000}{17{,}000 \times 17{\cdot}5} = 11{\cdot}10 \text{ sq. in.}$$

(*b*) To find the moment of resistance in compression and the corresponding amount of tensile reinforcement for an ell beam, the rib of which is 6 in. wide and extends 10 in. below a 7-in. slab. Span of beam = 14 ft. and available slab width exceeds 3 ft. Maximum stresses: 16,000 and 600 lb. per sq. in.

Effective breadth: (1) Available slab = 36 in. minimum.

(2) $\frac{\text{Span}}{3} = 56$ in.

(3) $4d_s + b = 4 \times 7 + 6 = 34$ in.

Hence $b_e = 34$ in.; effective depth, say, 15·5 in.

From *Table* No. 27, $n_1 \left(\text{for } r = \frac{16{,}000}{600}\right) = 0{\cdot}36 \times 15{\cdot}5 = 5{\cdot}6$ in.,

thus the neutral axis falls within the slab.

$$a_2 = 15{\cdot}5 - \frac{5{\cdot}6}{3} = 13{\cdot}6 \text{ in.}$$

$$\text{R.M. in compression} = 0{\cdot}5 \times 34 \times 5{\cdot}6 \times 13{\cdot}6 \times 600 = 770{,}000 \text{ in.-lb.}$$

$$A_T = \frac{770{,}000}{16{,}000 \times 13{\cdot}6} = 3{\cdot}54 \text{ sq. in.}$$

(*c*) To find the minimum distance from the supports of an end-span beam, loaded with uniformly distributed load, that two bars can be bent up to assist in shear resistance; the total number of equal diameter bars in the bottom at mid-span is six; span of beam = 20 ft.

For second bar of six, $k_2 = 0{\cdot}30$ and $k_3 = 0{\cdot}18$. Hence distance from inner support = 0·30 × 20 = 6 ft. and distance from outer support = 0·18 × 20 = 3 ft. 7 in.

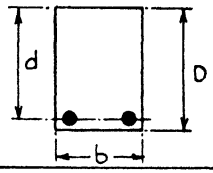
BEAM DATA — TABLE No. 32.

RECTANGULAR BEAMS.

PROPORTIONS :- MIN. $d = f \times$ SPAN. (PER L.C.C. REGS. 1915)

TENSILE STRESS :-	12,000	14,000	15,000	16,000	18,000
VALUE OF f :-	·0375	·044	·047	·05	·056

$D = b$ TO $3b$ (SEE CHAPTER XI)



FLANGED BEAMS

$b = \not< \frac{d_r}{3}$

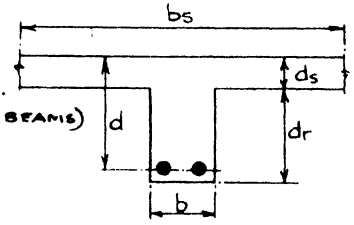
MAX. VALUE OF b_s = LEAST OF FOLLOWING :-

(1) DISTANCE BETWEEN CENTRES OF ADJACENT BEAMS.

(2) ONE-THIRD SPAN OF BEAM (ONE SIXTH FOR ELL BEAMS)

(3) TEE BEAM : $12d_s + b$

ELL BEAM : $4d_s + b$



LEVER ARM :

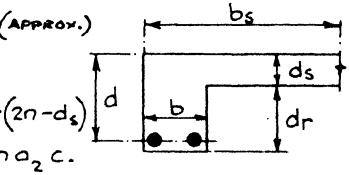
NEUTRAL AXIS BELOW SLAB : $a_1 = d - \cdot 5 d_s$ (APPROX.)

NEUTRAL AXIS WITHIN SLAB : $a_2 = d - \frac{n}{3}$

RESISTANCE MOMENT (COMPRESSION)

NEUTRAL AXIS BELOW SLAB : $R.M. = \frac{c b_s a_1 d_s}{2n}(2n - d_s)$

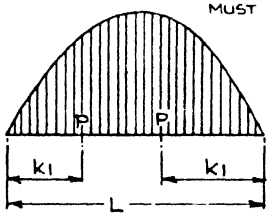
NEUTRAL AXIS WITHIN SLAB : $R.M. = \cdot 5 b_s n a_2 c.$



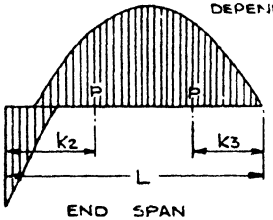
SECTIONS AT WHICH BOTTOM BARS CAN BE STOPPED (OR BENT UP).

UNIFORMLY DISTRIBUTED LOAD.

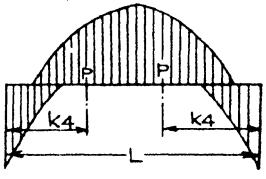
MAX. DISTANCE FROM SUPPORT TO $P = kL$ WHERE P = POINT OF STOPPING OR BENDING UP. IF BAR IS NOT BENT UP, SUFFICIENT ANCHORAGE LENGTH MUST BE PROVIDED ON SUPPORT SIDE OF P; ANCHORAGE LENGTH WILL DEPEND ON RATE OF CHANGE OF B.M.



FREELY SUPPORTED SPAN



END SPAN



INTERIOR SPAN

No. OF BARS AT MID SPAN	k_1						k_2						k_3						k_4					
	ORDER OF						STOPPING OFF OR						BENDING UP BARS											
	1ST.	2ND.	3RD.	4TH.	5TH.	6TH.	1ST.	2ND.	3RD.	4TH.	5TH.	6TH.	1ST.	2ND.	3RD.	4TH.	5TH.	6TH.	1ST.	2ND.	3RD.	4TH.	5TH.	6TH.
1	0	-	-	-	-	-	·11	-	-	-	-	-	0	-	-	-	-	-	·09	-	-	-	-	-
2	·15	0	-	-	-	-	·24	·11	-	-	-	-	·13	0	-	-	-	-	·21	·09	-	-	-	-
3	·21	·09	0	-	-	-	·30	·19	·11	-	-	-	·18	·08	0	-	-	-	·27	·16	·09	-	-	-
4	·25	·15	·07	0	-	-	·33	·24	·17	·11	-	-	·22	·13	·05	0	-	-	·30	·21	·15	·09	-	-
5	·27	·19	·12	·05	0	-	·35	·28	·21	·16	·11	-	·24	·15	·09	·04	0	-	·31	·24	·18	·13	·09	-
6	·30	·21	·15	·09	·04	0	·36	·30	·24	·19	·14	·11	·25	·18	·13	·08	·03	0	·33	·27	·21	·16	·12	·09
7	·31	·23	·17	·12	·08	·04	·39	·32	·26	·22	·18	·14	·29	·20	·15	·11	·07	·03	·34	·29	·23	·19	·15	·12
8	·32	·25	·19	·15	·10	·07	·40	·33	·27	·24	·20	·17	·30	·22	·17	·13	·09	·05	·35	·30	·25	·21	·18	·15

EXAMPLES OF USE OF TABLE No. 33.

(*a*) To design the shear reinforcement for the specified sections to take a shear force of 30,000 lb. Allowable stresses: bent-up bars and binders: 18,000 lb. per sq. in.; maximum unit effective shear stress on concrete 75 lb. per sq. in. Lever arm of all sections = 0·87 × effective depth.

(i) Effective depth = 40 in.; breadth = 15 in.

$$s = \frac{30{,}000}{0{\cdot}87 \times 40 \times 15} = 57{\cdot}4 \text{ lb. per sq. in.}$$

Therefore no shear reinforcement is required.

(ii) Effective depth = 30 in.; breadth = 12 in.

$$s = \frac{30{,}000}{0{\cdot}87 \times 30 \times 12} = 96 \text{ lb. per sq. in.}$$

If shear is taken on bent-up bars alone (arranged so as to give double system), shear value required is given by one $1\frac{1}{4}$-in. bar bent up at 45 deg.

(iii) Effective depth = 20 in.; breadth = 10 in.

$$s = \frac{30{,}000}{0{\cdot}87 \times 20 \times 10} = 173 \text{ lb. per sq. in.}$$

Therefore the whole shear would be taken on reinforcement.

Nominal binding, say, $\frac{5}{16}$-in. at 16-in. centres ($V = 173$) takes 173 × 0·87 × 20 = 3,010 lb., leaving 26,990 lb. to be taken on bent-up bars. If 1-in. bars are available for bending up, the resistance required is given by two 1-in bars at 30 deg. (double system).

(*b*) To calculate the resistance to shear of the section specified in (ii) if reinforced with one $\frac{3}{4}$-in. bar bent-up at 30 deg. (in single system) and $\frac{3}{8}$-in. double binders at 12-in. centres: maximum stress of 16,000 lb. per sq. in. on bent-up bars, and 14,000 lb. per sq. in. on binders.

Value of one $\frac{3}{4}$-in. bar bent-up at 30 deg. in single system

at 16,000 lb. per sq. in. $= \dfrac{3{,}900 \times 16{,}000}{18{,}000}$ = 3,470 lb.

$\frac{3}{8}$-in. double binders at 12-in. centres at 14,000 lb. per sq. in.

take $2 \times 0{\cdot}87 \times 30 \times 330 \times \dfrac{14{,}000}{18{,}000}$ = 13,400 ,,

Total = 16,870 ,,

The shear resistance of the concrete alone

$= 75 \times 0{\cdot}87 \times 30 \times 12 = 23{,}700$ lb.

Hence the shear resistance of the given section is controlled by the concrete and is 23,700 lb.

SHEAR REINFORCEMENT — TABLE Nº 33.

SHEAR VALUE OF SINGLE BINDERS (TWO ARMS)

$$V = \frac{S}{a} = \frac{A_T t}{p}$$

$t = 18{,}000$ LB./IN.^{2} PRO RATA FOR OTHER STRESSES.

DIAM. OF BINDER	SPACING OF BINDERS (INCHES) 2"	3"	4"	4½"	5"	6"	7"	7½"	8"	9"	10½"	12"	14"	16"	18"	24"	SHEAR RESISTANCE WHEN p = a
3/16"	496	331	248	221	198	166	142	132	124	110	96	82	72	63	56	41	995 LB.
¼"	882	587	441	391	355	294	252	235	221	196	168	147	126	110	98	74	1764 "
5/16"	1386	924	693	616	554	462	396	370	347	308	264	231	198	173	154	115	2772 "
⅜"	1980	1320	990	880	792	660	566	528	495	440	377	330	283	248	220	165	3960 "
7/16"	2700	1800	1350	1200	1080	900	772	720	675	600	514	450	386	338	300	225	5400 "
½"	3530	2350	1765	1567	1412	1175	1008	940	883	784	671	588	504	441	392	294	7060 "

MAXIMUM SPACING OF BINDERS:- NORMALLY $p \not> a$
BINDERS AROUND COMPRESSION BARS: $\not>$ 12 × DIA. OF BAR.
DITTO DITTO (STEEL BEAM THEORY) $\not>$ 8 × DIA. OF BAR.

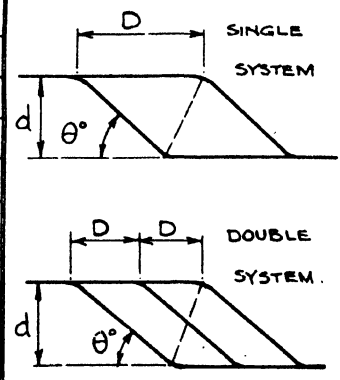
SHEAR RESISTANCE OF INCLINED BARS. (LB.)

SINGLE SYSTEM: $S = A_T t \sin\theta$ DOUBLE SYSTEM: $S = 2A_T t \sin\theta$

TABULATED VALUES ARE FOR $t = 18{,}000$ LB./INS.^{2} (PRO RATA FOR OTHER STRESSES.)

DIAM. OF BAR	θ = 45° SINGLE SYSTEM	θ = 45° DOUBLE SYSTEM	θ = 30° SINGLE SYSTEM	θ = 30° DOUBLE SYSTEM
½"	2,500	5,000	1,750	3,500
⅝"	3,900	7,800	2,750	5,550
¾"	5,600	11,200	3,900	7,800
⅞"	7,650	15,300	5,400	10,800
1"	10,000	20,000	7,050	14,100
1⅛"	12,650	25,300	8,950	17,850
1¼"	15,600	31,200	11,050	22,050
1⅜"	18,900	37,800	13,350	26,700
1½"	22,500	45,000	15,900	31,800
MAX^M VALUE OF D	1·41d	0·71d	2·00d	1·00d

NOTES



SEE CHAPTER XII FOR NOTES AND FORMULÆ.

SEE TABLE Nº 32 FOR POINTS AT WHICH TENSION BARS CAN BE BENT UP.

EXAMPLES OF USE OF TABLE No. 34.

To design the reinforcement for an 18-in. square beam, circular in plan, radius 12 ft., supported at equal intervals so that the angle subtended at the centre by each arc is 90 deg. The total load on a single span is 45,000 lb., uniformly distributed.

From *Table* No. 7c : (CASE 1):

Negative B.M. over the support $= 0{\cdot}137 \times 45{,}000 \times 12 \times 12 = 890{,}000$ in.-lb.
Positive B.M. at mid-span $=$ approximately 50 per cent. of 890,000 in.-lb. $= 450{,}000$ in.-lb.
Maximum shear at support $= 0{\cdot}5 \times 45{,}000 = 22{,}500$ lb.

When B.M. is a maximum, twisting moment is zero ; at point of contraflexure (B.M. = 0), twisting moment is a maximum and equal to $0{\cdot}021 \times 45{,}000 \times 12 \times 12 = 136{,}000$ in.-lb.

Point of contraflexure $= \frac{19\frac{1}{4}^\circ}{90^\circ}$, that is approximately at the fifth-point of the span.

Design of support section : With stresses of 16,000 and 600 and effective depth of approximately 16 in., the resistance moment with $A_C = A_T$ (but $c \not> 600$) $= 18 \times 50{,}000 = 900{,}000$ in.-lb. (from *Table* No. 28) which is satisfactory.

With actual effective depth of 16·5 in., $A_T (= A_C) = \frac{890{,}000}{16{,}000 \times 0{\cdot}88 \times 16{\cdot}5} = 3{\cdot}84$ sq. in., say, four $1\frac{1}{8}$-in. bars top and bottom.

These bars can be arranged by providing one $1\frac{1}{8}$-in. bar in each corner throughout the beam, bars from adjacent spans overlapping both in the top and bottom at the supports to take the negative bending moment.

Shear stress at support $= \frac{22{,}500}{18 \times 0{\cdot}88 \times 16{\cdot}5} = 86$ lb. per sq. in., which necessitates the whole shear being taken on the reinforcement, say binders ; V (required) $= \frac{22{,}500}{0{\cdot}88 \times 16{\cdot}5} = 1{,}553$; from *Table* No. 33 this is given by $\frac{3}{8}$-in. bouble binders at 5-in. centres (at 18,000 lb. per sq. in.).

Twisting moment at support section is zero. (CASE 1)

Design of midspan section : since the bending moment is approximately half that at the supports, half the support reinforcement is required, that is, two $1\frac{1}{8}$-in. bars top and bottom are satisfactory.

Shear and twisting moment at midspan are zero.

Design of section at fifth-point of span :

Shear force $= \frac{0{\cdot}5 - 0{\cdot}2}{0{\cdot}5} \times 22{,}500 = 13{,}500$ lb.

Shear stress $= 52$ lb. per sq. in., which does not require reinforcement.

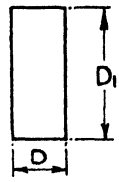
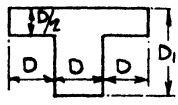
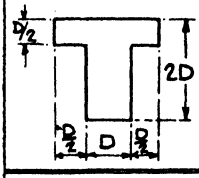
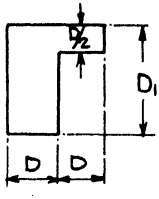
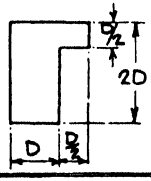
Bending moment is theoretically zero, but the four bars provided at the support would be continued beyond this section, thus allowing for negative bending moments that may be produced when adjacent spans only are subjected to live load ; a margin is also allowed for the longitudinal steel required to resist the twisting moment.

Twisting moment $= 136{,}000$ in.-lb. (see above). With rectangular section with $\frac{D_1}{D} = 1{\cdot}0$ and $D = 18$ in. (since B.M. being zero, whole section can be considered as uncracked), section coefficient varies from ·297 to 0·391 depending on the percentage of reinforcement. Section coefficient required $= R = \frac{T.M}{sD^3}$ and with $s = 75$ lb. per sq. in. (for 1 : 2 : 4 mix, *Table* No. 23) $R = \frac{136{,}000}{75 \times 18^3} = 0{\cdot}31$. Hence from *Table* No. 34, total percentage of torsional reinforcement required is 1·0. This can be divided into two parts : 50 per cent as longitudinal bars in the corners of the section and 50 per cent. as binding. Assuming that, of the four bars provided at top and bottom, the four corner bars are available for torsional resistance, the percentage is $\frac{3{\cdot}92 \times 100}{18^2} = 1{\cdot}21$, which is about double the amount required. The volume of binding per foot run required is $\frac{1{\cdot}25}{2} \times \frac{18^2 \times 12}{100} = 24{\cdot}3$ cu. in. The volume of one external $\frac{3}{8}$-in. binder $= 0{\cdot}11 \times 4 \times 16 = 7{\cdot}05$ cu. in. The spacing required $= \frac{12 \times 7{\cdot}05}{24{\cdot}3} = 3\frac{1}{4}$-in. centres.

(See p. 169.)

TORSION.

TABLE Nº 34.

SECTION		SECTION COEFFICIENT. (= R)						
SHAPE	$\frac{D_1}{D}$	TOTAL PERCENTAGE OF TORSION REINFORCEMENT. (EQUAL PROPORTIONS OF LONGITUDINAL BARS AND BINDERS)						
		¾%	1%	1¼%	1½%	1¾%	2%	2¼%
	-	0·280	0·293	0·310	0·321	0·336	0·352	0·368
	1·0	0·297	0·313	0·328	0·344	0·359	0·375	0·391
	1·5	0·612	0·644	0·677	0·708	0·740	0·772	0·805
	2·0	0·928	0·976	1·025	1·075	1·120	1·170	1·220
	3·0	1·595	1·675	1·76	1·84	1·92	2·01	2·095
	4·0	2·44	2·57	2·70	2·82	2·95	3·08	3·21
	2·0	1·060	1·115	1·170	1·225	1·280	1·335	1·395
	3·0	1·65	1·74	1·825	1·91	2·00	2·085	2·17
	4·0	2·44	2·57	2·70	2·82	2·95	3·08	3·21
	-	0·980	1·031	1·085	1·135	1·185	1·240	1·290
	2·0	0·980	1·031	1·085	1·135	1·185	1·240	1·290
	3·0	1·610	1·688	1·77	1·855	1·94	2·025	2·11
	-	0·980	1·031	1·085	1·135	1·185	1·240	1·290

TWISTING MOMENT RESISTANCE $= R\,s\,D^3$

R = SECTION COEFFICIENT AS TABULATED.
D = EQUIVALENT DIAMETER OF UNCRACKED SECTION.
S = SAFE SHEAR STRESS PER TABLE Nº 23. (= 0·10c)

EXAMPLES OF USE OF TABLE No. 35.

(*a*) To select the smallest column to carry 95 tons with minimum reinforcement and quality A concrete.

A 14-in. column with 0·8 per cent. and quality A 1 : 1 : 2 concrete will carry 97 tons (see *Table* No. 35). Reinforcement required

$= 0{\cdot}8$ per cent.

$$= \frac{0{\cdot}8}{100} \times 14^2 = 1{\cdot}57 \text{ sq. in.;}$$ this is given by four $\frac{3}{4}$-in. bars.

(*b*) To find the maximum safe load on an 18-in. by 12-in. column reinforced with six $\frac{7}{8}$-in. bars ($= 3{\cdot}6$ sq. in.), $1 : 1\frac{1}{2} : 3$ mix ordinary quality concrete, height of column 20 ft. continuing through several stories.

As a short column, $c_0 = 680$ lb. per sq. in.

and the safe load $= 680[(18 \times 12) + (15 - 1)3{\cdot}6] = 181{,}000$ lb.

For specified conditions, assume effective length $= 0{\cdot}8 \times$ actual length.

Hence ratio of effective length to least lateral dimension

$$= \frac{0{\cdot}8 \times 20 \times 12}{12} = 16.$$

Therefore reduction factor $R_L = 0{\cdot}97$ (*Table* No. 35).

$$\text{Safe load} = \frac{0{\cdot}97 \times 181{,}000}{2{,}240} = 78{\cdot}3 \text{ tons.}$$

COLUMNS. SAFE LOAD (TONS) TABLE Nº 35.

MIX.		1 : 2 : 4						1 : 1½ : 3						1 : 1 : 2					
QUAL.		ORDINARY			A			ORDINARY			A			ORDINARY			A		
COMPVE STRESS.		$C_o = 600$			760			680			880			780			1000 LB/IN²		
PER CENT REINFORC'T.		0·8	4·0	8·0	0·8	4·0	8·0	0·8	4·0	8·0	0·8	4·0	8·0	0·8	4·0	8·0	0·8	4·0	8·0
SIZE OF SQUARE COLUMN	10	30	42	57	38	53	72	34	47	64	44	61	83	39	54	74	50	70	95
	11	36	51	69	46	64	87	41	57	78	53	74	101	47	66	89	60	84	114
	12	43	60	82	54	76	104	49	68	93	63	88	120	56	78	106	72	100	136
	13	50	71	96	64	89	121	57	80	109	74	104	141	66	92	125	84	118	160
	14	59	82	111	74	104	141	66	93	126	86	120	163	76	107	145	97	137	185
	15	67	94	128	85	119	161	76	107	144	99	138	187	87	122	166	112	157	212
	16	76	107	145	97	136	184	87	121	165	112	157	213	99	139	189	127	178	242
	18	97	135	184	123	171	233	110	153	208	142	198	270	126	176	239	161	226	307
	20	119	167	227	151	216	288	136	189	258	175	245	333	155	217	295	199	278	378
	21	132	184	250	167	234	317	149	209	284	193	270	367	171	239	325	219	307	417
	22	144	202	275	183	256	348	164	229	311	212	297	403	188	263	357	241	337	458
	24	172	241	327	218	303	415	195	273	371	252	353	480	224	313	425	287	402	545
	27	218	305	414	276	386	524	246	345	469	319	447	607	283	396	538	363	508	690
	28	234	328	445	296	415	564	265	371	504	352	481	653	304	426	578	390	546	742
	30	268	376	511	340	476	647	304	426	579	394	552	749	349	489	664	447	626	851
	33	325	455	618	411	576	783	368	516	701	477	607	907	422	592	804	541	758	1030
	36	386	541	736	489	686	932	438	613	834	567	794	1079	502	704	954	644	902	1226

SAFE LOAD $= c_o D^2 \left[1 + \left(\frac{m-1}{100}\right)p\right] R_L$ (TABULATED VALUES ARE FOR $m = 15$; $R_L = 1{\cdot}0$).

LONG COLUMN REDUCTN FACTORS R_L																
	EFFECTIVE COLN LENGTH / LEAST RAD. OF GYRATION	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120
	REDUCTION FACTOR R_L	1·0	·95	·90	·85	·80	·75	·70	·65	·60	·55	·50	·45	·40	·35	·30
	EFFECTIVE COLN LENGTH / LEAST LATERAL DIMENSION	15	16	17	18	19	20	21	22	24	26	27	30	32	33	36
	REDUCTION FACTOR R_L	1·0	·97	·93	·90	·87	·83	·80	·77	·70	·63	·60	·50	·43	·40	·30

$R_L = 1{\cdot}5 - \frac{\text{EFFY LENGTH}}{100 \times \text{RAD. OF GYR}^{N}}$ $\qquad$ $R_L = 1{\cdot}5 - \frac{\text{EFFY LENGTH}}{50 \times \text{LEAST LAT}^{L}\ \text{DIM}^{N}}$

EFFVE COLUMN LENGTH.	COLUMNS OF ONE STORY		COLUMNS OF TWO OR MORE STORIES.	
	FIXED AT BOTH ENDS	0·75 × ACTUAL LENGTH	FIXED AT BOTH ENDS	0·75 × ACTUAL LENGTH
	HINGED AT BOTH ENDS.	ACTUAL LENGTH	HINGED AT BOTH ENDS PARTIALLY FIXED AT ONE OR BOTH ENDS.	BETWEEN ACTUAL LENGTH AND 0·75 × ACTUAL LENGTH.
	FIXED AT ONE END IMPERFECTLY RESTRAINED IN DIRECTION AND POSITION AT OTHER END.	BETWEEN ACTUAL LENGTH AND TWICE ACTUAL LENGTH DEPENDING ON EFFICIENCY OF IMPERFECT RESTRAINT.	FIXED AT ONE END AND FREE AT OTHER END	2·0 × ACTUAL LENGTH.

RADIUS OF GYRATION	SQUARE OR RECTANGULAR	CIRCLE.	OCTAGON.	ANNULAR
	0·288 D	0·25 × DIAM.	0·257 × DISTANCE ACROSS FLATS.	$0{\cdot}5\sqrt{R^2 + r^2}$

EXAMPLES OF USE OF TABLE No. 36.

(See also p. 290.)

A rectangular section is 18 in. deep and 12 in. wide and is reinforced with two 1-in bars in the top and three 1-in bars in the bottom. The centres of the bars are $1\frac{1}{2}$ in. from the top and bottom faces respectively. To find the stresses produced in the section under the conditions of direct thrust and moment given below, assuming a modular ratio of 15.

For all cases $D = 18$ in., $b = 12$ in., $d = 16{\cdot}5$ in., $f_1 = \dfrac{16{\cdot}5 - 1\frac{1}{2}}{16{\cdot}5} = 0{\cdot}909$.

$A_T = 2{\cdot}36$ sq. in.; $A_C = 1{\cdot}57$ sq. in.

(*a*) $M = 200{,}000$ in.-lb. $N = 100{,}000$ lb.

$$e = \frac{200{,}000}{100{,}000} = 2 \text{ in., that is, } < 0{\cdot}167D(= 3 \text{ in.}).$$

$A_1 = 14 \times 3{\cdot}93 = 55$ sq. in. $A_2 = 12 \times 18 = 216$ sq. in.

$$Z = \frac{2 \times 55}{18}(9 - 16{\cdot}5 + 15)^2 + (0{\cdot}167 \times 216 \times 18) = 988 \text{ in.}^3$$

$$c = \frac{100{,}000}{55 + 216} \pm \frac{200{,}000}{988} = 571 \text{ and } 165 \text{ lb. per sq. in.}$$

(*b*) $M = 200{,}000$ in.-lb. $N = 50{,}000$ lb.

$$e = \frac{200{,}000}{50{,}000} = 4 \text{ in., that is, } > 0{\cdot}167D(= 3 \text{ in.}) \text{ and } < 0{\cdot}5d(8{\cdot}25 \text{ in.}).$$

A_1, A_2, and Z as for (*a*).

$$c_{\text{min.}} = \frac{50{,}000}{55 + 216} - \frac{200{,}000}{988} = 184 - 203 = -19 \text{ lb. per sq. in.}$$

This is less than one-sixth of the allowable maximum stress $\left(\text{say } \dfrac{600}{6}\right)$ and therefore maximum compression stress $= 184 + 203 = 387$ lb. per sq. in.

(*c*) $M = 300{,}000$ in.-lb. $N = 15{,}000$ lb.

$$e = \frac{300{,}000}{15{,}000} = 20 \text{ in., that is, } > 0{\cdot}5d(8{\cdot}25 \text{ in.}) \text{ and } < 1{\cdot}5d(= 25 \text{ in.}).$$

$$F = \frac{20 - 9}{16{\cdot}5} + 1 = 1{\cdot}666. \quad \text{Assume } n_1 = 0{\cdot}55.$$

$J = 0{\cdot}55 \times 12 \times 16{\cdot}5 \times 0{\cdot}5 = 55.$

From *Table* No. 36: $G = 0{\cdot}224$. $H = 11{\cdot}5$ approx.

$$c = \frac{15{,}000 \times 1{\cdot}666}{(0{\cdot}224 \times 12 \times 16{\cdot}5) + (11{\cdot}5 \times 1{\cdot}57 \times 0{\cdot}909)} = 412 \text{ lb. per sq. in.}$$

$$t = \frac{412(55 + 11{\cdot}5 \times 1{\cdot}57) - 15{,}000}{2{\cdot}36} = 6{,}440 \text{ lb. per sq. in.}$$

Corresponding value of $n_1 = \dfrac{1}{1 + \dfrac{6{,}440}{412 \times 15}} = 0{\cdot}49$, compared with the assumed value 0·55. Re-working with an intermediate value of 0·51,

$J = 12 \times 16{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}51 = 50{\cdot}5.$

$$G = 0{\cdot}5 \times 0{\cdot}51\left(1 - \frac{0{\cdot}51}{3}\right) = 0{;}212.$$

$$H = \frac{14}{0{\cdot}51}(0{\cdot}51 + 0{\cdot}909 - 1) = 11{\cdot}5.$$

$$c = \frac{15{,}000 \times 1{\cdot}666}{(0{\cdot}212 \times 12 \times 16{\cdot}5) + (11{\cdot}5 \times 1{\cdot}57 \times 0{\cdot}909)} = 426 \text{ lb. per sq. in.}$$

$$t = \frac{426(50{\cdot}5 + 11{\cdot}5 \times 1{\cdot}57) - 15{,}000}{2{\cdot}36} = 6{,}020 \text{ lb. per sq. in.}$$

Corresponding value of $n_1 = \dfrac{1}{1 + \dfrac{6{,}020}{426 \times 15}} = 0{\cdot}514$; this is close enough to second trial value.

(*d*) $M = 100{,}000$ in.-lb. $N =$ direct pull of 20,000 lb.

$$e = \frac{100{,}000}{20{,}000} = 5 \text{ in. or } < 0{\cdot}5f_1d(= 7{\cdot}5 \text{ in.}).$$

$$\text{Tensile stress in } A_T = \frac{20{,}000}{2{\cdot}36}\frac{(7{\cdot}5 + 5)}{15} = 7{,}080 \text{ lb. per sq. in.}$$

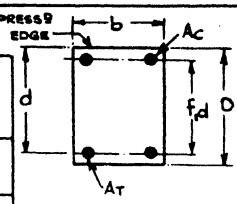
$$\text{Tensile stress in } A_C = \frac{20{,}000}{1{\cdot}57}\frac{(7{\cdot}5 - 5)}{15} = 2{,}120 \text{ lb. per sq. in.}$$

BENDING & DIRECT FORCE.

TABLE No. 36.

RECTANGULAR SECTIONS.

$e = \frac{\text{MOMENT}}{\text{DIRECT FORCE}} = \frac{M}{N}$



m = MODULAR RATIO.
$n_1 d$ = DEPTH OF NEUTRAL AXIS.
t = MAX. STEEL STRESS.
c = MAX. CONCRETE STRESS.

BENDING AND DIRECT COMPRESSION

$e < \cdot167D$

$A_1 = (m-1)(A_c + A_T)$; $z = \frac{2A_1}{D}(\cdot5D - d + f_1 d)^2 + \cdot167 A_2 D$

$A_2 = bD$ $\quad c_{MAX.}$ & $c_{MIN.} = \frac{N}{A_1 + A_2} \pm \frac{M}{Z}$

$e > \cdot167D$, $< \cdot5d$

COMPUTE C (MIN.) AS ABOVE; IF NEGATIVE AND $> \frac{\text{ALLOWABLE C}}{6}$ RECALCULATE BY METHOD GIVEN BELOW.

$e > \cdot5d$

EVALUATE $F = \frac{e - \cdot5D}{d} + 1$; ASSUME n_1; FIND G, H, AND $J = \frac{bdn_1}{2}$

SUBSTITUTE IN $c = \frac{NF}{Gbd + HA_c f_1}$ $\quad t = \frac{c(J + HA_c) - N}{A_T}$

CHECK ASSUMED VALUE OF n_1 FROM $n_1 = \frac{1}{1 + t/mc}$ AND REWORK IF MORE THAN 10% ERROR.

VALUES OF t/c, H, & G. $[H = 1/n_1(m-1)(n_1 + f_1 - 1)]$

VALUE OF n_1			·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70	·75
$G = \cdot5n_1(1 - n_1/3)$			·093	·114	·135	·155	·173	·191	·208	·224	·240	·254	·268	·281
m = 15	t/c		60	45	35	28	22·5	18·3	15	13·7	10	8	6·5	5
	VALUES OF f_1	·80	-	2·8	4·6	60	7·0	7·7	8·4	9·0	9·4	9·7	9·9	10·1
		·85	3·5	9·6	7·0	8·0	8·8	9·4	9·8	10·2	10·5	10·7	11·0	11·2
		·90	7·0	8·4	9·4	10·1	10·5	10·9	11·2	11·5	11·6	11·8	12·0	12·2
		·95	10·5	11·2	11·6	12·0	12·3	12·4	12·6	12·7	12·9	12·9	13·0	13·0
m = 12·5	t/c		50	37·5	29	23	18·8	15·2	12·5	10·2	8·3	6·8	5·4	4·2
	VALUES OF f_1	·80	-	2·3	3·8	4·9	5·7	6·3	6·9	7·4	7·7	8·0	8·1	8·3
		·85	2·9	4·6	5·7	6·6	7·2	7·7	8·0	8·4	8·7	8·8	9·0	9·2
		·90	5·7	6·9	7·7	8·3	8·6	9·0	9·2	9·5	9·6	9·8	9·9	10·0
		·95	7·8	9·2	9·5	9·9	10·1	10·2	10·4	10·4	10·6	10·6	10·7	10·7
m = 10	t/c		40	30	23·3	18·5	15	12·2	10	8·2	6·7	5·4	4·3	3·3
	VALUES OF f_1	·80	-	1·8	3·0	3·9	4·5	5·0	5·4	5·8	6·1	6·3	6·4	6·5
		·85	2·3	3·6	4·5	5·1	5·6	6·1	6·3	6·6	6·8	6·9	7·1	7·2
		·90	4·5	5·4	6·1	6·5	6·8	7·0	7·2	7·4	7·5	7·7	7·7	7·8
		·95	6·8	7·2	7·5	7·7	7·9	8·0	8·1	8·2	8·3	8·3	8·4	8·4
m = 8	t/c		32	24	18·6	14·8	12	9·8	8	6·5	5·4	4·3	3·4	2·6
	VALUES OF f_1	·80	-	1·4	2·3	3·0	3·5	3·9	4·2	4·5	4·7	4·9	5·0	5·1
		·85	1·8	2·8	3·5	4·0	4·4	4·7	4·9	5·1	5·3	5·4	5·5	5·6
		·90	3·5	4·2	4·7	5·1	5·3	5·5	5·6	5·8	5·8	6·0	6·0	6·1
		·95	5·8	5·6	5·8	6·0	6·2	6·2	6·3	6·4	6·5	6·5	6·5	6·5

$e > 1\cdot5d$

CALCULATE STRESSES c_1 AND t_1 DUE TO M ONLY AND DETERMINE $n = n_1 d$ FOR THESE STRESSES.

EVALUATE $C_2 = \frac{N}{bn + A_T m + A_c(m-1)}$; THEN $c = c_1 + c_2$

$t = t_1 - mc_2$

BENDING AND DIRECT TENSION.

$e < \cdot5f_1 d$

TENSILE STRESS IN $A_T = \frac{N}{A_T}\left(\frac{\cdot5f_1 d + e}{f_1 d}\right)$; TENSILE STRESS IN $A_c = \frac{N}{A_c}\left(\frac{\cdot5f_1 d - e}{f_1 d}\right)$.

$e > \cdot5f_1 d$

EVALUATE $L = \frac{e + \cdot5D}{d} - 1$; ASSUME n_1 AND FIND G, H, AND $J = \cdot5bdn_1$

SUBSTITUTE IN $c = \frac{NL}{bdG + HA_c f_1}$ $\quad t = \frac{c(J + A_c H) + N}{A_T}$

CHECK n_1 AS ABOVE

$e > 1\cdot5d$

CALCULATE STRESSES c_1 AND t_1 DUE TO M ONLY AND DETERMINE $n = n_1 d$ FOR THESE STRESSES.

EVALUATE C_2 AS ABOVE; THEN $c = c_1 - c_2$ $\quad t = t_1 + mc_2$

EXAMPLES OF USE OF TABLE No. 36A.

Consider a section in which $D = 20$ in., $b = 12$ in., $A_T = A_C =$ three 1-in. bars;

$$p = \frac{100 \times 4{\cdot}71}{20 \times 12} = 2{\cdot}0 \text{ per cent.}$$

With $1\frac{1}{2}$-in. cover, $f_1 = \dfrac{20 - (2 \times 2)}{20} = 0{\cdot}8$. Thus *Table* No. 36A can be used directly for the determination of the maximum stresses in the following examples, in each of which the direct load is 120,000 lb.

(*a*) B.M. = 240,000 in.-lb.; $e = \dfrac{M}{N} = \dfrac{240{,}000}{120{,}000} = 2$ in.; $e_1 = \dfrac{e}{D} = \dfrac{2}{20} = 0{\cdot}10$.

This falls in Case I and from *Table* No. 36A, for $e_1 = 0{\cdot}1$ and $p = 2$ per cent., $Q_1 = 1{\cdot}17$.

$$c_{(max.)} = \frac{Q_1 N}{bD} = \frac{1{\cdot}17 \times 120{,}000}{12 \times 20} = 585 \text{ lb. per sq. in.}$$

$$c_{(min.)} = \frac{2N}{[1 + (m - 1)p]bD} - c_{(max.)} = \frac{2 \times 120{,}000}{1{\cdot}28 \times 12 \times 20} - 585 = 350 \text{ lb. per sq. in.}$$

(*b*) B.M. = 720,000 in.-lb.; $e = \dfrac{720{,}000}{120{,}000} = 6$ in.; $e_1 = \dfrac{6}{20} = 0{\cdot}3$; this falls in Case II; for $e_1 = 0{\cdot}3$ and $p = 2{\cdot}0$ per cent. from *Table* No. 36A, $n_0 = 0{\cdot}80$. For $p = 2$ per cent. and $n_0 = 0{\cdot}80$, $Q_2 = 0{\cdot}153$.

$$\text{Concrete stress } c = \frac{720{,}000}{0{\cdot}153 \times 12 \times 20^2} = 980 \text{ lb. per sq. in.}$$

$$\text{Steel stress: } t = mc\left(\frac{0{\cdot}9}{n_0} - 1\right) = 15 \times 980\left(\frac{0{\cdot}9}{0{\cdot}8} - 1\right) = 1{,}840 \text{ lb. per sq. in. (tension).}$$

NOTES.

(i) In borderline cases between Cases I and II, that is when $c_{(min.)}$ in accordance with Case I is tensile, but of low value, and when it is imperative that all tension on the concrete should be ignored, the maximum compressive stress will be higher than that obtained by the solution of Case I, its approximate values being given by $c_{(max.)} = \dfrac{NQ_1}{0{\cdot}96D}$.

(ii) *Table* No. 36A applies only when the cover ratio is 0·1 for both A_T and A_C (that is, $f_1 = 0{\cdot}80$), but the table can be used if the following adjustment is made. If any cover ratio r_1 is greater than 0·1, substitute for the actual value of D_1 the value $(1{\cdot}1 - r)D$. If r is less than 0·1, the maximum concrete stress calculated from the table will be slightly in excess of the actual stress; therefore the table can be used to determine a stress which will not be exceeded.

BENDING MOMENT AND DIRECT THRUST. TABLE No. 36A

NOTE:– THIS TABLE APPLIES WHEN $A_C = A_T$ WITH $m = 15$, AND $f_1 = 0{\cdot}80$.

$$p = \frac{100(A_T + A_C)}{bD}$$

$$e_1 = \frac{\text{B.M.}}{D(\text{THRUST})} = \frac{M}{ND}$$

e_1	PERCENTAGE OF REINFORCEMENT ($= p$)						
	0·5	1·0	1·5	2·0	2·5	3·0	4·0
·04	1·14	1·07	1·00	–	–	–	–
·06	1·25	1·16	1·09	1·01	–	–	–
·08	1·36	1·25	1·17	1·09	1·02	–	–
·10	1·47	1·35	1·25	1·17	1·10	1·04	–
·12	1·58	1·44	1·33	1·24	1·17	1·10	0·99
·14	1·69	1·53	1·42	1·32	1·24	1·17	1·05
·16	1·80	1·62	1·51	1·40	1·31	1·24	1·10
·18	(1·90)	1·72	1·60	1·48	1·39	1·30	1·17
·20	(2·00)	(1·82)	(1·69)	1·57	1·45	1·36	1·22
·30	·70	·75	·77	·80	·82	·84	·86
·40	·54	·60	·65	·67	·69	·71	·74
·50	·44	·52	·56	·59	·62	·64	·66
·60	·39	·46	·51	·54	·56	·59	·62
·70	·35	·43	·47	·50	·53	·55	·59
·80	·33	·41	·45	·48	·51	·52	·55
·90	·31	·38	·43	·46	·48	·50	·53
1·0	·30	·37	·41	·44	·46	·48	·51
2·0	·25	·31	·35	·38	·40	·42	–
3·0	·23	·29	·33	·36	·40	–	–
n_0							
·20	·100	·162	·220	·280	·340	·400	–
·30	·100	·140	·180	·220	·260	·300	·380
·40	·102	·132	·164	·192	·222	·252	·311
·50	·108	·130	·156	·180	·203	·227	·273
·60	·110	·130	·150	·170	·190	·210	·250
·70	·111	·127	·144	·161	·179	·196	·230
·80	·109	·124	·139	·153	·169	·182	·213
·90	·104	·118	·130	·143	·158	·170	·195

VALUES OF Q_1

CASE I

COMPRESSIVE STRESSES ONLY (FOR VALUES IN BRACKETS A SMALL TENSION IS DEVELOPED).

FIND VALUE OF Q_1 FOR GIVEN VALUES OF e_1 AND p

SUBSTITUTE Q_1 IN

$$c = \frac{Q_1 N}{bD}$$

(c = MAX. COMPVE STRESS.)

VALUES OF n_0

CASE II

COMPRESSIVE AND TENSILE STRESSES

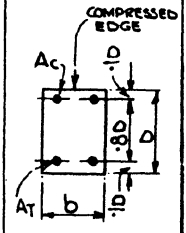
$$n_0 = \frac{n_1 d}{D}$$

FIND VALUE OF n_0 FOR GIVEN VALUES OF e_1 AND p

WITH THIS VALUE OF n_0 AND GIVEN VALUE OF p FIND Q_2 AND SUBSTITUTE IN

$$c = \frac{M}{Q_2 b D^2}$$

VALUES OF Q_2



EXAMPLES OF USE OF TABLE No. 37.

(*a*) To convert 17 ft. $8\frac{5}{16}$ in. into metric units:

17 ft. 8 in. = 5·385 metres.
$\frac{5}{16}$ in. = 0·0079 „

17 ft. $8\frac{5}{16}$ in. = 5·393 „

(*b*) To convert 4·067 metres into English units:

4·067
4·064 = 13 ft. 4 in.

0·003 = $\frac{1}{8}$ (At foot of table: $\frac{1}{8}$-in. = 3·18 mm.)

13 ft. $4\frac{1}{8}$ in.

(*c*) To express $\frac{1}{2}$-in. bars at 8-in. centres in metric units:

$\frac{1}{2}$ in. = 12·7 mm. 8 in. = 0·203 m.
Nearest practical values: 12-mm. bars at 20-cm. centres.

(*d*) To express 600 lb. per sq. in. in metric units:

$$= \frac{600}{14{\cdot}25} = 42 \text{ kg. per sq. cm. approximately.}$$

(*e*) To convert 6 lb. per sq. yd. to metric units:

$$6 \text{ lb.} = \frac{6}{2{\cdot}21} = 2{\cdot}71 \text{ kg.}$$

$$1 \text{ sq. yd.} = \frac{1}{1{\cdot}196} = 0{\cdot}836 \text{ sq. m.}$$

$$\therefore 6 \text{ lb. per sq. yd.} = \frac{2{\cdot}71}{0{\cdot}836} = 3{\cdot}24 \text{ kg. per sq. m.}$$

NOTES.

Conversion table is based on

1 in. = 2·539 cm.
1 kg. = 2·21 lb.

To convert moment of resistance factor Q in British units to Q_M in metric units,

$$Q_M = 0{\cdot}0703Q$$

$R.M.$ in Kg. cms. = $Q_M bd^2$ (where b and d are in cms.).

METRIC EQUIVALENTS (METRES) TABLE Nº 37

FT	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"	10"	11"
0	–	0·025	0·051	0·076	0·102	0·127	0·152	0·178	0·203	0·229	0·254	0·279
1	0·305	0·330	0·360	0·381	0·406	0·432	0·457	0·483	0·508	0·533	0·559	0·584
2	0·610	0·635	0·660	0·686	0·711	0·737	0·762	0·787	0·813	0·838	0·864	0·889
3	0·914	0·940	0·965	0·991	1·016	1·041	1·067	1·092	1·118	1·143	1·168	1·194
4	1·219	1·245	1·270	1·295	1·321	1·346	1·372	1·397	1·422	1·448	1·473	1·499
5	1·524	1·549	1·575	1·600	1·626	1·651	1·676	1·702	1·727	1·753	1·778	1·803
6	1·829	1·854	1·880	1·905	1·930	1·956	1·981	2·007	2·032	2·057	2·083	2·108
7	2·134	2·159	2·184	2·210	2·235	2·261	2·286	2·311	2·337	2·362	2·388	2·413
8	2·438	2·464	2·489	2·515	2·539	2·565	2·591	2·616	2·642	2·667	2·692	2·718
9	2·743	2·769	2·794	2·819	2·845	2·871	2·896	2·921	2·946	2·972	2·997	3·023
10	3·048	3·073	3·099	3·124	3·150	3·175	3·200	3·226	3·251	3·277	3·302	3·327
11	3·353	3·378	3·404	3·429	3·454	3·480	3·505	3·531	3·556	3·581	3·607	3·632
12	3·658	3·683	3·708	3·734	3·759	3·785	3·810	3·835	3·861	3·886	3·912	3·937
13	3·962	3·988	4·013	4·039	4·064	4·089	4·115	4·140	4·166	4·191	4·216	4·242
14	4·267	4·293	4·318	4·343	4·369	4·395	4·420	4·445	4·470	4·496	4·521	4·547
15	4·572	4·597	4·623	4·648	4·674	4·699	4·724	4·750	4·775	4·801	4·826	4·851
16	4·877	4·902	4·928	4·954	4·978	5·004	5·029	5·055	5·081	5·105	5·131	5·156
17	5·182	5·207	5·232	5·258	5·283	5·309	5·334	5·359	5·385	5·410	5·436	5·461
18	5·486	5·512	5·537	5·562	5·588	5·613	5·639	5·664	5·690	5·715	5·741	5·766
19	5·791	5·817	5·842	5·867	5·893	5·918	5·944	5·969	5·994	6·020	6·045	6·071
20	6·096	6·121	6·147	6·172	6·198	6·223	6·248	6·274	6·299	6·325	6·350	6·375
21	6·401	6·426	6·452	6·477	6·502	6·528	6·553	6·579	6·604	6·629	6·655	6·680
22	6·706	6·731	6·756	6·782	6·807	6·833	6·858	6·883	6·909	6·934	6·960	6·985
23	7·010	7·036	7·061	7·087	7·112	7·137	7·163	7·188	7·214	7·239	7·264	7·290
24	7·315	7·341	7·366	7·391	7·417	7·442	7·468	7·493	7·518	7·544	7·569	7·595
25	7·620	7·645	7·671	7·696	7·722	7·747	7·772	7·798	7·823	7·849	7·874	7·899
26	7·925	7·950	7·975	8·001	8·026	8·052	8·077	8·102	8·128	8·153	8·179	8·204
27	8·229	8·255	8·280	8·306	8·331	8·356	8·382	8·407	8·433	8·458	8·483	8·509
28	8·534	8·560	8·585	8·610	8·636	8·661	8·687	8·712	8·737	8·763	8·788	8·814
29	8·839	8·864	8·890	8·915	8·941	8·966	8·991	9·017	9·042	9·068	9·093	9·118
30	9·144	9·169	9·195	9·220	9·246	9·271	9·297	9·322	9·348	9·373	9·399	9·424
31	9·449	9·474	9·500	9·525	9·551	9·576	9·602	9·627	9·653	9·678	9·704	9·729
39	11·887	11·912	11·938	11·963	11·989	12·014	12·040	12·065	12·091	12·116	12·142	12·167
40	12·192	12·217	12·243	12·268	12·294	12·319	12·345	12·370	12·396	12·421	12·447	12·472
50	15·240	15·265	15·291	15·316	15·342	15·367	15·393	15·418	15·444	15·469	15·495	15·520

FRACTIONS OF INCH					
1/16"	1·59 M/M	7/16"	11·11 M/M	13/16"	20·64 M/M
1/8"	3·18 "	1/2"	12·7 "	7/8"	22·23 "
3/16"	4·76 "	9/16"	14·29 "	15/16"	23·81 "
1/4"	6·35 "	5/8"	15·88	0·10"	2·54 "
5/16"	7·94 "	11/16"	17·46	0·20"	5·08 "
3/8"	9·53 "	3/4"	19·05	0·30"	7·62 "

TO CONVERT	MULTIPLY BY
METRES TO YARDS:	1·094
SQ. MTS. TO SQ. YDS:	1·196
CU. MTS. TO CU. YDS:	1·306
KGMS/CM^2 TO LBS/INS.^{2}:	14·25
KGMS/M^3 TO LBS/FT.^{3}:	·0128

EXAMPLES OF USE OF TABLE No. 38.

(*a*) To express 10 ft. $5\frac{5}{32}$ in. as a decimal of a foot:

10 ft.	=	10·00000
$5\frac{5}{32}$ in.	=	0·42969
		10·43 ft.

(*b*) To convert 0·6732 ft. to inches (nearest $\frac{1}{32}$nd):

0·6732 ft.		
0·67187 ft.	=	$8\frac{1}{16}$ in.

Difference = 0·00133 ft. which is slightly more than $\frac{1}{64}$ in. Therefore 0·6732 ft. = $8\frac{3}{32}$ in.

NOTES.

DECIMAL EQUIVALENTS

TABLE Nº 38.

INS.	0	1	2	3	4	5	6	7	8	9	10	11
	-	·08333	·16666	·25	·33333	·41666	·5	·58333	·66666	·75	·83333	·91666
1/32	·00260	·08594	·16927	·2526	·33594	·41927	·5026	·58594	·66927	·7526	·83594	·91927
1/16	·00521	·08854	·17187	·25521	·33854	·42187	·50521	·58854	·67187	·75521	·83854	·92187
3/32	·00781	·09114	·17448	·25781	·34114	·42448	·50781	·59114	·67448	·75781	·84114	·92448
1/8	·01041	·09374	·17707	·26041	·34374	·42707	·51041	·59374	·67707	·76041	·84374	·92707
5/32	·01302	·09635	·17969	·26302	·34635	·42969	·51302	·59635	·67969	·76302	·84635	·92969
3/16	·01562	·09895	·18228	·26562	·34895	·43228	·51562	·59895	·68228	·76562	·84895	·93228
7/32	·01823	·10156	·18489	·26823	·35156	·43489	·51823	·60156	·68489	·76823	·85156	·93489
1/4	·02083	·10416	·1875	·27083	·35416	·4375	·52083	·60416	·6875	·77083	·85416	·9375
9/32	·02344	·10677	·1901	·27344	·35677	·4401	·52344	·60677	·6901	·77344	·85677	·9401
5/16	·02604	·10937	·1927	·27604	·35937	·4427	·52604	·60937	·6927	·77604	·85937	·9427
11/32	·02864	·11198	·19531	·27864	·36198	·44531	·52864	·61198	·6951	·77864	·86198	·94531
3/8	·03125	·11458	·19791	·28125	·36458	·44791	·53125	·61458	·69791	·78125	·86458	·94791
13/32	·03385	·11718	·20052	·28385	·36719	·45052	·53385	·61719	·70052	·78385	·86719	·95052
7/16	·03646	·11979	·20312	·28646	·36979	·45312	·53646	·61979	·70312	·78646	·86979	·95312
15/32	·03906	·12239	·20573	·28906	·37239	·45573	·53906	·62239	·70573	·78906	·87239	·95573
1/2	·04166	·125	·20832	·29166	·375	·45833	·54166	·625	·70832	·79166	·875	·95833
17/32	·04427	·1276	·21094	·29427	·3776	·46094	·54427	·6276	·71094	79427	·8776	·96094
9/16	·04687	·1302	·21353	·29687	·3802	·46353	·54687	·6302	·71353	·79687	·8802	·96353
19/32	·04948	·13281	·21614	·29948	·38281	·46614	·54948	·63281	·71614	·79948	·88281	·96614
5/8	·05208	·13541	·21874	·30208	·38541	·46875	·55208	·63541	·71874	·80208	·88541	·96875
21/32	·05469	·13809	·22135	·30469	·38802	·47135	·55469	·63802	·72135	·80469	·88802	·97135
11/16	·05729	·14062	·22395	·30729	·39062	·47395	·55729	·64062	·72395	·80729	·89062	·97395
23/32	·05989	·14323	·22656	·30989	·39323	·47656	·55989	·64323	·72656	·80989	·89323	·97656
3/4	·0625	·14583	·22916	·3125	·39583	·47916	·5625	·64583	·72916	·8125	·89583	·97916
25/32	·0651	·14844	·23177	·3151	·39844	·48177	·565	·64844	·73177	·8151	·89844	·98177
13/16	·06771	·15104	·23437	·31771	·40104	·48437	·56771	·65104	·73437	·81771	·90104	·98437
27/32	·07031	·15364	·23698	·32031	·40364	·48698	·57031	·65394	·73698	·82031	·90364	·98698
7/8	·07292	·15625	·23958	·32292	·40625	·48958	·57292	·65625	·73958	·82292	·90625	·98958
29/32	·07552	·15885	·24219	·32552	·40885	·49219	·57552	·65885	·74219	·82552	·90885	·99219
15/16	·07813	·16146	·24479	·32813	·41146	·49479	·57813	·66146	·74479	·82813	·91146	·99479
31/32	·08073	·16406	·24739	·33073	·41406	·49739	·58073	·66406	·74739	·83073	·91406	·99739
	0	1	2	3	4	5	6	7	8	9	10	11

DIVISIONS OF A FOOT EXPRESSED IN FEET

EXAMPLE OF USE OF TABLE No. 39.

To find the moment of inertia of a tee-beam of 20 ft. span with a rib 8 in. wide and 16 in. deep below a 4-in. slab.

Thus $D_s = 4$ in.; $D = 4$ in. + 16 in. = 20 in. $\frac{D_s}{D} = \frac{4}{20} = 0{\cdot}20$.

From *Table* No. 32 the width of the flange B_s can be taken as the smaller of

(i) $12D_s + B = (12 \times 4) + 8 = 56$ in.

(ii) $\frac{20 \text{ ft.}}{3} = 6$ ft. 8 in.

Hence $B_s = 56$ in. and $B = 8$ in.; $\frac{B}{B_s} = \frac{8}{56} = 0{\cdot}14$.

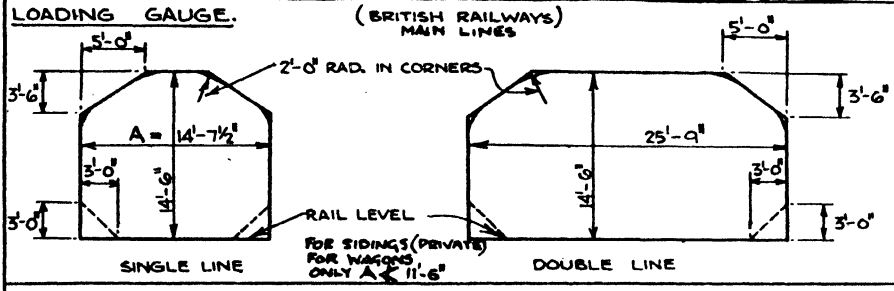
From *Table* No. 39, with $\frac{D_s}{D} = 0{\cdot}20$ and $\frac{B}{B_s} = 0{\cdot}14$, $C = 0{\cdot}175$.

Therefore moment of inertia $= 0{\cdot}175 \times 8 \times 20^3 = 11{,}200$ in.^{4}

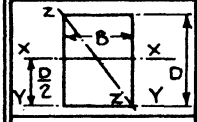
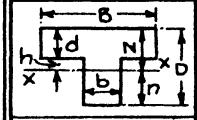
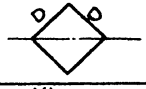
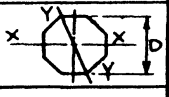
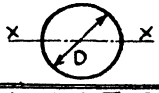
NOTES.

MISCELLANEOUS DATA. TABLE Nº39.

LOADING GAUGE. (BRITISH RAILWAYS) MAIN LINES



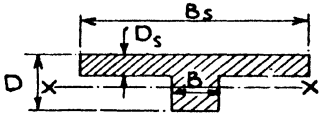
PROPERTIES OF SECTIONS.

SECTION	AREA	MODULUS	MOMENT OF INERTIA
	DB	XX. $\frac{BD^2}{6}$ YY. $\frac{BD^2}{3}$ ZZ. $\frac{B^2D^2}{6\sqrt{B^2+D^2}}$	XX. $\frac{BD^3}{12}$ YY. $\frac{BD^3}{3}$ ZZ. $\frac{B^3D^3}{6(B^2+D^2)}$
	$Bd + b(D-d)$	$\frac{M.\ of\ I}{n}$ $n = D - N$ $N = \frac{bD^2 + d^2(B-b)}{2[Bd + b(D-d)]}$	$\frac{1}{3}\left[BN^3 + bn^3 - h^3(B-b)\right]$
	D^2	$\cdot 118 D^3$	$\frac{D^4}{12}$
	$\cdot 828\ D^2$	XX. $\cdot 109\ D^3$ YY. $\cdot 1016\ D^3$	XX. $\cdot 055\ D^4$ YY. $\cdot 055\ D^4$
	$\cdot 7854\ D^2$	$\cdot 0982\ D^3$	$\cdot 0491\ D^4$

MOMENT OF INERTIA OF TEE SECTION

MOMENT OF INERTIA ABOUT AXIS X – X THROUGH CENTROID $= CBD^3$

VALUES OF C ARE TABULATED.



RATIO D_s/D	RATIO B/B_s																
	·12	·14	·16	·18	·20	·22	·24	·26	·28	·30	·35	·40	·45	·50	·60	·70	·80
0·10	·170	·161	·154	·147	·142	·137	·133	·129	·125	·122	·115	·110	·106	·102	·096	·092	·089
0·15	·180	·172	·165	·158	·152	·146	·143	·138	·135	·131	·124	·117	·111	·107	·100	·095	·090
0·20	·183	·175	·169	·163	·157	·152	·147	·143	·140	·137	·128	·121	·116	·111	·103	·097	·091
0·30.	·184	·176	·170	·164	·159	·155	·150	·146	·142	·139	·132	·125	·119	·114	·105	·099	·093
0·40	·190	·180	·173	·165	·160	·155	·151	·146	·142	·139	·132	·125	·119	·115	·106	·099	·093
0·45	·197	·185	·176	·169	·162	·156	·152	·147	·143	·140	·132	·125	·119	·115	·106	·099	·093
0·50	·208	·195	·183	·174	·167	·160	·155	·150	·146	·141	·133	·126	·120	·115	·106	·099	·093

ESSENTIAL TRIGONOMETRICAL FORMULÆ.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}.$$

Applicable to any triangle ABC in which $AB = c$; $BC = a$; $AC = b$;

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{Area} = \frac{bc\sin A}{2} = \frac{ac\sin B}{2} = \frac{ab\sin C}{2} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$.

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

NOTES

ADDITIONAL USEFUL FORMULÆ:—

$$\sin\phi + \sin\theta = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$
$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$
$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$
$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

ROOTS OF QUADRATICS:

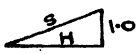
$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

APPLICATIONS:—

(a) ROOF SLOPES: $S = \sqrt{1 + H^2}$

LIMITING SLOPE FOR INCLINED ROOF LOADING (SEE TABLE No 2) = 20°

$H = \cot 20° = 2{\cdot}7475$



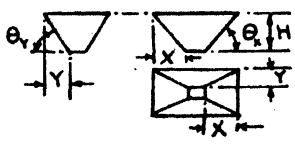
THEREFORE LIMITING SLOPE = 1 : 2¾

(b) EARTH PRESSURES: (SEE TABLE No 5)

$$k_2 = \frac{1 - \sin\theta}{1 + \sin\theta} = \tan^2\left(45° - \frac{\theta}{2}\right)$$

$$\frac{1}{k_2} = \frac{1 + \sin\theta}{1 - \sin\theta} = \tan^2\left(45° + \frac{\theta}{2}\right)$$

(c) HOPPER BOTTOM SLOPES:



SPECIFIED MINIMUM SLOPE IN VALLEY = ϕ

$$\frac{Y}{X} = R \qquad x = \frac{H\cot\phi}{\sqrt{1 + R^2}} \qquad Y = RX$$

$$\tan\theta_x = \tan\phi\sqrt{1 + R^2} \qquad \tan\theta_Y = R\tan\phi\sqrt{1 + R}$$

(SEE TABLES No 22 AND 43)

TRIGONOMETRICAL RATIOS. TABLE Nº 40.

ANGLE DEGREES	SINE	TANGENT	COTANGENT	COSINE	-
1	·0175	·0175	57·2900	·9998	89
2	·0349	·0349	28·6363	·9994	88
3	·0523	·0524	19·0811	·9986	87
4	·0698	·0699	14·3007	·9976	86
5	·0872	·0875	11·4301	·9962	85
6	·1045	·1051	9·5144	·9945	84
7	·1219	·1228	8·1443	·9925	83
8	·1392	·1405	7·1154	·9903	82
9	·1564	·1584	6·3138	·9877	81
10	·1736	·1763	5·6713	·9848	80
11	·1908	·1944	5·1446	·9816	79
12	·2079	·2126	4·7046	·9781	78
13	·2250	·2309	4·3315	·9744	77
14	·2419	·2493	4·0108	·9703	76
15	·2588	·2679	3·7321	·9659	75
16	·2756	·2867	3·4874	·9613	74
17	·2924	·3057	3·2709	·9563	73
18	·3090	·3249	3·0777	·9511	72
19	·3256	·3443	2·9042	·9455	71
20	·3420	·3640	2·7475	·9397	70
21	·3584	·3839	2·6051	·9336	69
22	·3746	·4040	2·4751	·9272	68
23	·3907	·4245	2·3559	·9205	67
24	·4067	·4452	2·2460	·9135	66
25	·4226	·4663	2·1445	·9063	65
26	·4384	·4877	2·0503	·8988	64
27	·4540	·5095	1·9626	·8910	63
28	·4695	·5317	1·8807	·8829	62
29	·4848	·5543	1·8040	·8746	61
30	·5000	·5774	1·7321	·8660	60
31	·5150	·6009	1·6643	·8572	59
32	·5299	·6249	1·6003	·8480	58
33	·5446	·6494	1·5399	·8387	57
34	·5592	·6745	1·4826	·8290	56
35	·5736	·7002	1·4281	·8192	55
36	·5878	·7265	1·3764	·8090	54
37	·6018	·7536	1·3270	·7986	53
38	·6157	·7813	1·2799	·7880	52
39	·6293	·8098	1·2349	·7771	51
40	·6428	·8391	1·1918	·7660	50
41	·6561	·8693	1·1504	·7547	49
42	·6691	·9004	1·1106	·7431	48
43	·6820	·9325	1·0724	·7314	47
44	·6947	·9657	1·0355	·7193	46
45	·7071	1·0000	1·0000	·7071	45
-	COSINE	COTANGENT	TANGENT	SINE	

BRIDGE DESIGN NOTES — TABLE No. 41

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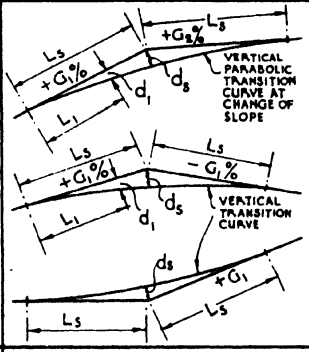
ROAD PROFILE (MAIN ROADS)



$S = G_1 - G_2$

$S = G_1 + G_2$

$S = G_1$

$L_s = 12{\cdot}5\,(S + 10)$ FEET.

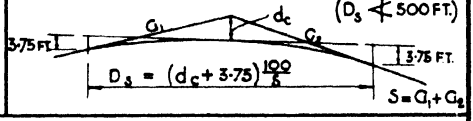
WHERE S = ALGEBRAIC DIFFERENCE OF SLOPES (EXPRESSED AS PERCENTAGE, E.G. SLOPE 1 : 20 $= \frac{100}{20} = 5\% = G$)

$d_s = \frac{SL_s}{400}$

AT ANY POINT: $d_1 = \left(\frac{L_1}{L_s}\right)^2 d$

MAX. GRADIENT FOR MAIN ROADS = 1 : 30 ; G = 3⅓%

SITE DISTANCE OVER CROWN OF BRIDGE (ROADWAY):- ($D_s \nless 500$ FT.)



HINGES AND BEARINGS

CONTACT CURVES.

$\frac{L}{W} = \frac{1}{K}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ $\quad \frac{L}{W} = \frac{1}{KR_1}$ $\quad \frac{L}{W} = \frac{1}{K}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

L = LENGTH OF BEARING (INS.)
K = 342 FOR 1:1½:3 CONCRETE
= 2,840 FOR CAST STEEL (PER W.L. SCOTT)
R = RAD. OF BEARING (IN.)
W = TOTAL LOAD ON BEARING (LB.)

PERMISSIBLE STRESSES ON STEEL BEARINGS.

PINS.	BENDING	= 13½ TON PER SQ. IN.
	BEARING	= 15 " " "
	SHEAR	= 6½ " " "

ROLLERS: D = DIA. OF ROLLER (IN). $\nless 3\frac{3}{4}''$
PRESSURE ON STEEL ROLLERS $\ngtr 0{\cdot}28D$ TON PER LIN. IN.

PERMISSIBLE PRESSURES UNDER BEDPLATES.

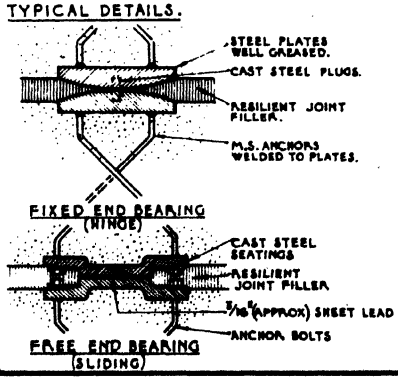
PLAIN CONCRETE	1:4 MIX	24 TON PER FT. SQ.	THESE PRESSURES ASSUME LOAD INCLUDES DEAD + LIVE + IMPACT
	1:6 "	20 " " " "	
REINFORCED CONCRETE	1:4 "	36 " " " "	INCREASE BY 20% IF WIND, CENTRIFUGAL FORCE, LONGITUDINAL FORCES, AND TEMPERATURE ARE INCLUDED IN LOAD.
	1:6 "	30 " " " "	

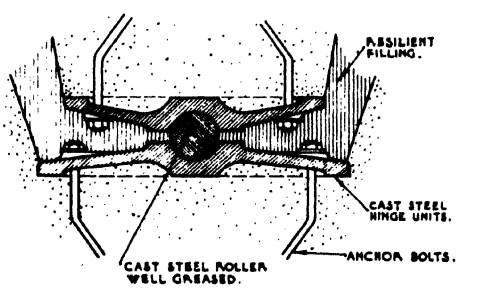
LONGITUDINAL FORCES DUE TO BEARINGS.

FRICTIONAL RESISTANCE OF ROLLERS 3 PER CENT.
" " SLIDING PLATES 25 " " (STEEL ON STEEL OR CAST IRON)

EXPANSION OR CONTRACTION DUE TO TEMPERATURE: ALLOWED FOR BY CONSIDERING FRICTION ON ONE EXPANSION BEARING UNDER DEAD LOAD AS ADDITION THRUST OR PULL IN BOOM OF GIRDER TO WHICH BEARING IS ATTACHED

TYPICAL DETAILS.





MECHANICAL HINGE

ADDITIONAL EXAMPLES.

The following examples have been compiled to indicate the use of a series of the foregoing tables in the design of a number of complete structural parts, together with examples of taking-off and approximately pricing bills of quantities.

(i) TYPICAL DESIGN OF FLOOR PANEL—BEAM AND SLAB CONSTRUCTION (page 303).
(ii) TYPICAL DESIGN OF FLOOR PANEL—FLAT SLAB CONSTRUCTION.
  (*a*) WITH DROPPED PANELS (page 306).
  (*b*) WITHOUT DROPPED PANELS (page 308).
(iii) TYPICAL DESIGN OF HOPPER BOTTOM (page 310).
(iv) TYPICAL EXAMPLE OF QUANTITIES AND COST ESTIMATING (page 313).
(v) TYPICAL DESIGN OF HOLLOW-TILE SLABS (page 315).

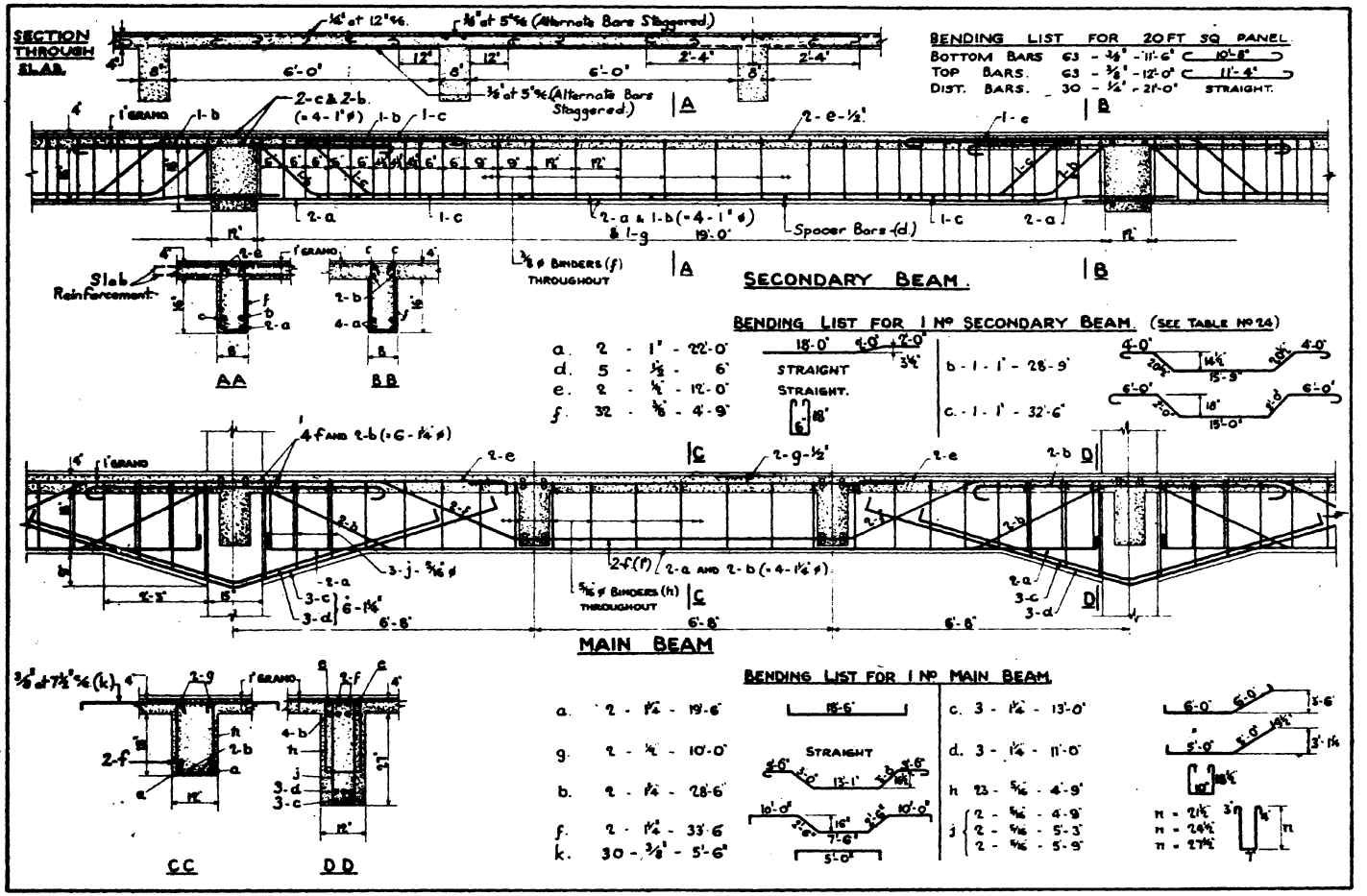


Fig. 50.—Typical Detail of Beam and Slab Construction.

(i) TYPICAL DESIGN OF FLOOR PANEL—BEAM AND SLAB CONSTRUCTION

(See *Fig.* 50 for details and general arrangement.)

Warehouse floor in panels 20 ft. square ; 1-in. granolithic finish not to be included in resistance section.

Adopt 1 : 2 : 4 ordinary concrete (see *Table* No. 23) and as working stresses :

Concrete in compression	= 750	lb. per sq. in.
Steel in tension	= 18,000	,, ,, ,,
Binders	= 18,000	,, ,, ,,
Bent-up bars	= 18,000	,, ,, ,,

With 18,000 and 750 (from *Table* No. 27 with stress ratio = 24), lever arm $= a_1 = 0{\cdot}87$; neutral axis depth ratio $= n_1 = 0{\cdot}39$, for $m = 15$.

Adopt live load = 224 lb. per sq. ft. ($\nless$ 200 lb. per *Table* No. 2).

SLAB.—Dead load : 1-in. granolithic (*Table* No. 1) = 12 lb. per sq. ft.

4-in slab = 4 × 12	= 48	,, ,, ,,
Total dead load	= 60	,, ,, ,,
Live load :	= 224	,, ,, ,,
Total load	= 284	,, ,, ,,

Ratio of live to dead load $= \frac{224}{60} = 3{\cdot}73$. Since the ratio of live to dead load exceeds 2, it will be necessary to provide top steel throughout ; hence adopt detail similar to that in *Fig.* 38(*b*). With B.M. coefficients given on *Table* No. 10 :

Midspan and support B.M. $= 0{\cdot}083 \times 284 \times 6{\cdot}67^2 = 1{,}045$ ft.-lb.
= 12,540 in.-lb.

Slab thickness, from *Table* No. 31, for 18,000 and 750, = 4 in.

Effective depth $= d = 3{\cdot}25$ in.

$$A_T = \frac{12{,}540}{18{,}000 \times 0{\cdot}87 \times 3{\cdot}25} = 0{\cdot}246 \text{ sq. in.}$$

Refer to *Table* No. 25 and provide $\frac{3}{8}$-in. bars at 5-in centres.

Distribution steel : 20 per cent. of 0·246 = 0·050 sq. in., say, $\frac{1}{4}$-in. at 12-in. centres.

Top bars over main beams $= \frac{0{\cdot}3}{100} \times 12 \times 4 = 0{\cdot}144$ sq. in., given by $\frac{3}{8}$-in. bars at $7\frac{1}{2}$-in. centres.

SECONDARY BEAMS.—Effective span = 20 ft. ; clear span = 19 ft. Assume net section = 16 in. by 8 in. ; $d = 18{\cdot}5$ in.

Dead load : Slab = 6·67 × 60	= 400	lb. per ft. run.
Beam rib, say	= 130	,, ,, ,,
Total dead load	= 530	,, ,, ,,
Live load : 6·67 × 224	= 1,500	,, ,, ,,
Total load	= 2,030	,, ,, ,,

Ratio of live to dead load is approximately 3. Referring to *Table* No. 10, the beam can either be designed for a B.M. coefficient of $\frac{1}{12}$ at midspan and support, or for 0·073 at midspan and 0·104 at supports ; since the secondary beams have main beams for their supports, adopt the former coefficients.

B.M. at midspan and support
$= 0{\cdot}083 \times 2{,}030 \times 20^2 \times 12 = 812{,}000$ in.-lb.

Maximum breadth of flange (*Table* No. 32):

(i) 6·67 ft. = 80 in.

(ii) $\frac{20}{3} \times 12$ = 80 in.

(iii) $(12 \times 4) + 8 = 56$ in.; therefore $b_t = 56$ in.

$n = 0{\cdot}39 \times 18{\cdot}5 = 7{\cdot}2$ in., and the neutral axis is below the slab; $a = 16{\cdot}5$ in.

R.M. of compression at midspan $= \dfrac{750 \times 56 \times 16{\cdot}5 \times 4}{2 \times 7{\cdot}2}(14{\cdot}4 - 4) = 2{,}003{,}000$ in.-lb., which is ample.

Midspan $A_T = \dfrac{812{,}000}{18{,}000 \times 16{\cdot}5} = 2{\cdot}73$ sq. in.

Four 1-in. bars = 3·14 sq. in.

At support: $A_T = \dfrac{812{,}000}{18{,}000 \times 0{\cdot}87 \times 17{\cdot}5} = 2{\cdot}98$ sq. in.

Four 1-in. bars = 3·14 sq. in.

R.M. of compression at support: From *Table* No. 29, R.M. of concrete only = 8 × 38,000, say, 304,000 in. lb. which is insufficient. With an increased compressive stress of 950 lb. per sq. in. at the support, and $A_C = A_T$, R.M. = 8 × 130,000 = 1,040,000 in. lb., which is satisfactory if $A_T = A_C = 8 \times 0{\cdot}45$ (say) = 3·6 sq. in. (3·14 sq. in. have been provided). Alternatively, by "steel-beam theory,"

$$A_T = A_C = \frac{812{,}000}{18{,}000 \times 15} = 3{\cdot}01 \text{ sq. in.}$$

Maximum shear force = 2,030 × 19 × 0·5 = 19,300 lb.

$$s = \frac{19{,}700}{8 \times 15} = 160 \text{ lb. per sq. in. Satisfactory.}$$

From *Table* No. 33, one 1-in. bar bent-up at 45 deg. at 18,000 lb. per sq. in. takes 20,000 lb. arranged in "double system." This is sufficient with nominal binding.

At end of second bent-up bar (3 ft. from support) shear force = 2,030 × 7 = 14,210 lb. and $s = \dfrac{14{,}210}{8 \times 16{\cdot}5} = 108$ lb. per sq. in.; therefore, from *Table* No. 33, if this shear is taken on binders, the required value of V is $\dfrac{14{,}210}{16{\cdot}5} = 860$. This is provided by $\frac{3}{8}$-in. single binders at $4\frac{1}{2}$-in. centres at 18,000 lb. per sq. in.

MAIN BEAMS.—Effective span = 20 ft.; clear span = 18 ft. 9 in. (15-in. column).

Assume section: 18 in. by 12 in. net.

Dead loads. Point loads = 2 × 530 × 19 = 20,200 lb.

Uniformly distributed: Beam = 216 lb. per ft. run.
Slab, etc. = 60 ,, ,, ,,

Total = $18\frac{3}{4} \times 276 = 5{,}180$ lb.

Live loads. Point loads: = 2 × 1,500 × 19 = 57,000 lb.
Uniformly distributed (= live load on beam width)
= $18\frac{3}{4} \times 1 \times 336 = 6{,}300$ lb.

Considering the span as equivalent to the centre span of a five-span beam system, the moment coefficients can be taken from *Table* No. 8:

Midspan B.M.			in.-lb.
Dead-load	point loads:	0·061 × 20,200 × 20 × 12 =	296,000
,,	distributed load:	0·046 × 5,180 × 20 × 12 =	57,000
Live-load	point loads:	0·115 × 57,000 × 20 × 12 =	1,570,000
,,	distributed load:	0·086 × 6,300 × 20 × 12 =	130,000
		Total =	2,053,000

Support B.M.: in.-lb.

Dead-load	point loads:	$0{\cdot}106 \times 20{,}200 \times 20 \times 12 =$	513,000
,,	distributed load:	$0{\cdot}080 \times 5{,}180 \times 20 \times 12 =$	100,000
Live-load	point loads:	$0{\cdot}148 \times 57{,}000 \times 20 \times 12 =$	2,022,000
,,	distributed load:	$0{\cdot}111 \times 6{,}300 \times 20 \times 12 =$	168,000
		Total =	2,803,000

Midspan section: Maximum breadth of flange $= (12 \times 4) + 12 = 60$ in.
$n = 0{\cdot}39 \times 19{\cdot}75 = 7{\cdot}7$ in.; $a = 17{\cdot}75$ in. approximately.
R.M. of compression.
Flange (excluding stem)

$$= \frac{750 \times 48 \times 17{\cdot}75 \times 4}{2 \times 7{\cdot}7}(15{\cdot}4 - 4) = 1{,}890{,}000 \text{ in.-lb.}$$

Stem alone $= 126 \times 12 \times 19{\cdot}75^2 = 590{,}000$,,

Total = 2,480,000 ,,

Thus the assumed section provides sufficient compressive resistance at midspan. Allow for the positive B.M. being increased by 15 per cent. of 2,803,000 = 422,000 + 2,053,000 = 2,475,000 in.-lb. R.M. is therefore sufficient at midspan.

Midspan: $A_T = \dfrac{2{,}475{,}000}{18{,}000 \times 17{\cdot}75} = 7{\cdot}43$ sq. in.

Six $1\frac{1}{4}$-in. bars (= 7·36 sq. in.) (see below).
Support Section. B.M. = 2,803,000 − 422,000 = 2,481,000 in.-lb. R.M. (per inch width) required $= \dfrac{2{,}481{,}000}{12} = 207{,}000$ in.-lb.

With c not exceeding 950 lb. per sq. in. and $A_T = A_C$, this is given by a section with $d = 22$ in. Hence a 9-in. haunch is provided, and $A_T = A_C$, $d = 27$ in.

$a_c = 0{\cdot}87 \times 27 = 23{\cdot}5$ in.; $a_s = 23{\cdot}5$ in.

$$A_T = \frac{2{,}481{,}000}{18{,}000 \times 23{\cdot}5} = 5{\cdot}66 \text{ sq. in.}$$

Provide six $1\frac{1}{4}$-in. bars in top and bottom. (Note that this gives excessive area in the top but less than the required area is provided at midspan.)

Maximum shear force = one-half the total load = 44,340 lb. Two $1\frac{1}{4}$-in. bars bent-up at 30 deg. (double system) at 18,000 lb. take

$2 \times 22{,}050 = 44{,}100$ lb.

(*Table* No. 33). This shear remains practically constant up to the point load; thus the same resistance is required throughout.

Approximate shear stress at end of haunch $= \dfrac{44{,}340}{12 \times 17{\cdot}75} = 208$ lb. per sq. in., which is allowable for main beams.

Maximum shear beyond point load $= \frac{1}{6}(5{,}180 + 6{,}300) = 1{,}900$ lb., which can be safely taken on the concrete at $\not> 75$ lb. per sq. in.

The anchorage lengths of, and types of, hooks for the various bars are calculated from the data on *Table* No. 24, and the points of bending-up or stopping-off are determined in accordance with *Table* No. 32. When the detailed arrangement of the bars has been finally determined, as in *Fig.* 50, the effective depths and lever arms of the various sections may be slightly different from the values assumed in the calculations, but these divergences would make no difference to the steel areas provided.

(ii) TYPICAL DESIGN OF FLOOR PANEL—FLAT SLAB CONSTRUCTION

(*a*) With Dropped Panels (*Fig.* 51).

Floor panel 20 ft. square, with finish, loading, stresses, etc., as in Example (i). See *Table* No. 42 for special formulæ, etc. Minimum size of dropped panel = 0·33 × 20 = 6 ft. 8 in.; maximum size = 0·5 × 20 = 10 ft.

Make drop panel 10 ft. square.

Minimum diameter of column head = 0·2 × 20 = 4 ft.; maximum diameter = 0·25 × 20 = 5 ft. Make column head 4 ft. 6 in. diameter.

$B = L = 20$ ft.

$S_B = S_L = 20 - (0{\cdot}67 \times 4{\cdot}5) = 17$ ft.

Loading:	Live	= 224 lb. per sq. ft.
	1-in. granolithic	= 12 ,, ,, ,,
	Dead (say)	= 114 ,, ,, ,,
	w	= 350 ,, ,, ,,

Total positive B.M. = 0·038 × 350 × 20 × 17^2 × 12 = 885,000 in.-lb.

(*a*) 42 per cent. to be provided in middle strip $= \dfrac{0{\cdot}42 \times 885{,}000}{10} = 37{,}200$ in.-lb. per ft.

(*b*) Remaining 58 per cent. to be provided in column strip

$$= \frac{0{\cdot}58 \times 885{,}000}{10} = 51{,}300 \text{ in.-lb. per ft.}$$

Total negative B.M. = 0·062 × 350 × 20 × 17^2 × 12 = 1,445,000 in.-lb.

(*c*) 26 per cent. to be provided in middle strip $= \dfrac{0{\cdot}26 \times 1{,}445{,}000}{10} = 37{,}400$ in.-lb. per ft.

(*d*) Remaining 74 per cent. to be provided in column strip

$$= \frac{0{\cdot}74 \times 1{,}445{,}000}{10} = 106{,}900 \text{ in.-lb. per ft.}$$

Moments (*a*), (*b*), and (*c*) will control the thickness of the slab between the dropped panels; from *Table* No. 31, effective depth required for 18,000 and 750 is about 6·25 in.

Slab 7½ in. thick with ⅝-in. bars gives d = 6·56 in. for bottom layer of steel and 5·94 in. for second layer (average = 6·25 in.).

Moment (*d*) will control the thickness of the dropped panel: from *Table* No. 31 an effective depth of about 8·6 in. is required.

Making the dropped panel 10 in. thick gives d = 9·06 in. for the top layer of bars and 8·44 in. for second layer of bars (with ⅝-in. bars).

The slab thicknesses provided give a dead load not exceeding that assumed.

Calculation of steel areas.

In middle strip:

in bottom at midspan and in top over column centre line, area

$$= \frac{37{,}500}{5{\cdot}94 \times 0{\cdot}87 \times 18{,}000} = 0{\cdot}404 \text{ sq. in.}$$ Say, ⅝-in. bars at 7½-in. centres.

In column strips:

in bottom at midspan, area $= \dfrac{51{,}300}{5{\cdot}94 \times 0{\cdot}87 \times 18{,}000} = 0{\cdot}552$ sq. in.

say, ⅝-in. bars at 6-in. centres;

in top of drop panel over column centre line, area

$$= \frac{106{,}900}{8{\cdot}44 \times 0{\cdot}87 \times 18{,}000} = 0{\cdot}806 \text{ sq. in.}$$

Say, ⅝-in. bars at 6-in. centres and ⅝-in. bars at 12-in. centres.

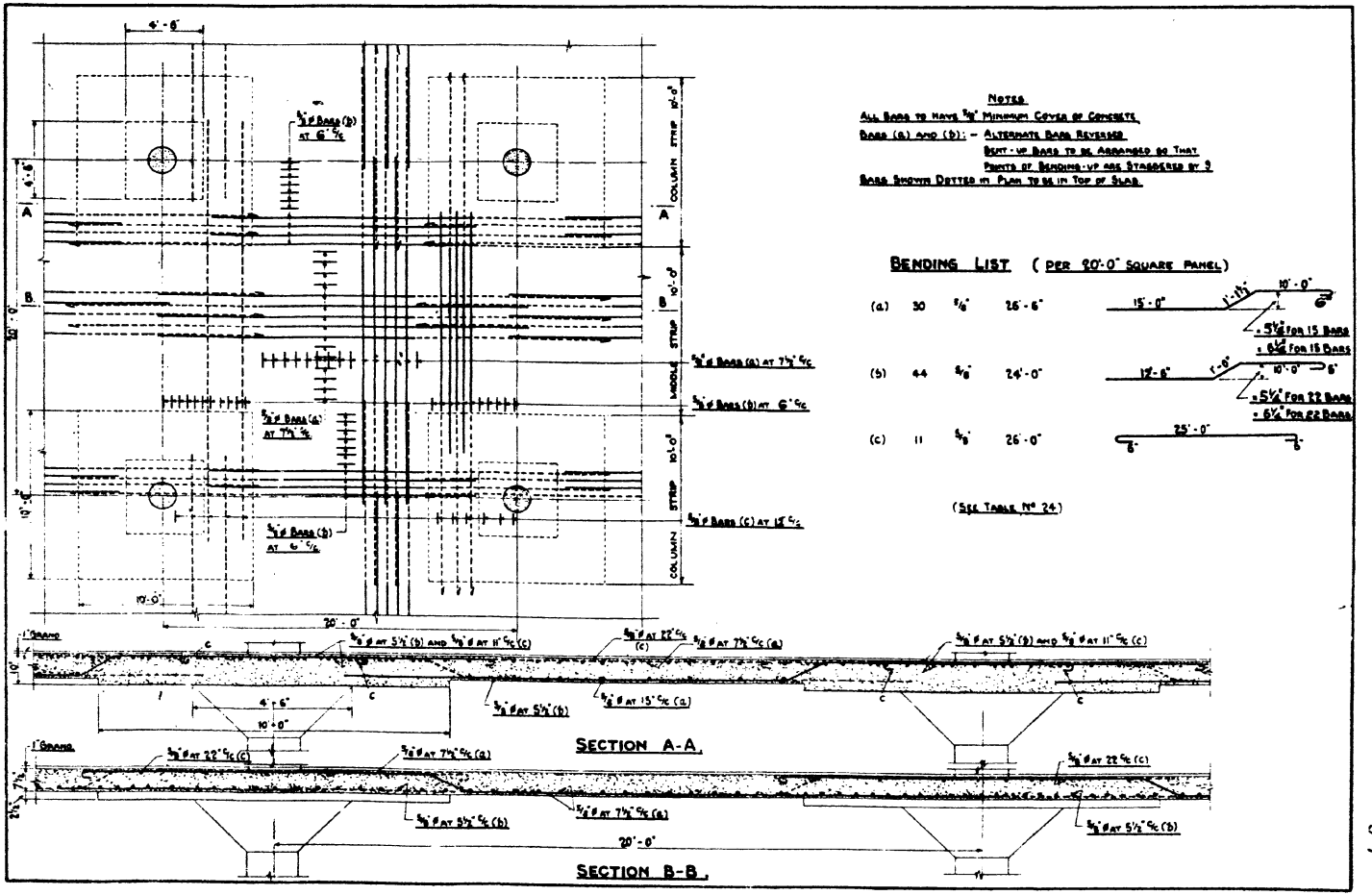


Fig. 51.—Typical Detail of Flat Slab Construction with Dropped Panels.

(*b*) Without Dropped Panels (*Fig.* 52).

Assume value of loading is 350 lb. per sq. ft. as in example with dropped panels. By calculations similar to previous example, the following moments are obtained :

Total positive B.M. = 933,000 in.-lb.
(*a*) 45 per cent. to be provided in middle strip = 42,000 in.-lb. per ft.
(*b*) 55 per cent. to be povided in column strip = 51,300 in.-lb. per ft.
Total negative B.M. = 1,400,000 in.-lb.
(*c*) 30 per cent. to be provided in middle strip = 42,000 in.-lb. per ft.
(*d*) 70 per cent. to be provided in each outer quarter = 98,000 in.-lb. per ft.
Moment (*d*) will control slab thickness : from *Table* No. 31, effective depth required is about 8 in.

A $9\frac{1}{2}$-in. slab with $\frac{1}{2}$-in. bars gives an effective depth of 8·75 in. for the first layer of bars and 8·25 in. for the second layer.
Calculation of steel areas :

In middle strip :

in bottom at midspan and in top over column centre line

$$\frac{42{,}000}{8{\cdot}25 \times 0{\cdot}87 \times 18{,}000} = 0{\cdot}325 \text{ sq. in.}$$

Use $\frac{1}{2}$-in. bars at 6-in. centres.

In column strips :

in bottom at midspan $\dfrac{51{,}300}{8{\cdot}25 \times 0{\cdot}87 \times 18{,}000} = 0{\cdot}400$ sq. in. Use $\frac{1}{2}$-in. bars at $4\frac{1}{2}$-in. centres (to suit negative moment steel),

in top over column centre line $\dfrac{98{,}000}{8{\cdot}25 \times 0{\cdot}87 \times 18{,}000} = 0{\cdot}760$ sq. in.

Use $\frac{1}{2}$-in. bars at $4\frac{1}{2}$-in. centres and $\frac{1}{2}$-in. bars at 9-in. centres.

FLAT SLABS. (BEAMLESS FLOORS) TABLE No. 42.

SEE ALSO CHAPTER IV

L & B = LENGTHS OF PANEL SIDES (FT.)
L $\not<$ B AND $\not>$ $1\frac{1}{3}$B.

SIZE OF DROPPED PANEL $\not< \frac{L}{3} \times \frac{B}{3}$ AND $\not> \frac{L}{2} \times \frac{B}{2}$

DIA. OF COLUMN HEAD = D $\not<$ 0·20L & $\not>$ 0·25L

EFFECTIVE SPANS : $S_L = L - 0{\cdot}67D$; $S_B = B - 0{\cdot}67D$

BENDING MOMENTS (FT.-LB.)	WITHOUT DROPS.	WITH DROPS.
TOTAL POSITIVE PARALLEL TO L :	$0{\cdot}04\,\omega L S_B^2$	$0{\cdot}038\,\omega L S_B^2$
DITTO DITTO B :	$0{\cdot}04\,\omega B S_L^2$	$0{\cdot}038\,\omega B S_L^2$
DITTO TO BE PROVIDED IN MIDDLE STRIP:-	45 PER CENT.	42 PER CENT.
TOTAL NEGATIVE PARALLEL TO L :	$0{\cdot}06\,\omega L S_B^2$	$0{\cdot}062\,\omega L S_B^2$
DITTO DITTO B :	$0{\cdot}06\,\omega B S_L^2$	$0{\cdot}062\,\omega B S_L^2$
DITTO TO BE PROVIDED IN MIDDLE STRIP:	30 PER CENT.	26 PER CENT.

END SPANS:- POS^{ve} B.MS TO BE INCREASED BY 25 PER CENT.

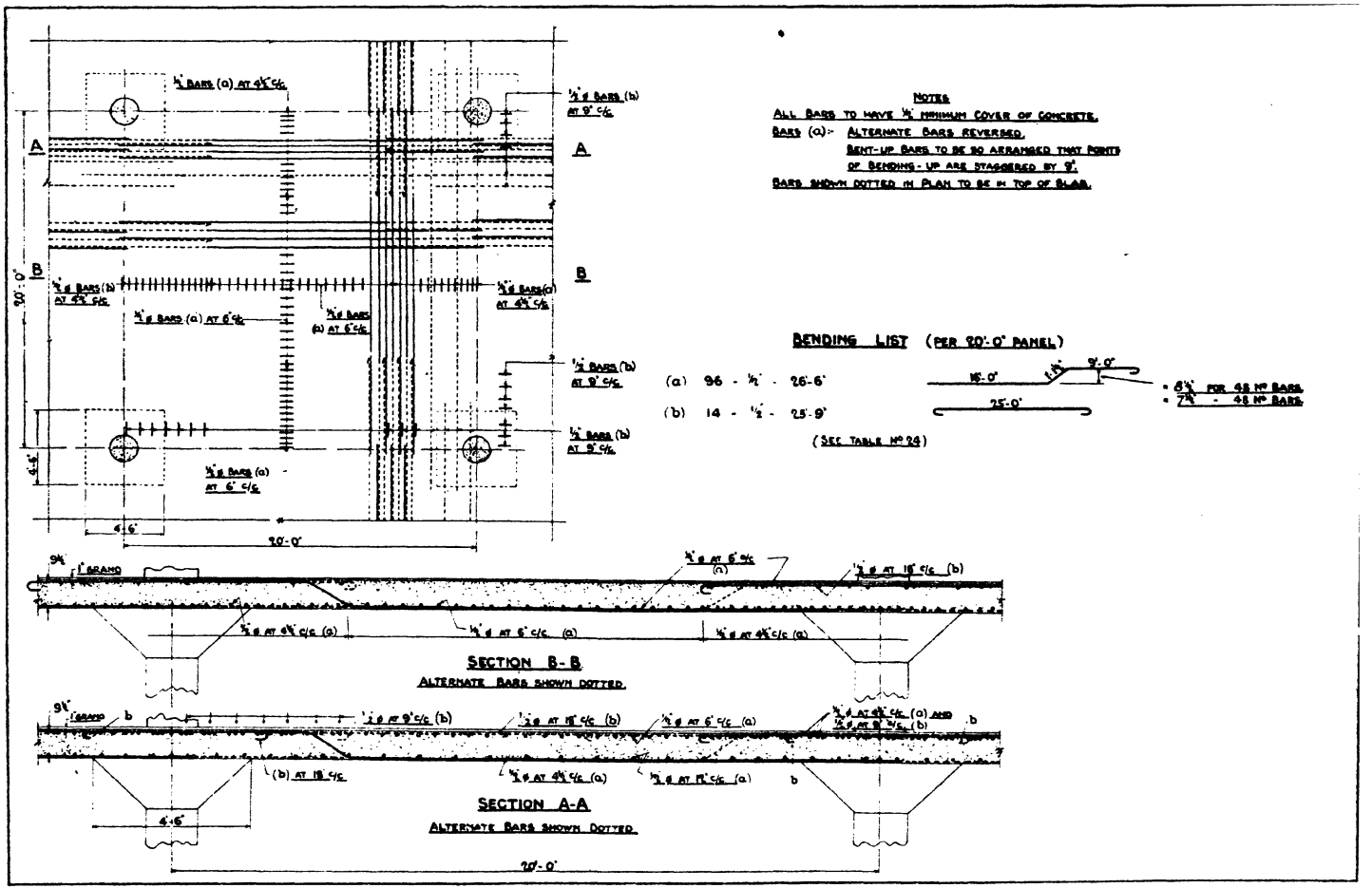


Fig. 52.—Typical Detail of Flat Slab Construction without Dropped Panels.

(iii) TYPICAL DESIGN OF HOPPER BOTTOM.

(See *Fig.* 53 for details and general arrangement. See *Table* No. 22 for formulæ.)

To design the hopper bottom of a coal bunker 15 ft. square, that is, $L = B = 15$ ft.; $a = b = 1$ ft. 6 in.; $H = 12$ ft.; $E = 7$ ft. 6 in.

Hence by calculation or scaling, $d = 10$ ft.

$$\text{and } \sin\theta = \frac{7{\cdot}5}{10{\cdot}0} = 0{\cdot}75$$

$$\cos\theta = \frac{6{\cdot}75}{10{\cdot}0} = 0{\cdot}675.$$

By drawing the inscribed circle on a normal plan of one side of the bottom:

$D = 8$ ft. and $D_1 = 9$ ft. 6 in.

$h = 15$ ft.; $S = 4$ ft. 6 in.; and $l = 9{\cdot}5$ ft.

Assuming a 5-in. slab, $w_s = 60$ lb. per sq. ft.

From *Table* No. 5, for coal, $w = 50$ lb. per cb. ft.; $k_2 = 0{\cdot}271$.

$$p_n = 50 \times 15(0{\cdot}271 \times 0{\cdot}75^2 + 0{\cdot}675^2) + (60 \times 0{\cdot}675)$$
$$= 495 \text{ lb. per sq. ft. normal to slab.}$$

w_1 = wt. of bottom below C = 8,500 lb. approximately.

w_2 = wt. of complete bottom = 22,500 lb. approximately.

$$W = 50[2{\cdot}5(1{\cdot}5^2 + 15^2 + \sqrt{15^2 \times 1{\cdot}5^2}) + (15^2 \times 12)] + 22{,}500$$
$$= 188{,}750 \text{ lb.}$$

$$W_1 = 50\left[\frac{4{\cdot}5}{3}(1{\cdot}5^2 + 9{\cdot}5^2 + \sqrt{1{\cdot}5^2 \times 9{\cdot}5^2}) + (9{\cdot}5^2 \times 15)\right] + 8{,}500$$
$$= 93{,}650 \text{ lb.}$$

At critical sections B.M. $= 0{\cdot}375 \times 495 \times 8^2 = 12{,}000$ in.-lb. Neglecting effect of direct tension, effective depth required $= 0{\cdot}0926\sqrt{\frac{12{,}000}{12}}$, say, 3 in. with stresses not exceeding 700 and 17,000 lb. per sq. in. Thus 5 in., as assumed, is ample, but it is not practicable to make the slab much thinner.

Horizontal reinforcement: $N = 0{\cdot}5 \times 495 \times 0{\cdot}75 \times 9{\cdot}5 = 1{,}760$ lb. per ft.

Adopting the method given in Chapter XIV for combined stresses,

$$e = \frac{12{,}000}{1{,}760} = 6{\cdot}8 \text{ in.}$$

$$e_s = 6{\cdot}8 - \frac{5}{2} + 0{\cdot}75 = 5{\cdot}05 \text{ in.}$$

$$B_S = 1{,}760 \times 5{\cdot}05 = 8{,}900 \text{ in.-lb.}$$

$$A_T = \frac{8{,}900}{0{\cdot}87 \times 4{\cdot}25 \times 17{,}000} = 0{\cdot}143 \text{ sq. in.}$$

$$A_{T'} = \frac{1{,}760}{17{,}000} = 0{\cdot}103 \quad ,,$$

$$0{\cdot}246 \quad ,,$$

Provide ½-in. bars at 9-in. centres.

Longitudinal reinforcement in bottom at centre of slope:

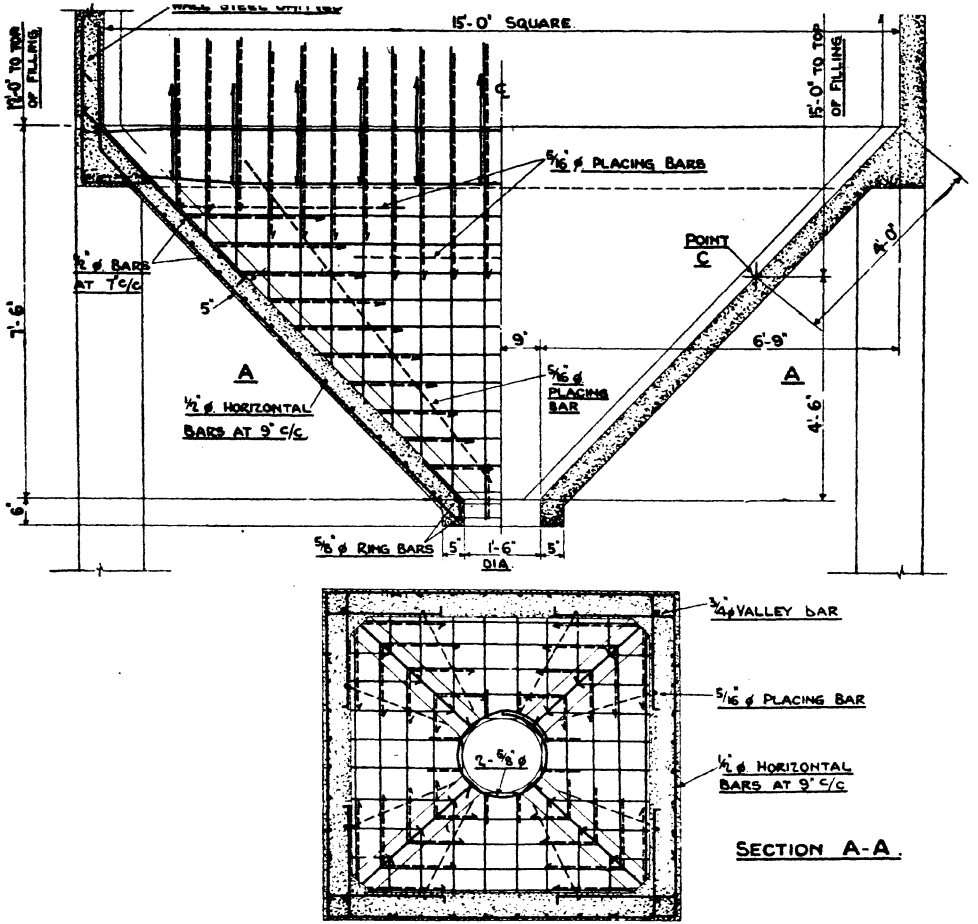
$$N = \frac{93{,}650}{2 \times 0{\cdot}75 \times 2 \times 9{\cdot}5} = 3{,}300 \text{ lb. per ft.}$$

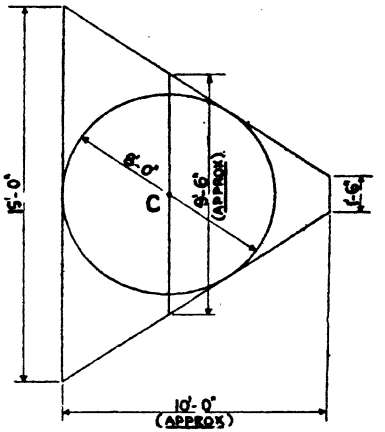
$$e = \frac{12{,}000}{3{,}300} = 3{\cdot}63 \text{ in.}$$

$$e_s = 3{\cdot}63 - 2{\cdot}5 + 0{\cdot}75 = 1{\cdot}88 \text{ in.}$$

$$A_T = \frac{3{,}300}{17{,}000}\left(\frac{1{\cdot}88}{0{\cdot}87 \times 4{\cdot}25} + 1\right) = 0{\cdot}293 \text{ sq. in.}$$

Provide ½-in. bars at 7-in. centres.





NORMAL PLAN OF SIDE OF HOPPER.
(SINCE BUNKER IS SQUARE ON PLAN AND OUTLET IS PLACED CENTRAL, DETAILS OF ALL SIDES OF HOPPER ARE IDENTICAL)

NOTE:- ALL BARS TO HAVE 1½" MINIMUM COVER OF CONCRETE.

Fig. 53.—Typical Detail of Hopper Bottom.

Longitudinal reinforcement in top of slab at top of slope.

$$N = \frac{188{,}750}{2 \times 0{\cdot}75 \times 2 \times 15} = 4{,}150 \text{ lb. per. ft.}$$

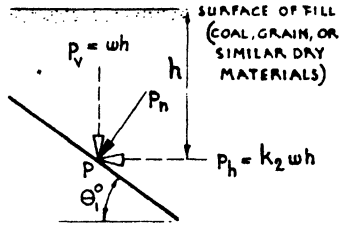
$$e = \frac{12{,}000}{4{,}150} = 2{\cdot}9 \text{ in.}$$

$$e_s = 2{\cdot}9 - 2{\cdot}5 + 0{\cdot}75 = 1{\cdot}15 \text{ in.}$$

$$A_s' = \frac{4{,}150}{17{,}000}\left(\frac{1{\cdot}15}{0{\cdot}87 \times 4{\cdot}25} + 1\right) = 0{\cdot}320 \text{ sq. in.}$$

Provide $\frac{1}{4}$-in. bars at 7-in. centres.

PRESSURES ON INCLINED SURFACES. TABLE Nº 43



FORMULA PER TABLE Nº 22.

$$P_n = wh\left(k_2 \sin^2\theta_1 + \cos^2\theta_1\right) + w_s \cos\theta_1$$

= NORMAL PRESSURE (LB/FT.^{2}) AT P.

w = WEIGHT OF FILLING (LB/FT.^{3})

h = DEPTH OF POINT P BELOW SURFACE

$k_2 = \dfrac{1 - \sin\theta}{1 + \sin\theta}$ (SEE TABLE Nº 5)

θ = ANGLE OF REPOSE OF FILLING

θ_1 = ANGLE OF SLOPE OF SURFACE

w_s = WEIGHT OF INCLINED SLAB (LB/FT.^{2})

VALUES OF $k_2 \sin^2\theta_1 + \cos^2\theta_1$

ANGLE OF SLOPE θ_1	ANGLE OF REPOSE = θ						
	30°	32·5°	35°	37·5°	40°	42·5°	45°
10°	0·98	0·98	0·98	0·98	0·98	0·98	0·98
20°	0·92	0·92	0·92	0·91	0·91	0·91	0·90
30°	0·83	0·83	0·82	0·81	0·81	0·80	0·79
35°	0·78	0·77	0·76	0·75	0·74	0·73	0·73
40°	0·72	0·71	0·70	0·69	0·68	0·67	0·66
45°	0·67	0·65	0·64	0·62	0·61	0·60	0·58
50°	0·61	0·59	0·57	0·55	0·54	0·52	0·51
55°	0·55	0·53	0·51	0·49	0·47	0·46	0·44
60°	0·50	0·48	0·45	0·43	0·42	0·40	0·38
65°	0·45	0·42	0·40	0·38	0·36	0·33	0·32
70°	0·41	0·38	0·36	0·33	0·31	0·29	0·26
80°	0·35	0·32	0·29	0·26	0·24	0·21	0·18

(iv) TYPICAL EXAMPLE OF QUANTITIES AND COST ESTIMATING.

Quantities are taken-off for one 20-ft. square panel of floor of beam and slab construction, as detailed in *Fig.* 50 (columns excluded).

	Concrete.		Shuttering.	
Slab	20′ 0″		20′ 0″	
	20′ 0″		20′ 0″	400
	4″	133·3		
Secondary Beams.	2/19′ 0″		2/2/19′ 0″	
	1′ 4″		1′ 4″	101
	8″	33·9		
	18′ 9″		2/18′ 9″	
	8″		1′ 4″	50
	1′ 4″	16·7		
Main Beams.	18′ 9″		2/18′ 9″	
	1′ 0″		1′ 6″	
	1′ 6″	28·1		
				56
	2/½/ 2′ 3″		2/2/½/ 2′ 3″	
	1′ 0″		6″	2
	9″	1·7		
	Total	213·7 cu. ft.		609
	say = 8·0 cu. yard.		Deduct 1′ 3″	
			1′ 3″	2
			Total	607 sq. ft.
			say = 68 sq. yd.	

1-in. granolithic:	20′ 0″	
	20′ 0″	400
Deduct	1′ 3″	
	1′ 3″	2
	398 sq. ft. = 45 sq. yd.	

Reinforcement (see descriptive bending list of details in *Fig.* 50.)

Diameter	¼ in.	5/16 in.	⅜ in.	½ in.	1 in.	1¼ in.
Slab (20 ft. sq.)	630		735			
			756			
Secondary beams (3 No.)			408	108	132	
				8	86	
					98	
Main beam (1 No.)		109	465	20		39
		9				59
		11				67
		12				39
						33
Total length:	630	141	2,354	136	316	237 ft.
Weight per ft.:	0·167	0·261	0·376	0·668	2·67	4·173 lb. (See *Table* No. 26)
Total weight (net)	105	37	885	91	845	990 lb.
Gross weight	1·0		8½	1·0	17 cwt.	

SCHEDULE OF QUANTITIES.

Beam and Slab Construction.

Item No.	Description.		Quantities.	Rate.	Total. £	s.	d.
1.	Mixing and placing concrete, 1 : 2 : 4 mix, in slabs and beams (rough screeding only).	cu. yd.	8·0	42*s.*	16	16	0
2.	Close-boarded shuttering to soffits of slabs, and sides and soffits of beams and haunches.	yd. sup.	68	4*s.* 9*d.*	16	3	0
3.	Provide and lay 1-in. granolithic surfacing.	yd. sup.	45	2*s.* 6*d.*	5	12	6
4.	Supply, cut if necessary, bend, and fix reinforcement in bars $\frac{5}{8}$ in. diameter and over, including tying wire.	cwt.	17	18*s.*	15	6	0
5.	Ditto $\frac{1}{2}$ in. diameter.	cwt.	1	19*s.*	0	19	0
6.	Ditto $\frac{3}{8}$ in. and $\frac{5}{16}$ in. diameter	cwt.	$8\frac{1}{2}$	20*s.*	8	10	0
7.	Ditto $\frac{1}{4}$ in. diameter	cwt.	1	22*s.*	1	2	0
	Total estimated cost per panel 20 ft. square.				£64	8	6

Inclusive estimated cost = 29*s.* per sq. yd.

These inclusive rates for both the concrete and the shuttering can be derived as follows :

Concrete :

		s.	*d.*
4-in. slab :	45 sq. yd. at 4*s.* 0*d.* =	180	0
Beams :	80 cb. ft. at 1*s.* 7*d.* =	126	8
		306	8

Average = $\frac{306s.\ 8d.}{8}$ = 38 *s.* 4 *d.* per cu. yd.

Add for hoisting, etc. = 3 *s.* 6 *d.* ,, ,,

Total 41 *s.* 10 *d.* ,, ,, say 42*s.* per cu. yd.

Shuttering :

	s.	*d.*
Slab soffit : 340 sq. ft. at 5*d.*	141	8
Beam sides and soffits : 260 sq. ft. at $6\frac{1}{2}$*d.*	140	10
Extra for haunches : 5 lin. ft. at 3*d.*	1	3
Forming chamfers : 160 lin. ft. at 3*d.*	40	0
	323	9

Average = $\frac{324}{68}$, say, 4*s.* 9*d.* per sq. yd.

For flat slab construction as illustrated in *Figs.* 51 and 52, some of the price rates would differ from those adopted for beam and slab construction, the modifications being due to the following considerations :

Concrete : rate reduced as placing in slabs only (without screeding) is cheaper than in beams.

Shuttering : rate reduced as there is less labour in making flat soffits than in forming beams and haunches ; rate for flat slab without dropped panels is less than rate with dropped panels.

(v) TYPICAL EXAMPLE OF HOLLOW TILE SLAB DESIGN.

(See also Chapter VIII.)

There are two methods of designing hollow tile slabs:

(*a*) Designing for a midspan bending moment of $\frac{wL^2}{9}$ and for only a nominal bending moment over the supports. Since at the supports, the compressive resistance is only that provided by the narrow ribs, the resistance moment at end sections is small if the tiles are continued as close as possible up to the support.

(*b*) Design for a bending moment of $\frac{wL^2}{12}$ (when fully continuous, or $\frac{wL^2}{10}$ when continuous at one end only) both at midspan and support, and provide a sufficient extent of solid slab to enable the compressive resistance near the supports to be obtained.

Design (*a*), which is followed in the example given below, gives a deeper slab, but may lead to a lighter floor since there will be a considerably greater area covered by tiles. The width of the rib is controlled largely by shear and the ability to accommodate the reinforcement. The depth of the rib and thickness of the slab are determined by the compressive moment of resistance required at midspan.

Example: To design a hollow-tile slab to span over a number of consecutive 12-ft. spans; assume office floor loads. According to the London Building By-laws, the ribs must be designed for the superimposed loads specified for slabs if the spacing apart does not exceed 2 ft. 6 in. (or 3 ft. in accordance with the Code of Practice). Assuming a 6-in. hollow-tile slab, the loading will be:

Superimposed load	= 80 lb. per sq. ft.	(*Table* No. 2)
6-in. hollow-tile slab	= 48 ,, ,, ,,	(*Table* No. 1)
Floor finish, say	= 10 ,, ,, ,,	
Ceiling finish, say,	= 6 ,, ,, ,,	
Partitions (minimum)	= 20 ,, ,, ,,	
Total	= 164 ,, ,, ,,	

With ribs 4 in. wide and tiles 12 in. wide (see *Fig.* 54), the distance between centres of ribs = 1 ft. 4 in. Hence load per rib = 1·33 × 164 = 219 lb. per ft. run.

$$\text{Midspan bending moment} = \frac{219 \times 12^2 \times 12}{9} = 42{,}000 \text{ in.-lb.}$$

Shear force on rib (at 12 in. from centre of support) = 5 ft. × 219 = 1,095 lb.

With 6-in. slab as in *Fig.* 52, effective depth = 4·69 in., lever arm = $4{\cdot}69 - \frac{2}{2}$ = 3·69 in.

With stresses of 18,000 and 750, neutral axis depth = 0·39 × 4·69 = 1·83 in.

$$\text{Shear stress on rib} = \frac{1{,}095}{4 \times 3{\cdot}69} = 74{\cdot}2 \text{ lb. per sq. in.}$$ (should not exceed 75 lb. per sq. in. without shear reinforcement).

$$A_T = \frac{42{,}000}{18{,}000 \times 3{\cdot}69} = 0{\cdot}63 \text{ sq. in.}$$ Provide two $\frac{5}{8}$-in. bars.

Check compression: If the depth of the neutral axis exceeds the thickness of the top slab (d_s), the minimum width of flange required is given by the expression

$$\frac{\text{B.M.}}{\frac{c}{2}\left(1 + \frac{n - d_s}{n}\right)d_s a}$$

In the present example, n = 1·83 in., which is less than the slab thickness of 2 in.; the moment of resistance of the compression of concrete (Q = 126 from *Table* No. 27) = 126 × 16 × $4{\cdot}69^2$ = 44,300 in.-lb., which is ample for the bending moment.

There is not usually any need to provide binders in the ribs, but a light mesh is desirable in the top slab. The arrangement of the reinforcement is shown in *Fig.* 52. One of the bars in the rib is bent up at each support to provide conveniently a nominal amount of steel over the supports.

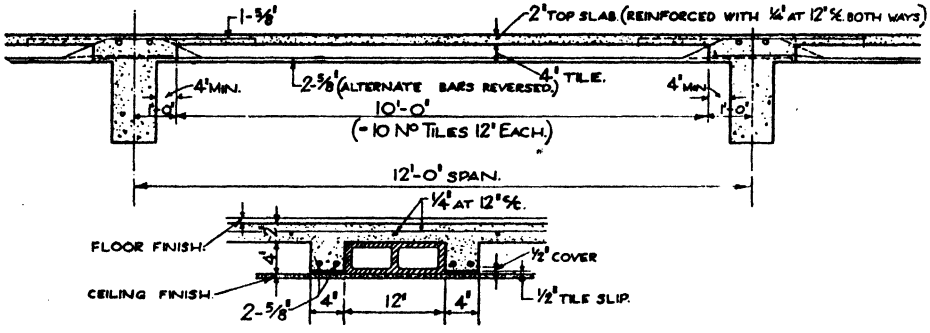


Fig. 54.—Detail of Hollow-Tile Slab.

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